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Modelling and control of multi-stage production-inventory systems.

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MODELLING AND CONTROL OF MULTI - STAGE

PRODUCTION - INVENTORY SYSTEMS.

Submitted by

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Dedication:

This thesis is dedicated to :

My Parents	who have taught me hard work and perseverance.
L.S.,Y.Y.,C.S.	for constant moral support.
E.C.	who supported me in many a ways too.

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Summary:

The research relates to the modelling and control of multi-stage production - inventory systems in high volume low-mix manufacturing industry. Examples of such class include typically the automotive and white goods industries.

The research has included the investigation of available mathematical control techniques in the "optimal" control of manufacturing systems, a study of their applicability and the practical implications of their use in a manufacturing environment.

Earlier work in the field of multivariable control theory has shown the potential of application in industrial management. In this thesis, previous work is extended whereby explicit consideration is given to some practical constraints existing in a typical manufacturing environment.

It is considered that the research carried out has contributed to the development of multivariable control theory as applied to practical control problems with constraints. This has been achieved by the use of structured canonical forms and the exploitation of their particular ordered properties, resulting in the development of practical control models.

The automotive industry has been used as a practical case study and modelled as a linear discrete-time control problem. The models have been developed in close liaison with a car manufacturing company in the U.K., and have been shown to produce practical control policies in the areas of both capacity requirements planning and inter-stage float levels. Particular attention is given to existing practical constraints of such systems.

The approach is extended to deal effectively with a more general

multi-product environment. It is noted that multi-product environment is of a more complex nature than single product since it involves the consideration of competition for the limited resources that have to be shared out "sub-optimally", in addition to providing smooth control of the responses.

The development and application of multivariable control theory as described in this thesis is shown to provide an effective methodology for the solution of dynamic production control problems of multi-stage production-inventory systems in both single and multi-product environments.

PART I

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PART I

PART I.

In this section of the thesis, the various facets of production control in Multi-Stage Production-Inventory systems are introduced. The modelling and control of such production-inventory systems is developed from an information feedback control point of view. It is shown how manufacturing information systems described in Chapter 1 complement the state variable feedback control theory in Chapter 2. Special emphasis is given to the practical implications of such control techniques in manufacturing systems. In particular, it is demonstrated how practical constraints such as production capacity and inter-stage buffers may be taken into consideration explicitly in the search of efficient control policies.

CHAPTER 1

1.1 INTRODUCTION.

Discrete manufacturing systems are characterised by both variety and complexity, and this fact excludes, at the outset, a single concept of manufacturing systems. Nevertheless an examination of the fundamental properties and characteristics of a whole range of systems reveals some general control features typical to a class of manufacturing. This thesis describes the research carried out on the control of a particular class of manufacturing, the Multi-Stage Production-Inventory system, where one of the characteristics is the presence of inter-stage buffers of assemblies at various stages in the production process. Examples of such class of manufacture are found in the automobile industries, consumer electrical goods, semiconductors and other high volume production based on discrete flow line process. Examples of classes not included in such a group are continuous process manufacturing(e.g. chemical plants) or job-shop manufacturing.

Decision making for control purposes has usually used the concept of information feedback, a technology borrowed from mathematical control theory. A major objective of the research has been therefore to investigate available mathematical control theory in the field of "optimal" control of manufacturing operations. In formalising the process of decision making with mathematical control theory, special attention has been given to the practical constraints and implications in a manufacturing environment. Such an approach not only highlights some of the pertinent features of manufacturing, resulting in a better insight of the control problem, but also contributes to a better framework for dynamic control of production - inventory systems.

An extensive literature exists in the area of computer - based production systems, production management, operational research techniques, control theory and other techniques in the analysis and control of manufacturing systems. A study of this literature reveals the numerous problem areas of manufacturing sytems and how these systems have been dealt with by previous workers. To a large extent, the work carried out in the area of production and inventory control has been concentrated on specific and narrow problems of manufacturing e.g. job - shop scheduling, inventory control. Less attempt has been made to study manufacturing as an integration of interacting systems. Furthermore, the problem of controlling and co-ordinating the various sub-systems on a dynamic basis has been given even less attention. Generally speaking, a voluminous literature abounds in the field of industrial management as a result of the development of management science and operational research. Whilst some studies are deterministic and goal optimising such as critical path analysis, mathematical programming as applied in scheduling techniques and inventory control, others have adopted an investigative - type of approach based on simulation studies. Unfortunately, actual industrial practice has not witnessed, to any significant extent, implementation of such techniques. This is especially so in the control problem of Multi - Stage Production Inventory (MSPI) systems. The main reasons are believed to be :

- Lack of skill in the problem formulation.
- Lack of comprehension of the techniques.
- Lack of appropriate data.
- Fear of delegation of power.

Some current trends indicate a promising change in this state of affairs and among these are:

- Cheaper and more powerful computing power with easier information processing and communication. This has led to the development of "manufacturing information systems", some with real time properties.
- An increase in "scientific" consciousness within the management of manufacturing as a result of complexity and competition.
- Introduction of more sophisticated and capital intensive equipment such as N.C. machine tools, and robotics leading to a necessity for optimising such resources.

The above factors have provided a favourable environment for the present research into the practicality and extended applications previous theoretical analyses.

1.2 Research Programme.

In carrying out the research, an initial requirement was the identification of the various characteristics of the class of manufacture to be studied. An explicit attempt has been made to investigate and rationalise the control features pertinent to Multi-Stage Production-Inventory (MSPI) systems. Obviously this exercise has to be carried out with an appreciation of the design features necessary for the associated with a control information system. This joint approach has contributed significantly to the establishment of a model that can be used as a basis for a more analytical approach as mathematical control theory. Mathematical control theory has shown the potential for ease of formulation and analysis of fairly complex manufacturing systems as typified by Multi-Stage Production-Inventory (MSPI) family. This has been demonstrated by Christensen and Brogan, (1971,/1/), where a discrete manufacturing system was formulated as a linear discrete - time model. A more recent practical study was carried out by Drew, (1975,/2/), using hierarchical control theory. Porter et al (1976,/3/) have also introduced a particular approach which is of substantial relevance in the work described in this thesis.

Since these new modelling techniqes require reliable and up-to-date information on the various states of production processes on a dynamic basis, progress in the application of such techniques has not been fast due to the lack of necessary information infrastructure. Moreover the recent advent of computer-based manufacturing information systems have highlighted the fact that many studies based on control theory, or a systems engineering approach, have not been able to take into consideration the various practical constraints existing in a manufacturing environment. A more detailed analysis of the work in this area is given subsequently. The initial stage of the research has been to identify the various potential areas of multivariable control theory in the control of MSPI systems. In so doing, various aspects and features of multivariable control theory have been analysed and extended so as to accommodate the practical realities of manufacturing systems. This exercise, of course has had to be carried out with a close working relationship with industry, in order to be of full practical relevance. Here, the cooperation of a few major manufacturing companies in the U.K. is gratefully acknowledged.

1.3 Design of a Control Framework.

In this section, a systematic approach is proposed in the design of information systems and control models for discrete manufacturing. This framework has been adopted as the base upon which the research has been subsequently developed. In presenting this framework, it is noted that to a large extent, various state-of-the-art developments in areas related to production control are synthesised together.

1.3.1 Concept of Manufacturing System.

At a global level, it is possible to visualise the manufacturing system as an input - output conversion system. "Input variables" from the environment are converted with added value by some technological process into "output variables". This concept is illustrated in Figure 1.1. A feedback control loop is included, so as to adjust when necessary the inputs, when random fluctuations disturb the input and the production process.

Different subsystems interact within the manufacturing system, some of these are:

- Sales / marketing.
- Manufacturing process.
- Planning and control.
- Design and research.
- Finance and accounting.

This interaction between the sub-systems is illustrated in Figure 1.2 which shows the information network that links the overall system. Such flow of informatio n is the basis for decision making in the goal of meeting the multitude of objectives of the firm in the face of the continuously changing conditions and constraints. The design and control of an information system for control purposes has therefore an important role in order that the manufacturing system shall achieve its objectives. The identification of the control features for such an information system can also contribute to the establishment of the framework necessary for future modelling and control purposes.

1.3.2 Information Systems in Manufacturing Industries.

It may be appropriate initially to identify the objectives of the manufacturing organisation and the parameters representative of performance so as to highlight the aspects of manufacturing on which information is required for control purposes. These include:

- Profitability.
- Reliable products / services.
- Economic manufacturing costs.
- "Optimisation" of manpower, machines, and finance.
- Maximum utilisation of resources.
- Short throughput time.
- Reliable delivery dates.
- Minimum inventory costs.
- Low WIP.
- Flexibility for change.
- Control of workflow.
- Efficient maintenance management.

This list is not exhaustive, nor in any priority order since this

may vary with time from company to company, and even from department to department. It is to be appreciated that while some of these are complementary, not all can be optimised or satisfied simultaneously. Therefore a trade - off has to be achieved on a dynamic basis according to prevailing circumstances and priorities.
Information systems in industry have traditionally ranged from a formal and highly structured form with periodic reports to informal word-of-mouth communication. The formal system is usually based on periodic reporting and some part of it may prove useful for planning purposes such as sales/marketing, cost accounting, financial analysis, etc. Nevertheless a substantial amount is usually too out of date to provide timely decisions for control. This is especially the case for manufacturing control. Therefore an informal parallel system has come into being which, because it is uncontrolled and "fire - fighting" in nature, may not allow the long term objectives to be fully appreciated, let alone fulfilled.

Since the late 1960's, there has been a drastic reduction in the cost of computing power. This has led to the development of a multitude of computer - based information systems, some of which are specifically designed for manufacturing industries. These are usually referred as "Production Control Packages". I. St Hugo (1979,/5/) gives a directory of suppliers of such packages in the international market, while Green and Hall (1977,/4/) discuss the various guidelines in the implementation of these packages. These consist of fixed and dynamic files, some of which are shown in Figure 1.3. Kochhar (1979,/6/) gives a very good treatise on the development of such files and also provides an extensive bibliography on this subject. The relative importance of such files will depend upon the type of the manufacture under consideration, e.g. whether job - shop, batch manufacture or flow line production. Some of the most relevant techniques that have evolved from the implementation of such systems are BOMP (Bills Of Materials Processing), MRP (Materials Requirement Planning), Capacity Planning, Master Production Scheduling. These are dealt with extensively in Kochhar (1979,/6/). Another important reference is Orlicky (1975,/7/) with particular emphasis on MRP. These new systems may be implemented from a periodic reporting and updating basis to on-line real-time monitoring. The timeliness required in the decision making will also determine the type of information system.

The design of a control system and the information system required to support it may vary in detail for different manufacturing systems. The introduction of computer-based information systems has no doubt led to a substantially improved control, nevertheless the situation is still, to a large extent, one of trial and error. Industrial experience with these packages has produced varied results as witnessed by Philips (1981,/8/), Prod. Eng.(1979,/9/), Donelson (1979,/10/), Burbidge (1980,/11/). It is believed that a structured concept is both essential and applicable in designing information systems and modelling manufacturing systems. Whilst the sub-modules of adopted in the present approach are not totally novel, together they provide a synthetic framework integrating the results of the present author and that of other workers.

1.3.3 Characteristics Of A Control System For Discrete Manufacturing.

The approach adopted in the present research is now described and it is shown how it can contribute to the development of a mathematical model with the use of control theory. Throughout the description, wherever relevant, the "state of the art" situations in the particular fields are brought out.

The proposed steps are:

a, Identification of subsystems and levels of control.

b, Input - output analysis of each subsystem.

c, Steady and dynamic analysis of each subsystem.

d, Collection of data and computer architecture.

e, Control of Manufacturing:

i) Integration and coordination of control.

ii) Individual control of subsystem..

The approach attempts to tackle the production control problem objectively and provide constructive guidelines. Local conditions and constraints from organisational structure have to be fully appreciated in the local context. Fitting the production systems to the task is most important, Miller (1981,/12/). Appleton (1979,/13/), Tricker (1969,/14/) give a good indication as how to look at manufacturing control requirements in their wider context before tackling the problem of production control in particular.

1.3.3.a Identification of Subsystems and levels of control.

Some of the major subsystems in a production system may be categorised in the following hierarchy:-

- Strategic level.

- Managerial level and production control.
- Process control.
- Materials flow control.

This hierarchical structure was first characterised by Hammond and Oh in (1973,/15/), and has since been widely adopted by subsequent workers (/16/ - /18/). This decomposition procedure reduces large complex problems into smaller ones with relatively easier solution procedures. This inter-relationship of the subsystems at the various levels is illustrated in figure 1.4.

- At the strategic level, the decisions are heuristic in nature and more geared to a long term horizon. The macro-economic environment needs to be taken into full account in the formulation of manufacturing strategies and policies.
- Within the guidelines from the above level, decision making at the managerial level is undertaken in order to control the overall response of the production system with respect to both external and internal disturbances. This involves the selection of control policies so as to match the disturbances. The results of such decisions include new or revised work plans, reallocation of resources, requirements planning, and acquisition of additional resources.
- Process control ensures that operations plans, requisition schedules and control measures are actually carried out. Periodic reporting updates the upper control levels.

1.3.3.b Input and output analysis.

Each subsystem has input variables and output variables in the form of information and/or physical material. Thus a control department will have input variables files of customer orders, quantities and delivery dates. Exploding them into the required parts, the planner/controller works out the required capacities, material and parts against the current committed ones. Output variables include work schedules, material requisitions, parts kitting and purchasing requirements. This is illustrated in Figure 1.5 where a production line is considered as a subsystem, the inputs are work schedules, raw material and manpower which are converted into goods.This input - output analysis has been dealt with in detail in Parnaby, (1979,/18/) and Buffa, (1966,/19/).

1.3.3.c Steady state and dynamic analysis.

At the steady state level, the analysis is carried out on an average basis, i.e. average orders, average production capacity, average production rate, average scrap rate, and average average work flow. The dynamic analysis considers the actual fluctuations and disturbances. Among these are sales fluctuations, varying performance rates of operators and machines, varying work flow, machine breakdown and other environmental changes. Control measures are therefore required locally for the subsystems, with reordering of work schedules, changing priorities and redeployment of resources. Such a state of affairs is quite common in actual manufacturing.

1.3.3.d Collection of Data and Consideration of Computer Hardware.

Once the most relevant input - output variables have been identified at their respective requirement levels, measures can be taken in the methods of obtaining them. This, of course, depends on the subsystems and the nature of the decision making involved. Some of the typical variations that need continuous or intermittent monitoring for production control purposes are:

- Level of WIP.
- Status of machine.
- Queue lengths.
- Stock levels.
- Scrap or reject rate.
- Utilisation rate.

The more recent method of data collection is through the use of digital keyboards or special punched cards. Shopfloor data collection terminals are presently available in a ruggedised form to suit the production environment.

A survey of computer systems carried out by Aronson in the U.S. in (1973,/20/) in the area of discrete manufacturing identified three main types:

- Centralised processing.
- Distributed processing.
- Hierarchical, multilevel with satellite computers.

While Kochhar, (1977,/21/) reported on a distributed network, other references tend to favour a multilevel hierarchical structure. In actual practice, there is still a large proportion of "centralised" computing in medium size companies in U.K. The increasing power of computers to support numerous terminals simultaneously on a real time basis, has permitted a hierarchical and "distributed" structure as far as the different users are concerned. Different users have access to different predetermined amounts of information and processing according to their needs. On the other hand, in a very large manufacturing organisation as automotive industries, where the manufacture of one assembly itself involves labour and equipment equivalent to a medium size company, the advantage of hierarchical approach with satellite computers are obvious. References /16/-/17/ describe some of such computer monitoring systems in automobile and computer industries in the U.S. Reference 9 relates to similar applications in batch manufacturing in the U.K. , Crumpton and Yeoh (1980,/22/) and Yeoh (1981,/23/) describe a practical case in automobile manufacture in the U.K.

1.3.3.e Control of the Manufacturing System.

When the performance departs from the expected average values in the dynamic state, control is necessary to bring the (sub)system into the desired operating state. Such control may mean increasing, decreasing, or reallocation of manufacturing capacities e.g.:

- introduction of appropriate overtime.
- subcontracting.
- reshuffling of job priorities.
- splitting work batches.
- checking suppliers.
- new maintenance schedules, etc.

Currently various commercial application packages exist for production monitoring systems: these may possibly provide accurate and timely information on the static and dynamic states of the manufacturing processes for planning and control. However due to the complexity of the numerous variables involved, it will be very difficult for the production controller to devise control measures that are the most cost - efficient and practical. Formulation of appropriate control policies do indeed create the most challenging areas of production control. Numerous workers have studied and proposed various production management techniques but less effort has been devoted to the control aspects, compared with the planning aspects of production systems. In an actual environment, the dynamic of the system will call more for control or replanning than "green-fields" planning. This control decision is made more difficult still by the fact that sub-systems are very much inter-related with each other, changes in one may affect others in the same or different control levels. Controlling one sub-system may well be at the expense of another. The problem is therefore two-fold:

(i) Integrating the control problem for the sub-systems.

(ii) Controlling individual systems.

Since the early 1970's, the concept of hierarchical control within an integrated context of manufacturing has been advocated, Hammond and Oh (1973,/15/). Parallel to this concept, the hardware for multilevel computer based production has also been developed so as to be able to encompass many of the necessary design characteristics. i.e. the necessary computer architecture has also evolved in a compatible manner to the decision making structure of manufacturing organisations. It is only recently that there has been a formalisation in the methodology of integrating production decisions. Relevant references in this area include Doumeinghts et al, (1978,/24/), Erschler et al (1976,/25/), Hansen et al, (1978,/26/), Drew (1975,/27/), Gunadson (1978,/28/), Wilson (1977,/29/), Doumeingts (1981,/30/). Common to their work is the adoption of hierarchical decomposition technique as first characterised by Mesarovic et al, (1970,/31/).

It is to be emphasised that the concept of "decomposition" does not rule out the concept of integration. The objective is of hierarchical decomposition technique to reduce the solution of large problem whose time characteristics are not homogeneous into the solution of more homogeneous sub-problems. In order to simplify control and to keep the system flexible, subproblems are allowed to retain independence while still remaining co-ordinated. The aim is to develop a coherency in the multi-level structure by bringing in at each level, through constraints rather than criteria, the solutions arising from a preceding level. The degree of autonomy at each level is used for more flexible decision making in the face of the actual events not included in the larger model.

1.3.4 Multi-Stage Production Inventory System.

The research carried out has concentrated on the multi-stage production-inventory type of manufacture. This is usually found in large volume, low mix manufacture. The various parts/assemblies are usually processed through the same general sequence of operations. These production stages may consist of single machines or a group of machines for machining, assembly or inspection. Due to the imbalanced nature of demand and supply between the production stages, inventory banks are allocated to absorb mismatch. Manufacturing systems with a collection of such production stages and inventory banks linked in series and/or in parallel are termed as Multi-Stage Production-Inventory systems in this analysis. Examples of such a class of manufacture are found in flow-line production, e.g. the automotive industry or semiconductor device manufacture as shown in Figure 6. Dynamic analysis of such systems involve the control of amplified responses of production quantities, interstage buffers that usually occur in such environment. Typical analyses in this field will include Forrester (1961,/32/), Fey (1961,/33/), Christensen and Brogan(1971,/1/), Drew (1975,/2/), Porter et al (1975,/3/), Kimura and Kerada (1979,/34/), Tabe et al (1980,/35/). Their different approaches will be discussed subsequently in the thesis.

If the automotive industry is taken as a MSPI system as shown in Figure 1.6, it can be demonstrated how the control problem in such an environment may lend itself to the control approach described.

At the strategic level, senior management issues target values for various car models to cater for the sales demands and forecast for both home and export markets. This decision is then passed down to the next level of management. This new level of control monitors the actual production of the various subsystems, and co-ordinates the flow of parts and assemblies between them. The sub-systems consist of both parallel and serially linked production-inventory stages as shown in Figure 1.6.

They are namely:

Stage 1 : Gear box assembly.

Stage 2 : Engine assembly.

Stage 3 : Power unit assembly.

Stage 4 : Body in white welding.

Stage 5 : Painted body production.

Stage 6 : Trim and final assembly.

Assemblies produced at the various stages may be fed directly in the production requirements or may be required as stand-alone products or may be put into inventory. At this control level, policies are made for the required production rates of gearboxes, engines and other assemblies so as to meet the final target of finished cars, allowing for reject and other stochastic disturbances. This is the integration or coordinating level of the control decision.
At the next level, the shift or daily or weekly target values are then broken down and applied to the processes feeding in the necessary parts. The new decision time interval will be shorter than the level above. Thus as regards the engine assembly, feeding lines bring in typically engine blocks, crankshafts, pistons that need to be regularly monitored. Here "individual" control is applied at each sub-system using the parameters obtained from the coordinating decision as new constraints. This process can, of course, be carried another step further since each individual subsystem in this particular case consists of still smaller subsystems.

The decomposition technique allows for :

- Decentralisation of decision making.
- Operation on a smaller amount of information.
- Use of mini - computers and decentralised ones.
- On - line capabilities in order to provide a dialogue between decision maker and model at various levels of

problem.

It is restated here that the concept of decomposition and decentralisation may not necessarily mean a physical separation with different decision makers at the various levels of the problem. Large manufacturing systems will, of course, involve more levels of decision making and decision makers. The methodology of looking at the problem in a decomposed hierarchical structure still applies.

It is believed that a strategy combining the above systematic approach and the hierarchical control decision making provide a practical and efficient framework for designing and implementing a computer based control system. Such a potential has arisen as a result of:

- the development of multivariable control theory. This will be discussed in detail in the next chapter.
- the formalisation of the concept of multilevel decision making.
- Availability of computer architecture to support the various forms of information and processing requirements.

1.4 Production Control in Manufacturing.

1.4.1 The Role Of Production Control In Manufacturing System.

Manufacturing management and production control in particular is among one of the most important factors contributing to the success of manufacturing industry. It is these functions that actually ensure that value is added to the raw material in the conversion to sub - assemblies and final products. It is necessary to ensure that goods are produced at the scheduled time and rate. The exercise of managing such a system would obviously be trivial were it not for the fact that disturbances, both internal and external, continuously affect the manufacturing operations. Some of the internal disturbances can be itemised as :

- Reject or scrap.
- Machine breakdown.
- Absenteeism, strikes.
- Material shortage.

and external disturbances are :

- New orders.
- New delivery dates.
- Sales fluctuations.

In view of the above unavoidable disturbances, it is therefore the role of production control to continuously decide on recovery plans and implement control measures at various stages of the production inventory system so as to meet the final sales requirements to the best of the given state of affairs. In so doing production control has to consider the reallocation of resources as manpower and equipment in order to provide for the necessary parts and subparts at the different production stages. Obviously innumerable policies of resource allocation and buffer storing exist as recovery actions and control measures. The problem is therefore how efficiently these decisions actually control the situation while making the most beneficial use of both currently and potentially available resources.

1.4.2 Mathematical Treatment Of The Production Control Problem.

1.4.2.a Simulation Studies.

The problem of production control has received a limited amount of attention during the past two decades. Previous work in control has been mainly based on a investigative type of analysis with the use of digital computer and analyses using corrective control measures were not developed until the early 1960's. The early simulation exercise involves the identification of the various relevant control features and aspects of the particular manufacturing system. These features are then modelled mathematically and their dynamic responses in a given time horizon are then analysed under varying conditions. It is a "what if" approach whereby the modeller varies the pertinent parameters to obtain solution guidelines and insight of the system. No control or optimisation is built into the model, the modeller is the actual controller in his/her ability to vary the parameters and to check whether the solution is satisfactory or not. Such simulation techniques have won, to an appreciable extent, the favour of industrial practitioners. This is mainly due to the logical and non-rigorous nature of the mathematics involved which results in easy comprehension on the part of potential users.
Such an approach is an example of feedback control, albeit that the

control is done externally by the modeller using his experience and insight of the system. The schema of this process is shown in Figure 1.7 . Some of the earlier attempts reported in the development of factory simulation system using actual operating data were at Hughes aircraft company, Earl Legrande, (1963,/37/); Steinhoff, (1963,/38/); Bulkin et al, (1963,/39/). Some workers have even developed generalised simulation languages for this purpose, e.g. GPSS, Generalised Simulator system : Silver,(1974,/40/). Currently, various commercial packages are available in the market providing this type of "what if" analyses, some of them with extensive graphics facilities. However exhaustive iterations are very likely before an acceptable solution is reached.

1.4.2.b The Concept of Control Feedback in Decision Making.

As outlined in the previous section, the concept of control feedback is the process of synthesising control upon the knowledge of the current states of the manufacturing processes. The present problem is to inbuild to a certain extent this control function within the model itself. A production process may be schematically represented with a set of input variables and another set of output variables as in Figure 1.8

Input variables:

- Production orders.
- Production plans.
- Manpower, equipment, material resources.

Output variables:

- Final products.
- Assemblies.

- Required inventory levels.

From a mathematical point of view, this exercise involves the design of a controller that is within the information feedback loop. The controller uses this feedback information to synthesise the necessary control measures in order to obtain a controlled output. In other words, weightings are necessary to assess the current states for control decision making. One of the earliest attempt of such an exercise is the work of Forrester, (1961,/32/). Although his work was based on production - distribution - inventory systems, the concept is identical for a MSPI system as identified by Buffa, (1963,/19/). Forrester devised numerous rules that are based from managerial experience and insight in the particular systems in conjunction with a servo-mechanism approach. While the non - linear approach may provide an extraction of a structured procedure that can be transferred to other manufacturing systems, serious difficulty is envisaged in the rebuilding of the controller. Fey, (1961,/33/) introduced a more formal servo-mechanism approach into Forrester's work in his study of a practical example in the electronic component industry. This approach provides one of the most comprehensive case of modelling dynamic systems and towards the understanding of system behaviour. It is an exercise of identifying relevant parameters that represent systems and in particular it focuses on the rules underlying decision making processes. This approach coined as Industrial Dynamics by Forrester has been further taken up by Coyle (1973,/42/) as Systems Dynamics. Examples of System Dynamics in an industrial environment are Sharp and Coyle (1976,/43/) and Keloharju (1974,/44/). Alkalay and Buffa (1963,/45/) proposed a general model with the use of differential equations relating the various functions and variables of the system. This approach was further extended in a more rigorous structural and mathematical framework for the analysis of a larger class of production system by Reissman and Buffa, (1963,/46).

The vast number of variables that a manufacturing manager has to contend with will obviously create a difficult situation in the search of an "optimum" or even "suboptimum" control solution of the manufacturing problem. Therefore there has been a need for a more structured formulation and analysis that can achieve the following objectives:

- Some optimising approach.
- Introduction of feedback.
- Integrating control decisions upon numerous variables or parameters from the inter-linked systems.

Some similar analysis involving the integration of production decisions is Bitran and Hax (1977,/47/), Singhal (1978,/48/), Erschler et al (1976,/25/) where the emphasis has been more on the planning side than on the control one. Nevertheless, the hierarchical decomposition, common to all of them is adopted in the present approach.

Recent developments of modern control theory and other optimisation techniques indicate the potential as an analytical tool to the various problem aspects of production control. The concept of feedback control is to a large extent inbuilt in the formulation and analysis of the new modelling techniques. The general objective behind the use of control theory is attempt to shift from a "what if" exercise to one of "what is best". The theoretical advantage of such an approach is that a methodology of solution seeking is presented that attempts to converge to a suboptimum control policy without the necessity of trying every possible solution. While a few workers, Christensen and Brogan, (1971,/1/); Drew (1975,/2/) have tried to assess the practical potential and significance of such new techniques in a manufacturing environment, a majority has concentrated on synthetic cases only. The fact that this new methodology has found only limited practical application or appraisal is not surprising due to the following reasons:

- High mathematical content.
- Communication problems.
- Necessity for a structured information system to support the model.

These features have been constantly borne in mind during the course of the research. In investigating the applicability of some of the control tools developed by other workers in the control of MSPI, it was found increasingly necessary to extend the earlier theoretical analyses so as to increase their practical value in a maufacturing environment. It ought to be stressed that in attempting to obtain the "what is best" solution, the current work does not regard it as the only alternative to the "what if" approach : these two are viewed as the the poles of a spectrum of solutions. The approach is to identify along that spectrum a balanced and practical solution between the two poles. In so doing, it is strongly believed that the gap between pure research and actual application can be bridged.

Design and analysis of production control in a multi-stage environment has been categorised by some workers into the Push or

Pull systems. (Kimura and Terada, 1979,/34/; Tabe et al, 1980,/35/).

(i) Push system: The control system calculates the required amount of parts and assemblies at each stage on the basis of demands (actual and predicted) made during the total production lead time at each stage. This exercise will involve a joint control of both the number of parts or assemblies to be produced and the inter-stage buffers. This has been the conventional approach.

(ii) Pull System: This control system is a direct one, whereby as a production process pulls one part (or a pallet) from the preceding buffer, it also initiates the preceding production process to fill the quantity at the time it is actually consumed. This practice is the one adopted in Toyota manufacture, and has recently attracted quite a substantial interest from the senior management of car manufacturing companies in Europe and the United States.

The conventional push system has been claimed to have numerous disadvantages among which are:

(i) The control system is usually inoperative due to many unforeseen disturbances.

(ii) The difficulty of obtaining up-to-date information, and the inability to process all the states related to production rate and inventory levels into timely decisions.

The two above factors usually lead to large amplifications, resulting in excess ive buffers, large requirements of capacity in some cases and idle production stages in others.

The approach adopted in this research is weighted towards the Push system. However, it will be shown that the approach presented overcomes the above-mentioned disadvantages by :

(i) Making use of the increasingly effective computer-based manufacturing information system as described in Section 1.3

(ii) Recent development of multivariable control theory that provides both a formulation and analytical tool for the control and co-ordinating of numerous parameters involved in such systems.

(iii) Presenting a new solution methodology developed from the two above features that will control the previous widely amplified responses in excessive production requirements or excessive in-process inventory.

1.4.3 Mathematical Formulation of MultiStage Production Inventory systems.

Dynamic control analyses of MSPI systems have not been very abundant due to the limitations of the formulation techniques. Thus most of the inventory models developed are concentrated on the control of raw material or finished goods. One field of work slightly related to the control of MSPI is the practical research carried out in the design of transfer lines and response behaviours of such lines under various parameters as :

- Number of production stages.
- Size of inter-stage buffers.
- Breakdown frequency.
- Repair time (down time) distribution.
- Variation of processing times.

However this approach is still based on a "what-if" investigative concept as opposed to the control simulation utilised in this work. Using the framework described in Section 1.3, the current approach considers the MSPI problem as one that requires control, both at the individual production-inventory stage and at the co-ordinating level.

Steady State.

At steady state a production system may be represented as in Figure 1.8 , where the input variables are :

- Customers orders.
- Manpower.
- Equipment.
- Material.

and the output variables :

- Final Products.
- Assemblies.
- Buffer levels.

Within the context of control theory, the above input variables are referred as "control variables" while the output variables are the "state variables", the latter being controlled by the former. At the steady state of performance the various resources are utilised at a constant level to obtain the required level of production. This is obviously a very idealised state of affairs.

Dynamic State:

In a real manufacturing environment, uncontrollable variables, i.e. disturbances are very likely to occur, altering the performance of the system by affecting the production rate, quality, quantity and schedules. These disturbances are either external or internal :

External :	Internal :
New orders.	Scrap.

New delivery dates. Machine breakdown.

Sales fluctuation. Absenteeism.

Material shortage.

Control can only be effected as a result of the availability of relevant information on the state variables, i.e. a decision for the necessary recovery plans is made from the feedback information. Such recovery plans may include:

- Adjustment of the production rates.
- Acquisition of extra resources by introducing overtime, subcontracting
- Improved buffer and inventory control.

Figure 1.8 shows the overall control problem.

As briefly pointed out in Section 1.4.2, the industrial dynamics of Forrester introduced the concept of feedback analysis with the notion of relating the systems states as "level equation" and "rate equation". The first equation relates the input and output balance of inter-production buffers while the second one denotes the actual production rates obtained as a result of injecting input resources. In the presented work here, the same general concept is initially adopted and developed more rigorously using mathematical control theory.

System equations.

(a) Level equation for interstage buffers.

Assuming a non-perishibility of the buffer, an equivalence equation can be constructed for each individual production-inventory system. This equation relates the amount of inventory at the current time period as equal to the amount of inventory before the beginning of the time period plus the amount produced from the preceding production stage less the amount withdrawn for the subsequent operation, i.e.:

$$i(k+1) = i(k) + p(k) - u(k) - s(k).$$

$i(k)$ - actual inventory at time k.

$p(k)$ - actual production rate at time k, at preceding stage.

$u(k)$ - desired operating rate at time k, at subsequent stage.

$s(k)$ - demand or reject rate at time k, at current stage.

(b) Production rate equation.

The actual production rate achieved at time k+1 at each production stage is the result of the desired production rate decided at time k.

$$p(k+1) = u(k).$$

The above two equations for each production-inventory stage can be written in the following vector-matrix equation:

$$x(k+1) = A\,x(k) + B\,u(k) + E\,d(k).$$

where $x(k)$ - State variable vector. $(p(k), i(k))$

$u(k)$ - Control variable vector.

$d(k)$ - Disturbance vector.

A - Plant matrix.

B - Input matrix.

E - Disturbance matrix.

The use of a matrix notation also allows a simultaneous and efficient analysis of all the production-inventory subsystems that make up the whole manufacturing system. Such a formulation has been widely adopted in all the work involving a control theory approach. However at the solution stage of such problems, a variety of approaches has been tried with varying degrees of success.

The control problem faced by the decision-maker is the formulation of a control policy at each time period for the allocation of resources to the various production stages in view of the current production rates, levels of buffers and demands. In other words, it is required to synthesise the values for vector u(k) with the use of some weighting function operating on the state variables x(k). This can be expressed mathematically as :

$$u(k) = F \; x(k).$$

F is the feedback matrix containing the different weights that are to be given to the individual state variables in the process of deciding the new desired production rates.

The control problem will be therefore how to decide on the values of matrix F. Whilst an arbitrary feedback matrix may be chosen from experience, with the help of empirical rules, it is the present intention to synthesise this matrix mathematically, in an attempt to arrive at an "optimum" or "sub-optimum" solution. The mathematical problem itself has been dealt with successfully by the use of optimal control theory. Practical applications of this mathematical concept is witnessed in mechanical, electro-mechanical systems, e.g. sevo-mechanisms, engine performance and flight control. In general, such applications of control theory in manufacturing systems have been limited to date.

1.5 Discussion.

In this chapter, various facets of production control in discrete manufacturing have been introduced. The particular area of manufacturing considered is the multi-stage production-inventory systems where low mix and high volumes are involved as in the automotive industry or the white consumer goods. Production control has been considered with particular emphasis on the feedback concept on two complementary levels:

(i) Feedback information obtained as a result of a properly designed control information system for the type of manufacturing concerned. A framework in the design of such control system has been discussed extensively, introducing the hierarchical approach in the information requirements for production control.

(ii) The development of feedback control theory in particular the multivariable control theory. The parallel between synthesis of production control decisions with information feedback and the concept of feedback theory has been discussed.

One major objective of the research has been to study the application of the two above-mentioned developments in actual manufacturing situations. This has led to the adoption of Multivariable Control Theory because of its potential to control and co-ordinate the numerous parameters simultaneously, through the design of an appropriate controller at a particular level of the control problem. In so doing, it was found increasingly necessary to further understand multivariable control theory and develop it to a state that could be applicable to the desired objective. This is particular so, in view of the fact that the main emphasis of the research has been on the practical applications of the currently developed control techniques.

Chapter 2 of this thesis is initially devoted to the development of such state variable feedback control theory when applied in actual Multi-Stage Production-Inventory systems. Subsequently in Chapters 3 and 4, it is demonstrated how this development has been used to analyse a particular automotive manufacturing company. The control of of both single and multi-product manufacturing environmentS is fully discussed.

CHAPTER 1
FIGURES

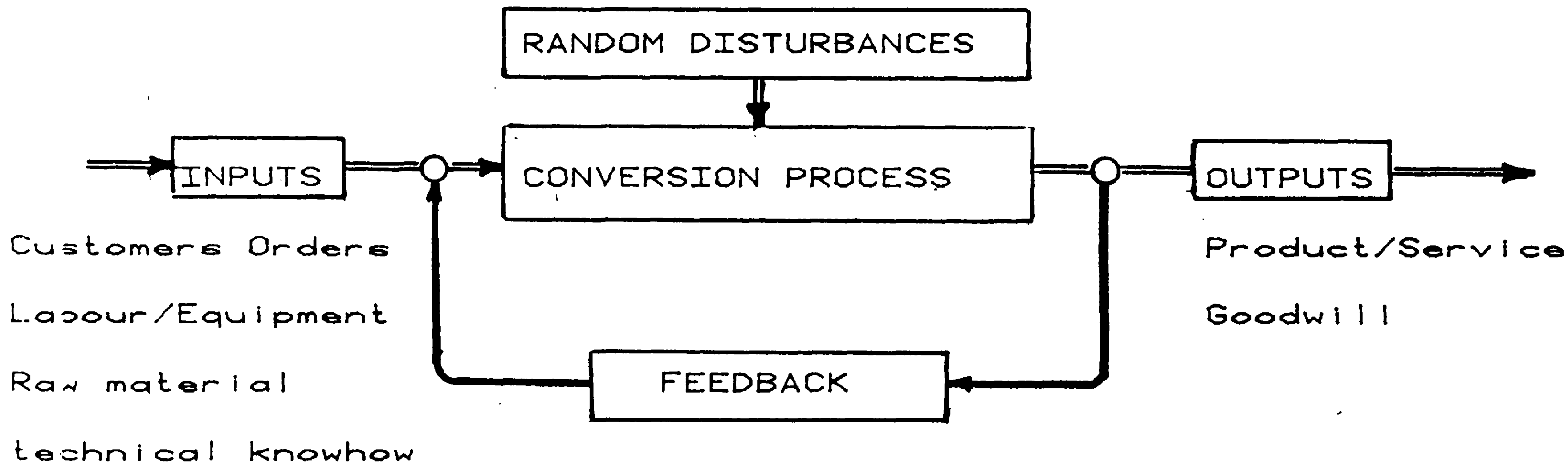


FIGURE 1.1 INPUT - OUTPUT MODEL

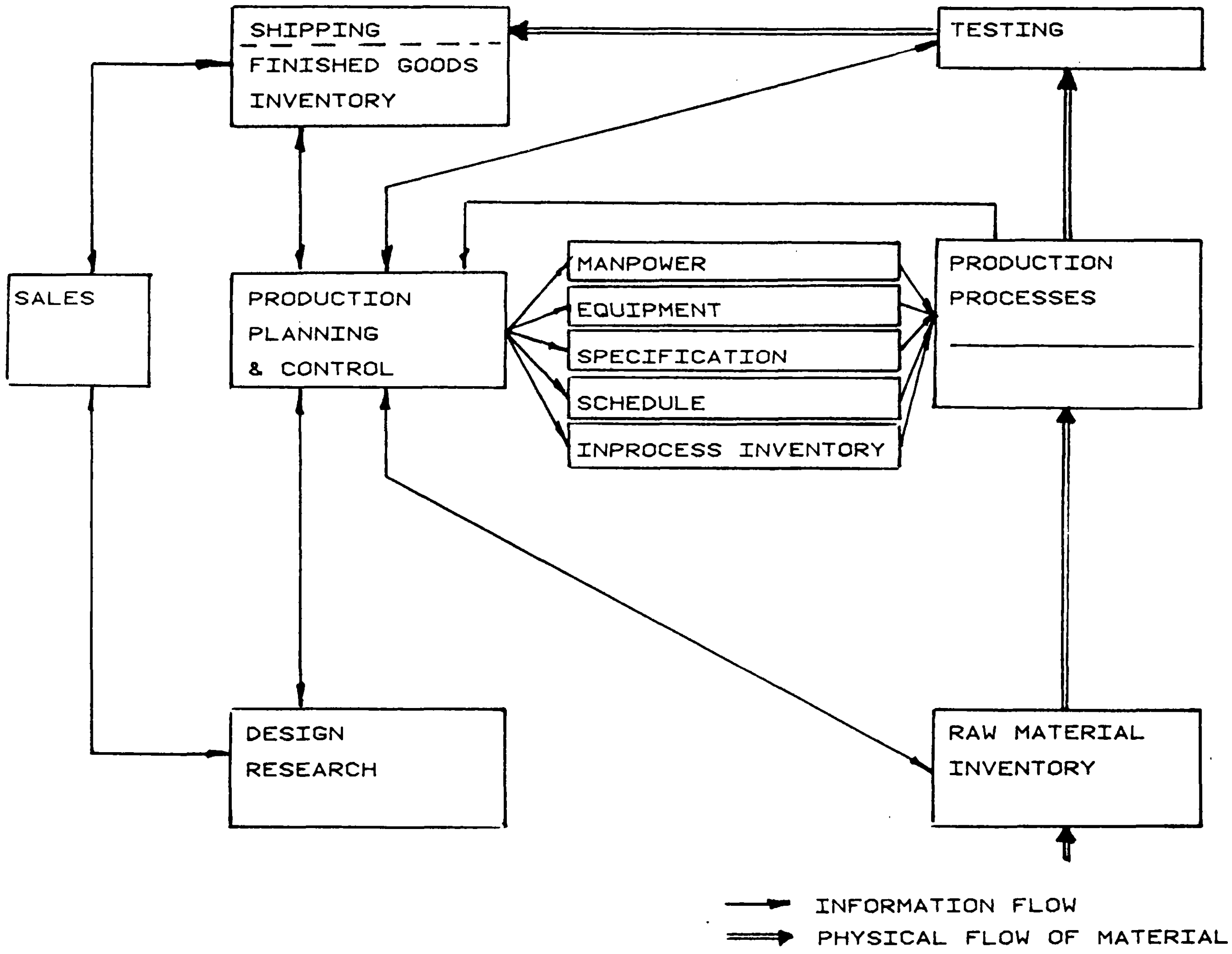


FIGURE 1.2 : MANUFACTURING SYSTEM

FIXED DATA

PARTS MASTER DATA	BILL OF MATERIALS	ROUTING DATA	SPECIAL INSTRUCTIONS DATA	RESOURCES AND RATES DATA	PLANNED COSTS DATA

DYNAMIC DATA

CONTRACT COMMERCIAL DATA	CONTRACT REQUIREMENTS DATA	PARTS REQUIREMENTS & TENTATIVE ORDERS DATA	PARTS STOCK & FIRM ORDER DATA	MASTER PRODUCTION SCHEDULE	WORK-IN-PROGRESS DATA	ACTUAL COSTS DATA	SHOP FLOOR STATUS DATA

FIGURE 1.3 : COMPUTER-BASED DATA FILES

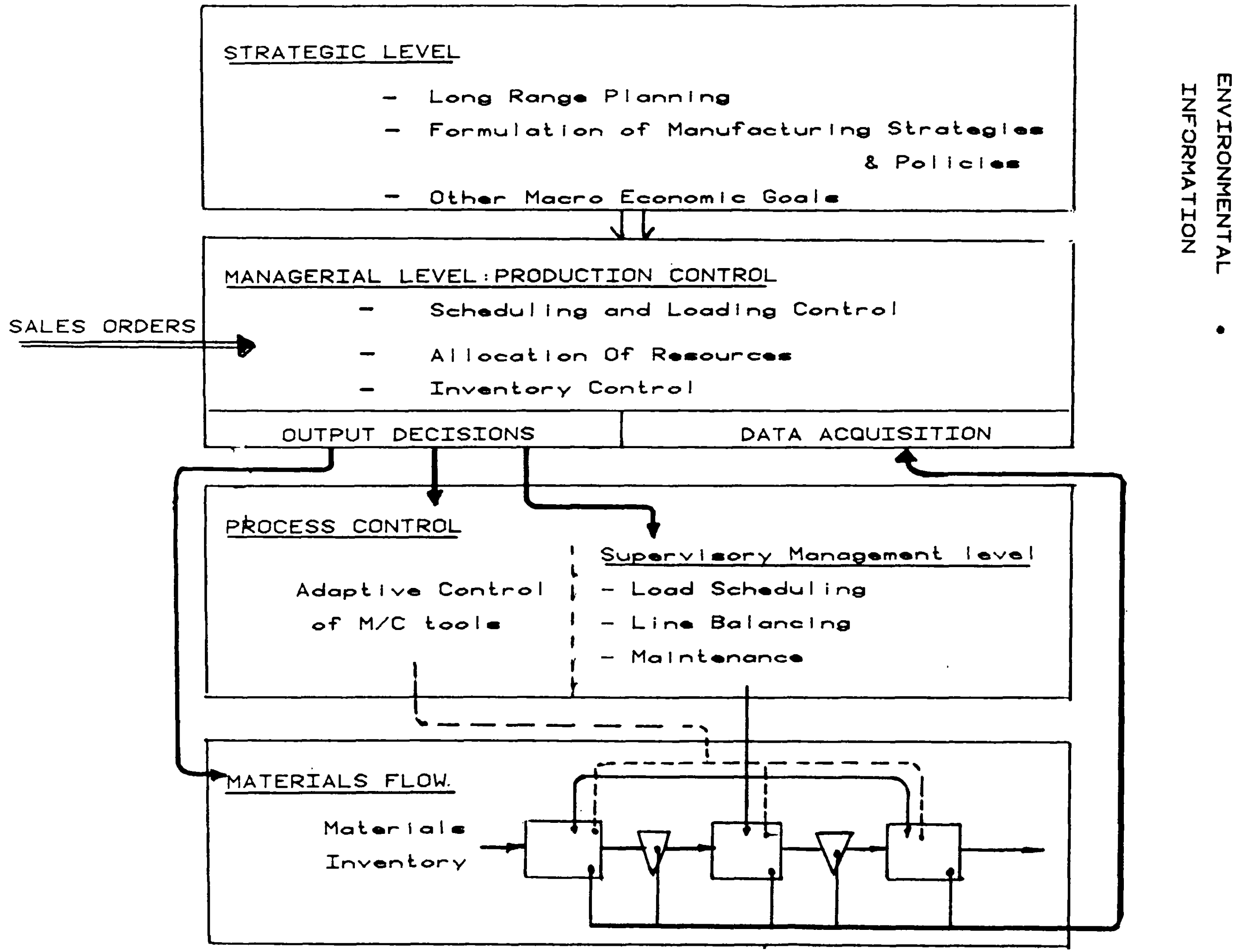


FIGURE 1.4 : LEVELS OF CONTROL IN MANUFACTURING

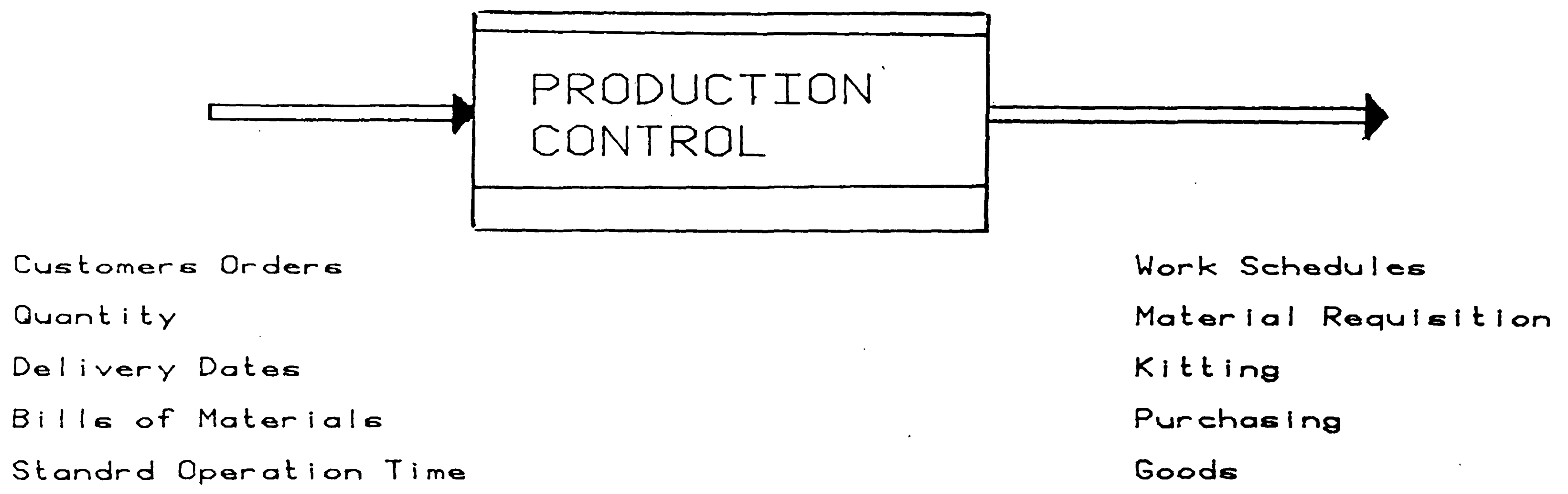


FIGURE 1.5 : PRODUCTION CONTROL.

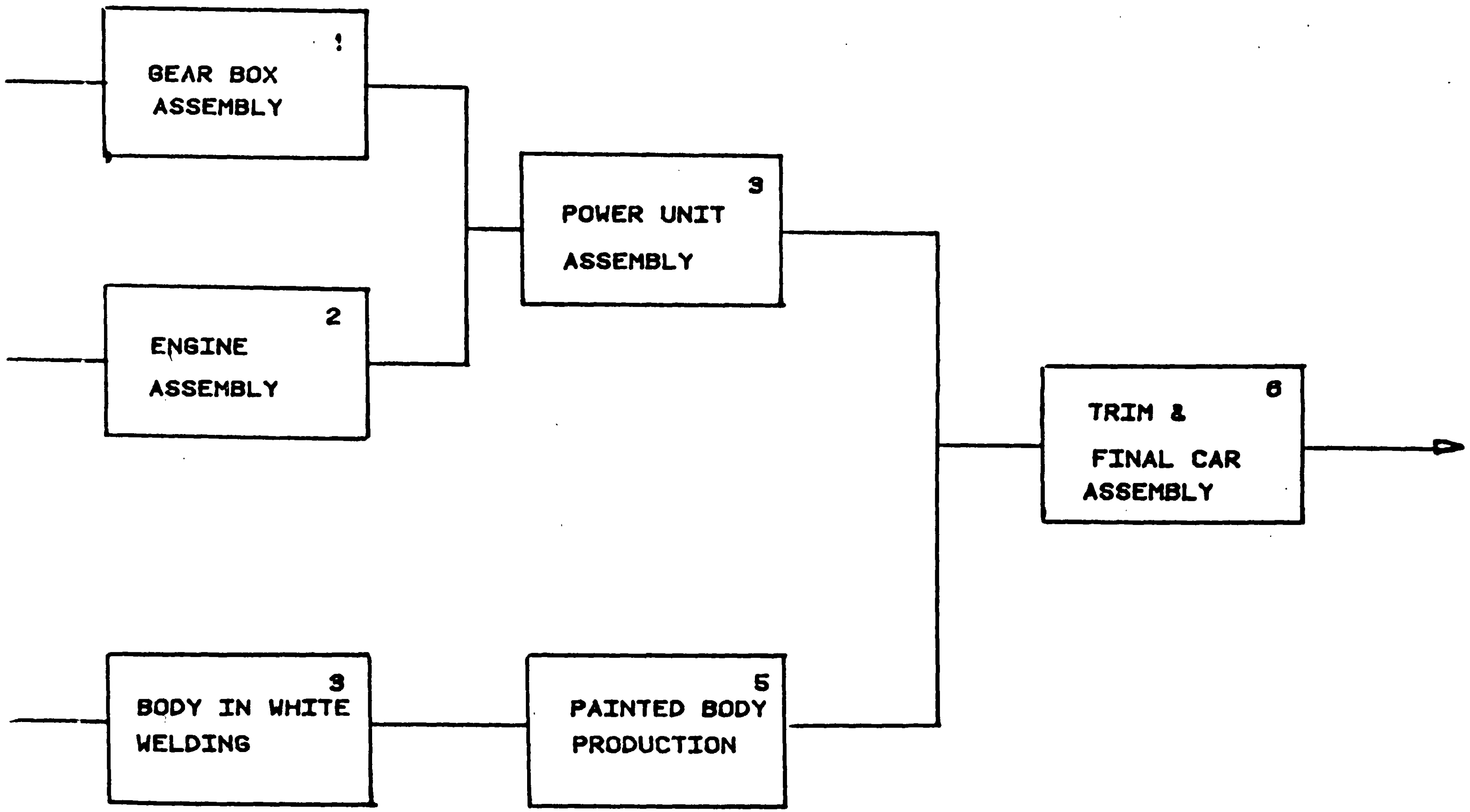


FIGURE 1.6 : CAR MANUFACTURING AS A MULTI-STAGE PRODUCTION-INVENTORY SYSTEM.

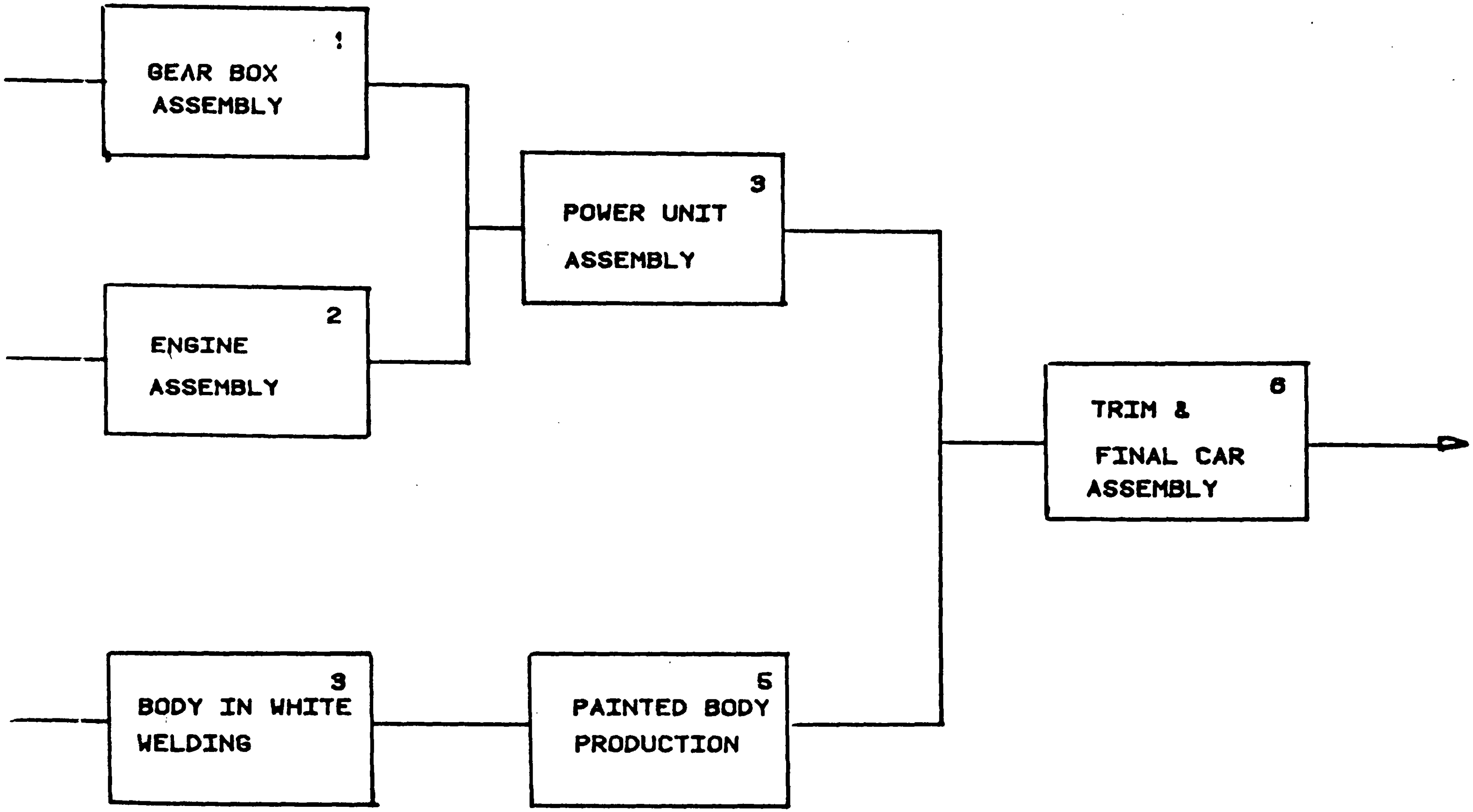


FIGURE 1.6 : CAR MANUFACTURING AS A MULTI-STAGE PRODUCTION-INVENTORY SYSTEM.

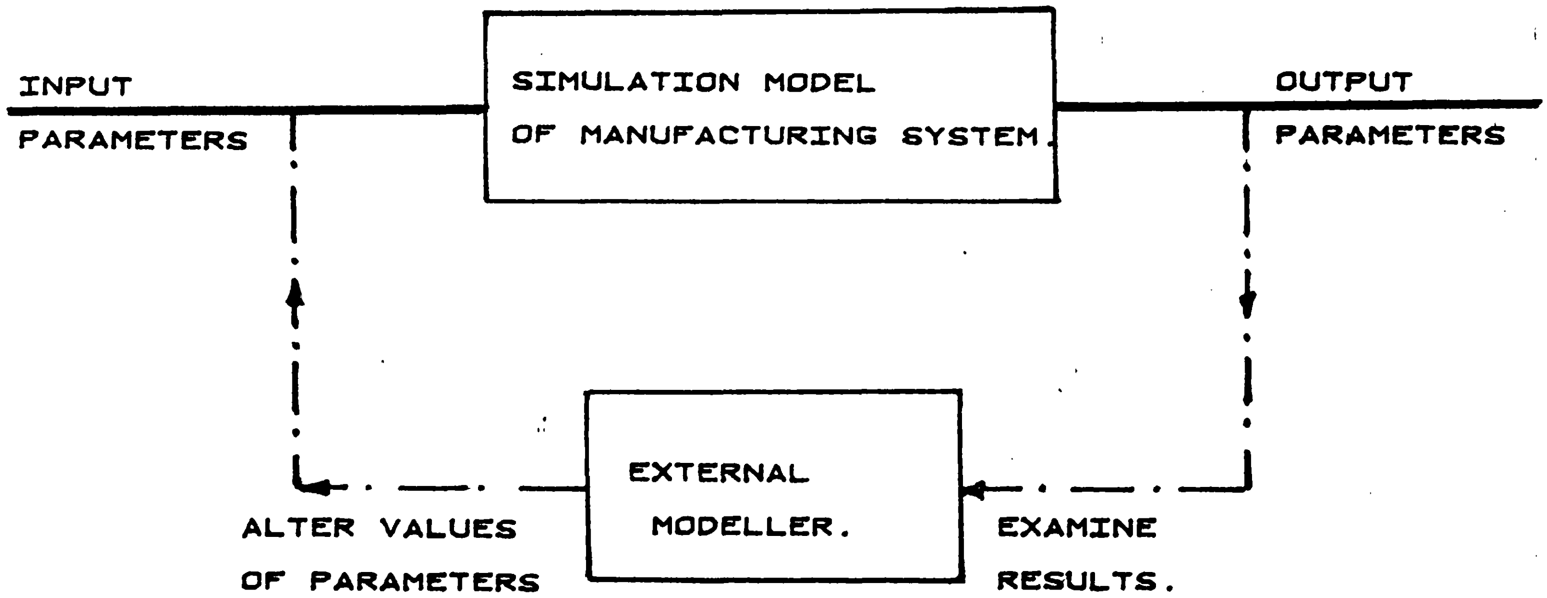


FIGURE 1.7 : FEEDBACK WITH EXTERNAL MODELLER.

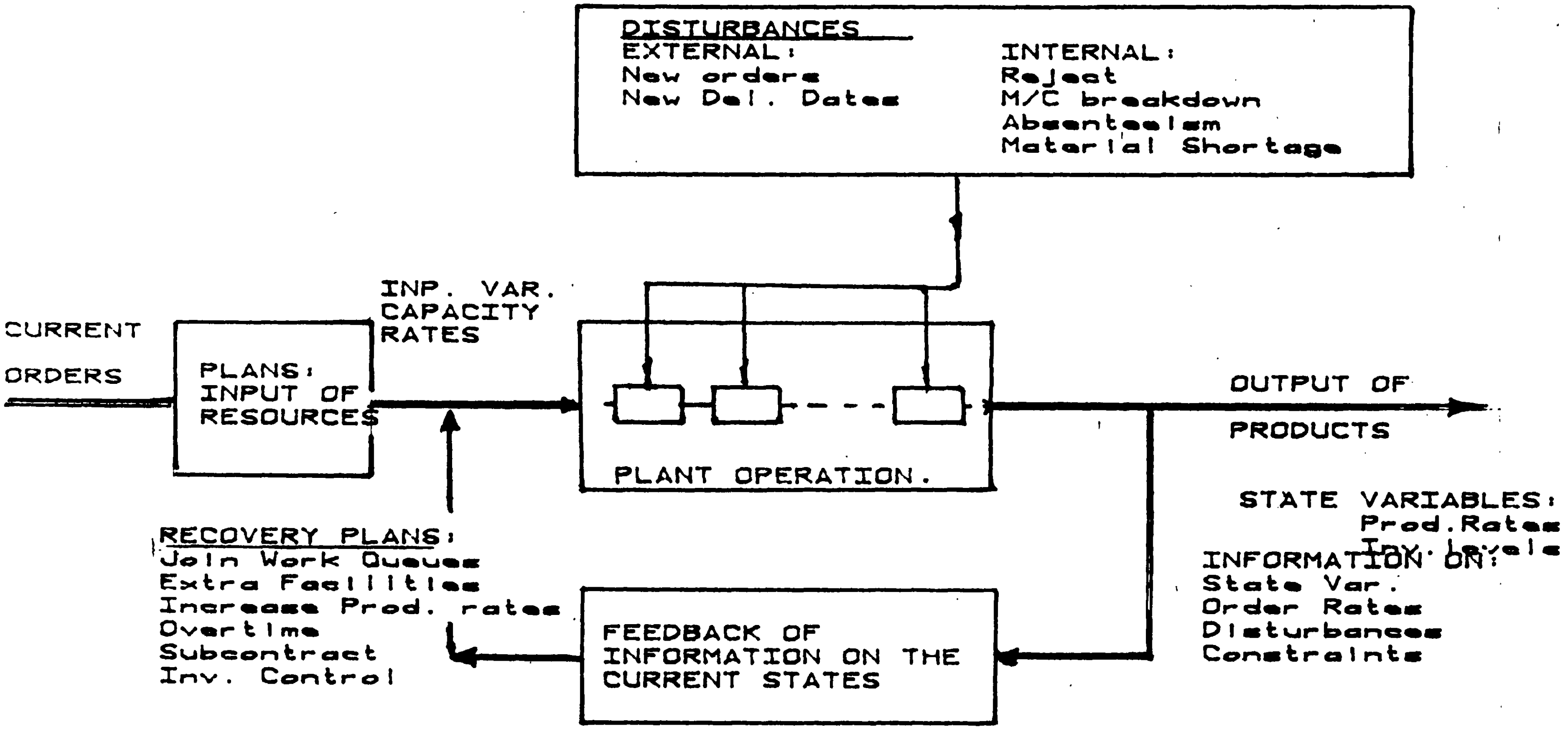


FIGURE 1.8 : OVERALL CONTROL MODEL

Computer Usage in Manufacturing Operations

	Online	Offline
Strategic Level	General MIS Profit, Company data Personnel data Stock Exchange Marketing	Simulation Studies Economic Environment Financial Modelling Product Demand
Production Control	Accounting Production Schedule Inventory Q/C Process Inventory	Cost Control Inventory Control Q/C and Reliability CAD/CAM
Process Control	Data Acquisition Mafg Operation Control	Mafg Support N/C preparation

TABLE 1.1 COMPUTER USAGE IN MANUFATURING OPERATIONS

CHAPTER 2

Chapter 2. Development of the Mathematical Control Model.

This chapter introduces some of the work carried out in the analysis of Multi-Stage Production-Inventory (MSPI) system with multivariable control theory (MVCT). The basic concept has already been outlined at the end of the previous chapter. A survey of previous relevant work is given and their practical merits discusse d. Introductory references to control theory are given in /49/ - /56/.

2.1 Previous Work.

2.1.1 Survey of Control Theory as applied to Manufacturing Systems.

One of the earliest study in manufacturing with modern control theory is that of Christensen and Brogan, (1971,/1/). The formulation described in Section 1.3.4 was adopted.

$$x(k+1) = A\,x(k) + B\,u(k) + E\,d(k) \qquad \text{---- } 2.1$$

The manufacturing system is modelled as a linear discrete time multivariable dynamical system with a disturbance input, d(k), introduced by sales fluctuation. The state variables x(k) consist of :

- Rates of flow of parts or sub-assemblies.
- Backlogs of parts awaiting processing.
- Inventory levels of final products.

Control variables, u(k), are the man-hours scheduled for various work processes.

The following performance criterion given by

$$J(x,u) = \sum_{k=0}^{N-1} \{ \hat{x}(k)^T Q\, \hat{x}(k) + \hat{u}(k)^T R\, \hat{u}(k) \} \qquad \text{---- } 2.2$$

is used.

The sign ^ being the deviations for the respective variables.

The control law equation is given as:

$$u(k) = F\,x(k) \qquad \text{---- 2.3}$$

where F is the feedback matrix is computed using the principle of optimality of Bellman, (1957/57/). Various feedback matrices $\hat{F}$, multiples of F, are then implemented and the corresponding responses analysed. These show characterised fluctuations in manpower requirements, production rates and inventory levels. In effect, this was still, to a certain extent, a control adopted on a "what if" basis. Moreover, no explicit methodology is present that attempts to consider any constraints such as capacity and inventory levels. The quadratic terms in equation 2.2 attempts to drive the control solutions away from the extreme values, thus into the feasibility region. Although a number of practicalities have not been fully considered, a sound approach has been formulated by Christensen and Brogan as the base for future work.

Drew, (1975,/2/) applied the control theory developed for large scale systems in an industrial management problem. The production control problem is decomposed into hierarchical sub-problems for decision making as first proposed by Mesarovic, (1970,/31/). The subproblems have their respective systems equations given as in equation 2.1:

$$x(k+1) = A\,x(k) + B\,u(k) + E\,d(k).$$

Costs functionals are given as:

$$J = \sum_{k=1}^{N} \{ \tfrac{1}{2}\|\hat{x}\|^2 Q + \tfrac{1}{2}\|\hat{u}\|^2 R \} \qquad \text{---- 2.4}$$

With the introduction of Lagrangian functions, optimal and suboptimal solutions are obtained using a modified version of goal-coordination algorithm first proposed by Tamura, (1973,/58/). Drew illustrated his approach in two problem aspects of manufacturing:

(i) Multi item inventory control.

(ii) Production control in a multi-batch environment. Constraints both in resource capacity and inter-stage buffers are fully considered. The results obtained are practical and very near optimal. It is believed that the theory of Drew is too highly advanced as to be able to gain immediate industrial attention.

Hitomi and Nakamura, (1976,/59/) also used the same formulation approach, i.e. state variables correspond to the number of machines at work (production rates) and inventory levels; control variables correspond to the increase or decrease of number of machines and working times. They used a quadratic cost function similar to that of Christensen and Brogan(1971,/1/).

$$J(x,u) = \sum_{k=0}^{N-1} \left\{ \hat{x}(k)^T Q \, \hat{x}(k) + \hat{u}(k)^T R \, \hat{u}(k) \right\}$$

At the solution stage Hitomi and Nakamura applied functional space analysis to the problem. The technique used was powerful as it could deal with :

- Constraints in resource capacity, e.g. number of extra machines or overtime.
- Constraints in inventory levels.
- Discrete random disturbances.

Nevertheless, some extensive development is believed to be necessary if it has to deal with MSPI consisting of both parallel and serially connected manufacturing systems.

Bedini and Toni, (1980,/60/) adopted the Pontryagin Maximum Principle (1962,/60A/) to analyse a manufacturing system. The Pontryagin principle provides the advantage of considering explicitly the constraints in the state and control variables, which is a very practical feature. The system analysed two production inventory systems linked together in series. It would have been interesting to know how the methodology would consider the interrelationship for the production-inventory systems linked both in series and in parallel. It is believed that a substantial extension in the methodology is required before it is applicable for a more involved real system. An earlier approach with the Maximum Principle was the work of Bhattacharyya et al (1969,/61/) who used a discrete-time formulation. (Fan and Wang, 1964,/62/).

Porter and Crossley , (1972,/63/) applied modal control theory to analyse the Christensen and Brogan model, (1971,/1/). With the use of eigenvalue assignment technique, control policies were synthesised for the manufacturing model. The structured properties of the modal matrices and Jordan canonical forms are used in identifying the corresponding elements necessary to control the required modes. The practical implications of having different controlling eigenvalues are demonstrated in the transient responses of production rate and inventory fluctuations. It was shown that if short settling time is a primary requisite, eigenvalues should be set close to the origin of unit circle, resulting in a high increase in demand for manpower. With eigenvalues chosen to lie just within the unit circle, the extra need for manpower is more gradual and results in a longer settling time. The selection of these eigenvalues is left to the discretion of management, depending on the amount of fluctuation they consider acceptable. Such a "what if" approach provides a substantial insight into the control features of the manufacturing organisation.

Bradshaw and Porter, (1975,/64/) extended the previous approach into a discrete - time analysis and introduced a series of discrete - time vector integrators into the controller, the practical significance of which is to drive backlog values to zero or inventory levels back to their original levels. The controllability conditions for such linear multivariable tracking systems were also established.

Bradshaw and Daintith, (1976,/65/) adopted a similar approach of assigning arbitrary eigenvalues to control cascaded production-inventory systems. In *this case, instead of the Jordan* canonical form previously used, the Brunovsky canonical form is introduced. It was also demonstrated that production inventory subsystems, when considered compositely, give a better response than when considered in isolation.

Porter et al, (1976,/3/) considered the same model with the extension of demands for both finished and semi-finished parts. Most of the research described above is dealt with in detail in Daintith, (1977,/66/).

2.2 Development of the Control Model.

2.2.1 Dynamics of A manufacturing System.

In this chapter, a four-stage serially connected MSPI is used to illustrate the potential application of multi-variable control theory. It is demonstrated how the work of Porter et al (1976/3/), and Daintith (1977,/66/) can be extended so as to deal with some of the practical constraints of a manufacturing system. The control problem considered is the situation where the production system is subjected to a sudden step in demand at the final product stage. There are three different ways in which the system can respond, namely:

(i) A cascaded start-up.

(ii) A simultaneous start-up.

(iii) A controlled start-up.

(i) Cascaded Start-up.

In a cascaded start-up situation as shown in Figure 2.1a, the first initial production stage starts production whilst the downstream production stages are idle if there are no interstage buffers. It is only after a finite length of time, such as a working shift, or after the completion of a batch, that the batch is passed to the next production stage. This procedure is repeated throughout the whole MSPI system. It is obvious that such state of affairs would be very undesirable and unpractical as a "control means". Moreover, should one of the production stages incur a high rate of reject or actually break down, the whole system is stopped. The need for certain buffer stocks at the various stages is thus obvious.

(ii) Simultaneous Start-up.

This control policy presumes both a knowledge of the necessary inter-stage buffers and their actual availability. This approach will therefore allow a simultaneous start-up at all the production stages (Figure 2.1b), once the demand for the final product is triggered. Whilst this policy may be both possible and acceptable, the inter-stage buffers will never be replenished, if no attempt is explicitly made to achieve this purpose. Replenishing the inter-stage buffers in such MSPI systems is very important so as to be able to deal with various heterogeneous stochastic disturbances. This is in order to avoid "starving" situations, i.e. production stages remaining idle as a result of the upstream buffers being empty and/or the upstream process being inoperative.

(iii) Controlled Start-up.

In this particular case, when the demand level is subjected to a step increase, the production capacities at the various stages are controlled in such a way as to:

(i) Manufacture the necessary parts to satisfy the demands of the final product.

(ii) Replenish the inter-stage buffers.

This type of response is shown in Figure 2.1c. The problem is therefore to determine how much of the resources should actually be used at each stage on a dynamic time basis in order to satisfy the above requirements.

The approach described in Porter et al (1976,/3/) provides a good formulation of such problem. In the present thesis, the same analytical approach is adopted as an initial starting point which is then further extended to cater for certain practical constraints existing in the manufacturing environment. Such an approach brings in the concept of "control variables" and "state variables" of the mathematical control theory as initially introduced in Section 1.3.4. In this present manufacturing context, the capacity rates of the production stages are the "control variables" that govern the production rates and the levels of inventories which are considered as the "state variables".

2.2.2. Mathematical Model.

The four-stage production system in Figure 2.2a, is given mathematically (Daintith (1976,/66/) in Figure 2.2b, and the relations between the variables are given in discrete-time basis as follows:

$$
\begin{aligned}
x_1(k+1) &= u_1(k) \\
x_2(k+1) &= u_2(k) \\
x_3(k+1) &= u_3(k) \\
x_4(k+1) &= u_4(k) \\
x_5(k+1) &= x_5(k) + x_1(k) - u_2(k) - d_1(k) \\
x_6(k+1) &= x_6(k) + x_2(k) - u_3(k) - d_2(k) \\
x_7(k+1) &= x_7(k) + x_3(k) - u_4(k) - d_3(k) \\
x_8(k+1) &= x_8(k) + x_4(k) - d_4(k) - d_5(k) \\
x_9(k+1) &= x_9(k) + x_5(k) \\
x_{10}(k+1) &= x_{10}(k) + x_6(k) \\
x_{11}(k+1) &= x_{11}(k) + x_7(k) \\
x_{12}(k+1) &= x_{12}(k) + x_8(k)
\end{aligned}
$$

The control variables "u" are the input capacity rates, given in number of parts that can be produced in one unit time interval.

The state variables x_1 to x_4 are the actual production rates at the respective stages 1 to 4, x_5 to x_8 represent the levels of inter-stage buffers and are given in number of parts. x_9 to x_{12} are state variable integrators, these are augmented variables that are included to drive the state variables to their original states, Porter et al (1976,/3/).

d_1 to d_4 are "disturbance" variables that represent reject or scrap rate, or separate demands at stages 1 to 4. d_5 is the demand rate of the final finished product. The units are given as number of parts per unit time. The system state equations as given in Section 1.3.4 may be represented in the following matrix equation.

$$x(k+1) = A\ x(k) + B\ u(k) + E\ d(k) \qquad \text{---- 2.5a}$$

This is represented in Figure 2.3

where A = plant matrix. B = Input matrix.

$$
A = \begin{array}{c}
\\
1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}\\
\left[\begin{array}{cccccccccccc}
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & \bigcirc & & \\
1 & & & & 1 & & & & & & & \\
 & 1 & & & & 1 & & & & & & \\
 & & 1 & & & & 1 & & & & & \\
 & & & 1 & & & & 1 & & & & \\
 & & & & 1 & & & & 1 & & & \\
 & \bigcirc & & & & 1 & & & & 1 & & \\
 & & & & & & 1 & & & & 1 & \\
 & & & & & & & 1 & & & & 1
\end{array}\right]
\end{array}
\qquad
B = \left[\begin{array}{cccc}
1 & & & \\
 & 1 & & \\
 & & 1 & \\
 & & & 1 \\
-1 & & & \\
 & -1 & & \\
 & & -1 & \\
 & & & \\
 & & & \\
 & & & \\
 & & & \\
 & & &
\end{array}\right]
$$

E = Disturbance Matrix.

$$
\begin{array}{c}
1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12
\end{array}
\left[\begin{array}{ccccc}
 & & & & \\
 & & & \bigcirc & \\
 & & & & \\
 & & & & \\
-1 & & & & \\
 & -1 & & & \\
 & & -1 & & \\
 & & & -1 & -1 \\
 & & & & \\
 & & & & \\
\bigcirc & & & & \\
 & & & &
\end{array}\right]
$$

The above matrices are shown with their non-zero entries.

$x(k)$, state vector =

$$\left[x_1(k), x_2(k), x_3(k), x_4(k), x_5(k) \;\; \cdots\cdots \;\; x_9(k), x_{10}(k), x_{11}(k), x_{12}(k)\right]^T$$

$u(k)$, Input vector =

$$\left[u_1(k), u_2(k), u_3(k), u_4(k)\right]^T$$

$d(k)$, disturbance vector =

$$\left[d_1(k), d_2(k), d_3(k), d_4(k), d_5(k)\right]^T$$

2.2.3 Analysis of Mathematical Model.

The control equation used to govern the above system is given by:

$$u(k) = F\,x(k) \qquad \text{---- 2.5b}$$

In effect, the control policies u(k), i.e. the capacity rates for the various production stages are decided by the use of a weighted function of the current states of production and inventories of the system. The inclusion of this control equation represents the feedback procedure, and is given in Figure 2.4. This weighted function in the form of the matrix F is mathematically derived to drive the response(s) optimally to a steady state as opposed to one derived empirically. The mathematical synthesis of the matrix F is usually treated in two ways:

(i) Without input constraints.

(ii) with input constraints.

Most of the theoretical and practical development has concentrated on the case of "without constraint" as for example in the quadratic optimisation feedback technique. The work of Porter et al (1976,/3/), is based on a pole-assignment technique which is also a non-constraint approach. It provides the solution for tracking the steady states and synthesises control inputs on a time-optimal basis. Non-time optimal solutions are shown to be very likely to lead to unstable systems. On the other hand, in the actual manufacturing environment, presence of limits in the availability of resources may not allow an implentation of time-optimal policies. The present control problem is one of class(ii) case. Mathematical treatment of such a control problem is usually dealt with by the Pontryagin's Maximum Principle. Bedini and Toni (1980,/60/) use such an approach but it needs to compute backwards in time, a feature which may be a disadvantage for real-time analysis. The approach discussed in this thesis relates more to the practical application of the work described in Porter et al (1976,/3/) and Daintith (1976,/66/) insofar as it considers the implications of certain practical constraints on the applicability of the mathematical solutions.

2.2.4 Mathematical Control Theory.

The concept of the approach adopted is discussed initially on a continuous-time basis, since the form is more familiar, before extending to the discrete-time analysis.

Consider a free response system as governed by the equation:

$$\dot{x}(t) = A\,x(t) \qquad \text{---- 2.6}$$

It can be shown that it is actually governed by the eigenvalues and eigenvectors of matrix A, Porter and Crossley (1972/63/). The general solution of such a system is given as :

$$x(t) = \exp(\lambda_i t)\, u_i v_i' x(0)$$

λ_i - eigenvalue.

u_i - eigenvector of A

v_i - eigenvector of transpose of A

Therefore as $t \rightarrow \infty$, $x(t) \rightarrow 0$ for asymptotic stability

if and only if $\mathrm{Re}(\lambda_i) < 0$

i.e. the eigenvalues have to be in the negative region of the s-plane.

In a discrete-time analysis :

$$x(t) = x(kT)$$

$$kT < T < (k+1)T$$

$$k = 0, 1, 2, 3, \ldots$$

$$x(k+1\ T) = [\exp(AT)]\ x(kT).$$

$$= \Phi\ x(kT)$$

$$x(kT) = \exp(k\lambda_i T)\ U\ V\ x(0)$$

as $k \rightarrow \infty$

$x(t) \rightarrow 0$ for asymptotic stability.

if and only if $|\exp(\lambda_i T)| < 1$ ($i = 1, 2, 3 \ldots n$)

i.e. the eigenvalues of the new plant matrix [exp(AT)] have to be within the unit circle.

The case for a forced input system follows the same analysis. Therefore arbitrary assignment of eigenvalues within the unit circle will ensure both asymptotic stability and convergence of solutions. This is assuming that the system is indeed controllable. The controllability conditions are discussed in Appendix 2.

Having established the states which the variables need to achieve so as to be considered as solutions, the problem is now how to design the necessary controller that will have this effect.

This is carried out by transforming the system equations 2.5a and 2.5b into their controllable companion forms by means of the following transformation.

$$x(k) = C\,\bar{x}(k)$$

$$\bar{x}(k) = C^{-1}x(k) \qquad \text{---- 2.7}$$

The derivation of the transformation matrix C can be achieved by the Prepelita algorithm (1971,/68/). This algorithm is given in Appendix 1.

$$C^{-1}x(k+1) = C^{-1}AC\,x(k) + C^{-1}Bu(k) + C^{-1}Ed(k)$$

$$\text{or} \quad \bar{x}(k+1) = \bar{A}\,\bar{x}(k) + \bar{B}\,u(k) + \bar{E}\,d(k) \qquad \text{---- 2.8a}$$

where

$$\bar{A} = C^{-1}AC$$

$$\bar{B} = C^{-1}B$$

$$\bar{E} = C^{-1}E$$

$$u(k) = F\,C\,\bar{x}(k)$$

$$u(k) = \bar{F}\,\bar{x}(k) \text{ where } \bar{F} = FC \qquad \text{---- 2.8b}$$

The companion matrices $\bar{A}$ and $\bar{B}$ are referred to as Brunovsky 's canonical forms (1966,/67/) and are given as follows:

$$
\bar{A} \qquad
\begin{array}{cc}
 & \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array} &
\left[\begin{array}{cccccccccccc}
0 & 1 & 0 & & & & & & & & & \\
0 & 0 & 1 & & & & & & & & & \\
0 & -1 & 2 & & & & & & & O & & \\
 & & & 0 & 1 & 0 & & & & & & \\
 & & & 0 & 0 & 1 & & & & & & \\
 & & & 0 & -1 & 2 & & & & & & \\
 & & & & & & 0 & 1 & 0 & & & \\
 & & & & & & 0 & 0 & 1 & & & \\
 & & & & & & 0 & -1 & 2 & & & \\
 & & & & O & & & & & 0 & 1 & 0 \\
 & & & & & & & & & 0 & 0 & 1 \\
 & & & & & & & & & 0 & -1 & 2
\end{array}\right]
\end{array}
\qquad
\bar{B} \qquad
\begin{array}{cc}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
 &
\left[\begin{array}{cccc}
 & O & & \\
1 & 0 & 0 & 0 \\
 & O & & \\
0 & 1 & 0 & 0 \\
 & O & & \\
0 & 0 & 1 & 0 \\
 & O & & \\
0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

Their ordered structures are clearly demonstrated.

Various relations exist between the new controllable matrices, their controllability properties and the eigenvalue assignability. This is discussed in more detail in Appendix 2, where the equivalence between controllability and eigenvalue assignability by state feedback is established. References /69/ - /78/ provide a good background literature on various aspects of multivariable control theory,and in particular some of the structural properties that are made used of in the research.

Numerous eigenvalue assignment algorithms have been devised for controllable multi-input time-invariant systems. Some of these examples include Davison and Chow (1963,/79/); Moore (1976,/80/). Related design methods incorporating optimisation techniques are in Porter and Crossley (1972,/55/); Lee and Jordan (1975,/81/). More recent work includes Porter et al (1976,/3/), Daintith (1976,/66/)and Porter (1977,/82/).

As noted in Porter (1977,/82/), most of the algorithms attempt to decompose multi-input systems to "equivalent" single input systems, and in so doing introduce difficulties not associated with the original problem. With the adoption of Brunovski canonical forms, it can be demonstrated that their fundamental structural properties lend themselves readily to the pole assignment procedure. Such an approach avoids some of the difficulties associated with previous techniques and even by-passes the need of calculating eigenvalues and eigenvectors. Furthermore the structured nature of the companion matrices allows an easy identification of submatrices controlling individual input variables as shown in Appendix 3. The relevance of this feature is far-reaching in that it provides a new opportunity to effect control on selected inputs so that they remain within the feasibility boundaries. All these advantages are enhanced by the fact that they render the necessary computational task very easy. The orderliness of the new structured forms allows the design of computationally attractve algorithms. This new approach thus provides a technique of multi-variable control theory to deal with problems where there are constraints on the input variables. These problems have been hitherto only been dealt with the Maximum Principle.

2.2.5 Computational Aspects of the Modelling Exercise.

The nature of the discrete-time control model lends itself readily for being implemented on a digital computer. The simulations of the control model has been carried out with the use of the TEKTRONIX 4052 graphic display desktop computer. The system consists of a computer with 56K memory, to which are attached a disc drive and a plotter. The language used is Basic with the additional feature of numerous in-built functions for matrix computations. This facility has been extensively used during the course of the research.

The actual control simulation exercise consists of 2 stages: an initial "static" one followed by a dynamic one. The static stage consists of establishing the various matrices e.g. Plant matrix, Input matrix and Disturbance matrix, specific to the configuration of the system. These are then transformed into their canonical forms as in equation 2.7. The transformation matrix C , is obtained from the Prepelita algorithm (1971,/68/) (Appendix 1.) This algorithm has been programmed in Basic language and is given in full in Appendix 1. The package consists of a suite of three programs of 10-15 K memory and takes 2-3 minutes to run depending on the size of the system matrices.

The synthesis of feedback matrices F, is described in Appendix 3, where it is also demonstrated that sub-matrices of F exist, which control individual responses of the system. The program for this exercise is given in the same Appendix.

For the ease of computation and modelling purposes, synthesis of the feedback matrix is not effected "in situ" during the simulation runs. Feedback (sub)matrices corresponding to the different production stages are pre-synthesised for the range of eigenvalues of 0.000 to

1.000 in increments of 0.025 . The response of each production - inventory stage has such a range of control feedback matrices to choose from. Each of these control will ensure asymptotic stability as first explained in section 2.2.4 . This library of feedback matrices is stored in a disc file that may be accessed directly during the actual computer simulation runs. Henceforth for the ease of discussion, control policies synthesised with eigenvalues of submatrices set at 0.000, 0.025, 0.050, 0.075 .... are referred to as CPN (Control Policy Number) 1, 2, 3, 4 ...respectively.

This approach of performing a major part of the computation before the actual modelling exercise offers many practical advantages and overcomes some aspects of the control theory that have previously been considered as severe limitations. The main benefit is in the reduction of the computation time for the control simulation. This fact allows the implementation of closed-loop feedback in the model instead of the open-loop approach which was adopted by some earlier workers in this field.

Drew (1975,/3/) also suggested such a pre-synthesis approach for the control vector u(k), but later discarded it, in view of the massive computer memory storage required. Moreover this approach is made more difficult, since values of vector u(k) at each time period, depends on the vector state x(k). It is therefore more practical to store the values of the operating matrix F, in the manner developed in this thesis. In addition, the structured property of canonical forms allows an easy and efficient search for the required control. It is noted that such feedback matrices are also determined by the number of production-inventory stages in the system and by the nature of the system configuration, e.g. serial, parallel or a combination of the two.

The dynamic stage then makes use of the above matrices in the actual iterative computations of the control exercise. The dynamic part can thus be run repeatedly for varying parameters such as limits on operating levels, sharp or sluggish control.

The iterative section consists of the following:

(i) Initialisation stage, where the relevant variables are set up and controllable matrices for the system obtained from external disc memory.

(ii) The iterative section consists of two main subroutines which calculate the responses of the system from the equations 2.8a and 2.8b.The one that computes 2.8a is referred as "@DIS/DYN" and is given in Appendix 4. Equation 2.8b is calculated using the feedback matrix as determined by the CPN. The feedback matrices as described earlier have been synthesised prior to the dynamic simulation

(iii) The output stage consists of a graphics subroutine for presenting the results. Here it is pointed out that the in-built graphic functions of the TEKTRONIX4052 computer have reduced the programming load substantially. An example of this subroutine is given in Appendix 5.

The structure of the overall simulation is given in Figure 2.5

2.2.6 Numerical Example:

The above development is now applied to the 4-stage MSPI currently considered. The ability to control individual inputs will be shown in two practical cases:

(i) Constraint in the availability of manufacturing facilities.

(ii) Constraint in the availability of buffer stocks.

A step input in demand of 250 units/time period at the final product is taken, i.e. d5 = 250. The problem is how to control the various individual stages to respond to the above disturbance.

For this illustration, CPN 25 is used for all stages, except at production stage 3, where CPN 23, 25 and 27 are used in 3 different runs. These results are shown in Figure 2.6. This figure consists of 4 pairs of graphs corresponding to the 4 production-inventory satges. Each pair of graphs represents two major features:

(i) Input capacity rate expressed in the number of units intended to be manufactured at each particular time period, one working shift.

(ii) The fluctuation of the inventory with respect to a certain arbitratry datum.

It is clearly demonstrated how the CPN provide the vehicle of controlling individual inputs. The effect of this individual control is also felt at the feeding buffer, i.e. stage 2. A higher value of CPN (as CPN 27) results in a slower build-up of resource requirements which depletes the feeding buffer only slightly. This will also mean that its own buffer will be depleted quite severely and takes a longer while to replenish. Conversely lower CPN (as CPN 23) responds with a sharper build-up of capacity requirements thereby depleting the feeding buffer drastically. On the other hand, its own buffer is depleted to a much lesser extent and is replenished faster. These features are clearly illustrated in Figure 2.6

2.2.6.a) Constraints in Input Variables.

In this exercise, the maximum operating level at stage 3 is taken as 378 units/time period (the maximum value as achieved with CPN 25), thère are economic and practical reasons to fully utilise the resource at this production stage. The current policy builds up gradually to a peak value before decreasing to a steady state level. An iterative algorithm named "@DIS/ICCR" is developed for this purpose and is shown in Figure 2.7 .

The essence of the algorithm is to find iteratively the CPN that will give the solution satisfying the input constraint (within an arbitrary range of tolerance) for each production-inventory stage. If the CPN provides a higher solution, the CPN is incremented by 1, and the new solution checked. Were the solution less than the limit, it is verified whether by decreasing the magnitude of the CPN will still provide an acceptable solution. This decrease in CPN value is also monitored to prevent a sharp response that would increase the transient effect. The iterative approach is analogous to the special branch of mathematics known as Numerical Analysis. Such an approach has become recently more established through the widespread use of digital computers.

The solutions of the new algorithm as applied to stage 3 only is shown in Figure 2.8, where the effectiveness of the algorithm is clearly demonstrated.

The resource at stage 3 is fully utilised at the initial period of the run. This results in substantial depletion of the feeding buffer and also in overproduction of parts. This is because the production at other stages has not been controlled to deal with the current situation. This exercise is to show the effect of controlling only

one stage so that the resource there is fully utilised. To be of practical value, an integration of the individual controls is obviously necessary and this is achieved by applying this algorithm at all stages. Thus, taking the values worked out from CPN 25 values, 441 , 409, 378, 347 as limits in the availability of the respective resources, the solutions would be in Figure 2.9 .
The total amounts of resources utilised at each production stage is the same for the two different runs, but the effects on the inventories are significantly improved. The relevance of the alogorithm "@DIS/ICCR" is very important practically, as will be shown throughout the rest of this thesis. An assessment of this feature is carried out in Section 2.3

2.2.6.b) Constraint in Inter-stage Buffers.

It is noted from the previous control simulation that no constraint in inter-stage buffers has been assumed. The runs are carried out with the assumption that these buffers are originally very large. From such an approach it is possible to state the required amounts of safe inventories. In an actual ca se where there is a finite amount of inventory, it can be shown how the ability to control individual inputs may be particularly relevant. A new algorithm, to be referred as "@DIS/ILCT" (Figure 2.10), is developed for this purpose. It is included as an additional subroutine in the overall control simulation as shown in Figure 2.11.
Supposing that the limits in inputs are as in the previous runs, a new constraint is introduced at the inter-stage buffer 2. Buffer values of 350, 250, 150 are chosen for three separate runs and the results are shown superimposed in Figure 2.12. The limits on the

buffer capacities at stage 2 are indeed respected, their effects on the subsequent production-inventory stage are also obvious.

A very small buffer at the stage 2 will cause a high depletion of inventory at stage 3, since there has not been enough parts to be worked at stage 3 in the original period of the time horizon.

In conclusion, the tractability of the different modes of the system through the CPN notation as derived from multivariable control theory has provided the ability to control individual inputs. The effectiveness of the algorithms developed are demonstrated in the synthesis of control within constraints. The practical implications in a manufacturing environment are substantial and will be discussed throughout the rest of this thesis.

2.3 Control Policies.

2.3.1 Structured Control Policies.

In this particular section, it is shown how some properties of the described new structured forms have further practical implications in manufacturing systems. In production control of MSPI systems, it is often required to control the acquisition of extra resources, and the reallocation of current resources and the interstage buffers. This is specially so, when the system is subjected to a step input in demand. In such a situation, it would be of practical and economic value if the control policies were formulated in such a way that they are "relatively proportional" to each other. By "relatively proportional", it is meant that the manufacturing resources (e.g. machine hours, labour hours) are allocated such that each interstage inventory is dynamically controlled towards a desired value. While this control is being effected, it still caters simultaneously for the demand in inputs of subsequent production stages. This is in order to achieve a co-ordinated balanced response of the system.

It should be mentioned that the control problem of MSPI currently studied and in particular the concept of structured control policies is to a certain extent related in some aspects to the line balancing problem. A simple definition of the line balancing problem will be the process of grouping operations into modules that have a relatively similar operation time. This exercise is usually carried out to investigate the efficiency of the system, typical features that are considered, include:

(i) Variability of the operation time.

(ii) Techniques in grouping the different manufacturing processes.

(iii) Effect of inter-stage buffers.

(iv) Failure rate and variation of repair time.

Some of the references specific to this field are /95/-/105/.

Whilst the above features are taken into consideration, the similarity stops here because the current analysis of MSPI is extended into a higher dimension, that of controlling the dynamic responses in the following policies.

(i) Requisition and reallocation of resources.

(ii) Inter-stage buffer control.

Both the control and dynamic nature of the problem is fully emphasised in the present thesis as opposed to the previous investigative simulation studies.

The objective of structured control policies is achieved by setting all the CPN's of the various modes to a similar value at the stage of selecting control policies. As described in Section 2.2.5, low values of CPN will give sharper response than those of higher values. The same basic manufacturing model described in Section 2.2 is used to illustrate the concept of the structured policies. It consists of 4 production-inventory stages linked in series as given in Figure 2.4 . Reject rates of 12.5%, 12.5%, 10% and 10% are assumed at the 4 production stages respectively with a final demand of 250 units/time period. A routine for calculating the cascaded effect of such disturbance is necessary and is given in Appendix 6 . The effect of this algorithm is to convert the reject rates (or conversely the efficiency rates) into absolute discrete units rejected on average, given a required operating rate. This will mean values of 50, 44, 31, 28 and 250 at the first, second, third, fourth and fifth entries of vector d(k).

2.3.2 Control Simulation Model.

It is stressed again that this modelling exercise is one with a control objective, i.e. it explicitly searches for control policies as opposed to previous "what-if" investigation. In this thesis, the exercise is referred to as control simulation.

The first part of the problem formulation is identical to the previous one and the same companion matrices as given in equations 2.7a and 2.7b are used. The dynamics of the runs are given in Figure 2.5.

The results of the control simulation for this system with CPN's 25 (RUN A1) are shown in Figure 2.13 . From these figures, it is observed how the capacity rates, at all production stages dynamically control and restore the inventories with very small amount of overshoot or undershoot. The practical significance of this feature are that the decision rules allocate the resources in such a way as to meet the various prevailing demands e.g.:

* Producing parts required for the next production stage.
* Replenishing the next buffer bank.
* Smooth restoration without overproduction.

Such an analysis, also provides a knowledge of the minimum amount of safe stocks required for such a problem.

At each production stage, there is a gradual build up in the requirements of resources towards a peak, then a subsequent scale down to the steady state. CPN of a higher order will give slower build-ups and lower peaks as demonstrated in Figure 2.14 where CPN

28 (RUN A2) is used. The same advantages listed above are obtained, except for the fact that the latter takes a longer time to reach the steady state.

In all cases, in the transient period there is the necessity to acquire extra manufacturing resources above the steady state values in order to replenish the inter-stage buffers. This control requirement may be achieved by releasing some facilities from some parallel production lines to the particular one needed. Another means would be by overtime or speeding up the production rate.

There is severe fluctuation capacity utilisation in the transient stage of the simulation. It is desirable in a practical environment that the production rates are smoothed. This may also permit the full utilisation of the production processes at the initial stage, (k=1).

2.3.3 Modified Control Model for Structured Control Policies.

In order to improve the control response, the mathematical model is modified to make more beneficial use of the available capacity. The structure of the new approach is exhibited in Figure 2.15 . The analysis undergoes a 2-step process:

(i) The first stage is similar to the original approach until the peak values are found after a short time horizon test run.

(ii) Once they are found, they are recorded and the simulation is restarted from time period k=1 using the peak values as limits.

Therefore the new run makes use of the subroutine "@DIS/ICCR" described in Section 2.2 to synthesise contol policies that are within the synthesised limits. This new approach will be referred to as the "RESET" technique in view of the fact that the simulation run is carried out twice: After a test trial the index for time period k is "reset" to unity again (k = 1) for the actual run. Moreover in the actual run, the subroutine "@DIS/ICCR" uses a resetting approach of the CPN in the search for appropriate control policies.

The results of the new approach are shown in Figures 2.16 and 2.17 for CPN 25 and 28 respectively (RUNS A3 and A4) where they are drawn out superimposed on the results of the previous approach. It is clearly seen that the resources are utilised in their maximum availability as well as having their previous fluctuations smoothed. These lead to a faster recovery of the inter-stage buffers. Thus, for example, the inventory level at stage 3, Figure 2.17 (for CPN 28) that took 24 time-periods to recover, currently needs only 9 time-periods. A very small amount of overshoot is noticeable, but the steady state is soon reached. The safe inventories required under the new approach are usually slightly higher (4 - 12 %), except for the last stage. Table 2.1 gives a summary of the results showing the maximum capacity rates and the safe inventories needed for RUN A1-A4.

2.4 Cost Benefit Analysis.

2.4.1 Development of a Cost Structure.

In order to assess quantitatively the practical significance of the various control responses of the system, a cost structure is developed. Such a costing procedure can thus provide a cost-benefit analysis for the different control approaches to be analysed and adopted during the course of the research. Three features of the control responses are analysed, and they are:

J1, Costs of Extra Inputs,

$$= \sum_{k=1}^{T_s} P\{(U(k) - Us)^{[2]}\} \qquad \text{---- 2.9}$$

J2, Costs of Inventory Held,

$$= Q\{(M)^{[2]}\} \qquad \text{---- 2.10}$$

J3, Replenishment Delay Penalty Cost,

$$= \sum_{k=t_r}^{T_s} R\{I(k)^{[a + 0.1(k - t_r)]}\} \qquad \text{---- 2.11}$$

where

[ ]		upper square brackets contain the exponent to which all elements are raised.
m	-	Number of production stgaes in system.
U(k)	-	m X 1 column vector for control input at time (k).
Us	-	steady state value of U(k).
Ts	-	Time at which steady state is reached.
P	-	weighting matrix for U(k) (1 X m)
M	-	m X 1 column vector for inventory held.
Q	-	weighting matrix for M. (1 X m)
I(k)	-	m X 1 column vector that need to be replenished at the inventories.

t_r - time period in which replenishing starts.
R - weighting matrix for I(k) (1 X m)
a - arbitrary constant.

The absolute magnitude of the values of the weighting matrices P, Q and R are chosen such that each respective resulting cost, namely J1, J2 and J3 is on a similar scale, i.e. the costs have the same units. This is done so as to make inter-comparison possible. The choice of these values is effected from a combination of both their financial values where appropriate and their relative importance as viewed by the management.

a, Costs of Extra Inputs, J1.

This is the cost associated with the extra capacity required to restore the depleted inventories to the original levels while simultaneously providing for the next production stages. At the initial start-up, inventories are usually severely depleted by the downstream production stages which are also operating at the above normal level. A quadratic cost function instead of a linear one is used so as to penalise high excess values of inputs. This is to emphasise the higher costs involved at high values. This can be easily shown in Figure 2.18 where a linear and a quadratic function are shown. While this cost function may not be a real one, the practical implication is important: it highlights the intention of avoiding the additions of manufacturing resources such as excessive overtime, speeding up production lines drastically, subcontracting, etc.

In addition to the economic disadvantage of excessive overtime, measures to increase or fluctuate the production rates will find very little acceptance from the workforce. The quadratic cost approach has been adopted to a very large extent in optimal control theory e.g. quadratic optimisation technique for synthesis of feedback. A more common example of this approach is in the minimisation of least squares techniques. Some workers in production control (e.g. Christensen and Brogan, 1971,/1/; Drew, 1975,/2/) have also adopted such an approach. Moreover it will be practically very difficult to determine such costs explicitly, given the fact that details as exact availability of production units, skill of individual operators, bonus schemes etc, have to be taken into account.

The values of the weighting matrix P are arbitrarily chosen as $[6 \quad 10 \quad 12 \quad 14]$. The increasing magnitude is to reflect the increased importance associated with the input variables as they approach the final production stage.

b, Costs Of Inventory held, J2.

It is to be noted that the simulation runs are carried out on the assumption that no limits exist at the interstage assembly buffers. From these runs, it is possible to know the minimum amount of buffers required for the different scenarios in the availability of resources. A quadratic cost function is again used to represent the intention of avoiding excessive holding of inventories. It will be noticed that the time during which the inventories are actually held, is not included in the equation. Since all the inventories have to be kept for the same hypothetical length of time before the disturbance, the values of Q are assumed to have incorporated this feature. In this particular case, the weighting matrix Q is chosen as :

$$[3 \quad 3 \quad 3 \quad 4]$$

c, Replenishment Delay Penalty Costs, J3.

This is the penalty cost attributed to the depletion of inventories. While it is appreciated that the role of holding assemblies is to act as buffers, it is also required to replenish them so as to face the next change in operating conditions or in re-starting of production lines. Such need to replenish the buffers is also vital in view of the other stochastic disturbances that perturb the manufacturing system. Therefore the cost function is structured in such a way that it varies with two parameters.

(i) The relative amount left to be replenished, i.e. the ratio of the absolute value to the desired value. The larger this value is, the higher the penalty cost will be.

(ii) The time elapsed in a depleted state. This is achieved by the structure of the exponent $a + 0.1(k - t_r)$ which increases with k, the time period, i.e. it continuously increases the penalty related to the delay in replenishment.

It will be noticed that large injections of capacity at various production stages will also mean more severe depletion of the preceding buffers and longer delays in replenishing. Low inputs of capacity will lead to the same results at the succeeding stages. This situation will then create an opportunity to find a local sub-optimum in the various strategies of control.

The weighting matrix, R used is : 1 2 3 4

The increasing magnitude of the values is to represent the higher importance given to the buffer with increased work content.

It has to be mentioned however that this cost J3 has been one which has previously received least attention by management as compared to cost functional J1 and J2, and still is. Cost functionals J1 and J2 associated with extra capacity requirements (e.g. overtime, float labour, etc) and buffer inventories are concepts realistically compatible to the production control management. This is so because they are more readily assessable in direct financial values while J3 is a "hidden" cost corresponding to the idle labour as a result of parts shotage, lost sales, and decline in goodwill. Parts shortage arises when one production stage has to stop operating because the feeding buffer is out of parts. Lost sales and decline of goodwill are also penalties that are difficult to assess directly, and have therefore been largely ignored by industrial management. The cost structure developed for this replenishment delay is believed to be original and has been considered to be representative of practical circumstances by industrial management of the companies consulted during the course of this work.

The above cost structure developed is applied to the runs A1 - A4 and the results are given in Figure 2.18a - 2.18d

2.4.2 Assessment of Revised Structured Control Policies.

a, Cost of Extra Inputs.

Comparing the costs A1 with A3 and A2 with A4, it is observed that the results of the "resetting" technique give a higher cost of 16% and 9% respectively. It is noted that the actual aggregate requirements of resources within the same time horizon are exactly the same. This increase in costs is due to the cost structure which favours a more widespread distribution of resources. The restting technique which attempts to maximise the utilisation of resources will therefore lead to a higher concentration of resource requirements at the beginning of the runs. This state of affairs will obviously be penalised by the structure of the cost involved.

b, Cost of Holding Inventory.

For this particular cost item, there is a marked improvement in the costs, 14% and 30% for CPN 25 and 28 respectively. This is in spite of the fact that 3 of the inventories as required under the resetting technique are slightly higher than with the non-resetting approach. It is the inventory of the finished product that is drastically improved in the revised approach. (Figure 2.19b)

c, Replenishment Cost.

Here the improvement obtained is much more significant because the cost is structured such that it penalises two features: the states of depletion and the length of time actually in the depleted state. This resetting policy, in maximising the available resources, drives the inter-stage buffers rapidly to their desired levels. It is also seen that the improvement in costs J3, if compared directly with the other costs J1 and J2, is far above both of them. (Figure 2.19c) d,Total costs.

The total costs of the three above cost components is given as :

$$J = J1 + J2 + J3 \qquad ---- \quad 2.12$$

The results are as shown in Figure 2.19d . The improvements obtained from the resetting technique are shown to be substantially positive.

2.4.3 Modified Cost Structure.

Some parts of the costing approach described above, i.e using a direct weighting method of the absolute values, has been used to a limited extent by a few workers who have tried to present a cost-benefit analysis to their respective approach, e.g. Fey (1961,/33/). The global structure as presented in this thesis is believed to be original. Nevertheless some practical difficulties may be encountered in adopting such a cost structure. Whilst it is relatively easy to compare the cost benefits of one cost component e.g. the results of J3 from one run to those in another runs, this is not particularly so for the inter-comparison between different cost components. Thus the decision as how much importance is to be given to cost J1 as compared to costs J2 and J3 is quite subjective and may prove to be very difficult to assess in practice. This is because of the fact that in this particular case, each individual cost is not only a financial measure of a certain event, but a combination of both the financial value and the subjective importance attributed to the particular event. The use of weights and the structure of the cost equation has to a certain extent alleviated this problem. In order to further reduce this problem of intercomparison, the different variables are considered as ratios of their respective steady state values as opposed to their absolute values. It is believed by the current author that this new approach will allow a more realistic framework of inter-comparison for actual practical purpose.

The new cost structure is presently given as:

$$\text{Total cost} \quad J = J1 + J2 + J3. \qquad \text{---- 2.13}$$

$$J1, \text{ Costs of Extra Inputs} = \sum_{k=1}^{T_s} P\,\{(U(k) - Us)_n^{[2]}\} \qquad \text{---- 2.14}$$

$$J2, \text{ Costs of Inventory held} = Q\,(M)_n^{[2]} \qquad \text{---- 2.15}$$

$$J3, \text{ Replenishment Delay Cost} = \sum_{k=t_r}^{T_s} R\,\{I(k)_n^{[a + .1(k-t_r)]}\} \qquad \text{---- 2.16}$$

where

[ ]		upper square brackets contain the exponent to which all elements are raised.
$(\ )_n$	-	The elements of the vector are normalised, divided by their steady state values.
U(k)	-	m X 1 column vector for control input at time (k).
Us	-	steady state value of U(k).
Ts	-	Time at which steady state is reached.
P	-	weighting matrix for U(k) (1 X m)
M	-	m X 1 column vector for inventory held.
Q	-	weighting matrix for M. (1 X m)
I(k)	-	m X 1 column vector that need to be replenished at the
R	-	weighting matrix for I(k) (1 X m)
t_r	-	time when production stage starts replenishing

Here again, the absolute values of weighting matrices P, Q and R are chosen such that each respective resulting cost is on a similar scale. This is to make the inter-comparison of the different costs possible.

The new cost structure is used to compare the responses of run A3 and A4 against A1 and A2, i.e. the modified and non-modified control responses, and the results are given in Figure 2.20 .

a, Cost of Extra Inputs

In an attempt to alleviate the difficulty of deciding on the weights assigned to the heterogeneous variables as the absolute extra capacity, a normalisation approach is adopted. The fluctuations are now considered in their relative ratios with respect to their individual steady state values. The resetting approach still gives a higher cost for both CPN 25 and 28, although the latter one is marginal 6% as compared to 18% in the former one. This is illustrated in Figure 2.19a. It is again mentioned that the amount of extra capacity actually allocated in the time horizon considered is similar, it is the way they are distributed that are currently assessed. The increase in costs is explained by the fact that the modified control response makes use of more extra capacity to which a higher cost is attached than in the non-modified response.

b, Costs of Inventory Held, J2.

In this case, the "normalisation" procedure consists of dividing the values of the inter-stage buffers are divided by the steady state values of the next downstream production stages. The resetting technique provides improvements of 19% and 35% for CPN 25 and 28 respectively as shown in Figure 2.20b c, Replenishment Delay Costs, J3

In this particular case, the cost function considers the dynamic inventory level as a fraction of the safe inventory that has to be held at each stage. The improvements are quantified as 41% and 47 % for CPN 25 and 28. (Figure 2.20c).

d, Total Costs, J

The results of the total costs are given in Figure 2.19d.
It is shown from this Figure that practical and economic benefits are indeed possible from the modified control model. This approach has been the one adopted during the rest of the research.

2.5 Conclusion.

In Sections 2.1 and 2.2, the earlier developments of control theory as applied to the control of manufacturing have been considered in both synthetic and practical cases. Multivariable control theory as developed by Porter et al (1976,/3/) has been analysed and further extended. This theory is based on the arbitrary pole-assignment technique to the control canonical forms of the system matrices. It has been shown in this chapter how the structured property of the control forms may be of practical significance in its ability to allow control of individual modes of the system responses. An original approach is adopted whereby discrete policies synthesised from discrete values of eigenvalues are used. These discrete control policies are referred to as CPN (Control Policy Number) in the analysis. The practical significance of the above approach presents the possibility of effecting control measures that can take into consideration the various practical constraints that exist in the manufacturing environment. These are namely :

(i) Constraint in individual input variables.

(ii) Constraint in inter-stage buffers.

In Section 2.3, the concept of structured control policy has been introduced by the use of a uniform CPN for all the modes of the system. This results in a co-ordinated and balanced injection of resources and the usage of the inter-stage buffers. Specific algorithms have been developed to achieve this co-ordinated control response in a practical environment of manufacturing.

In order to assess the cost-benefits of the various control strategies derived from the different approaches, a cost structure is developed in Section 2.4. The analysis considers the costs of extra inputs, inventory held, and the replenishment of buffers. The cost equation identifies the "resetting" algorithm as more effective and cost-economic alogorithm than the one developed in Section 2.3 Finally, the cost structure is further modified to deal with "normalised" values as opposed to the absolute values. For various reasons, as inter-comparison of the different cost components, and the fact that the costs are based on both financial values and relative importance of the events per se, the "normalised" cost structure is considered as the one more suited for the present purpose.

CHAPTER 2

FIGURES

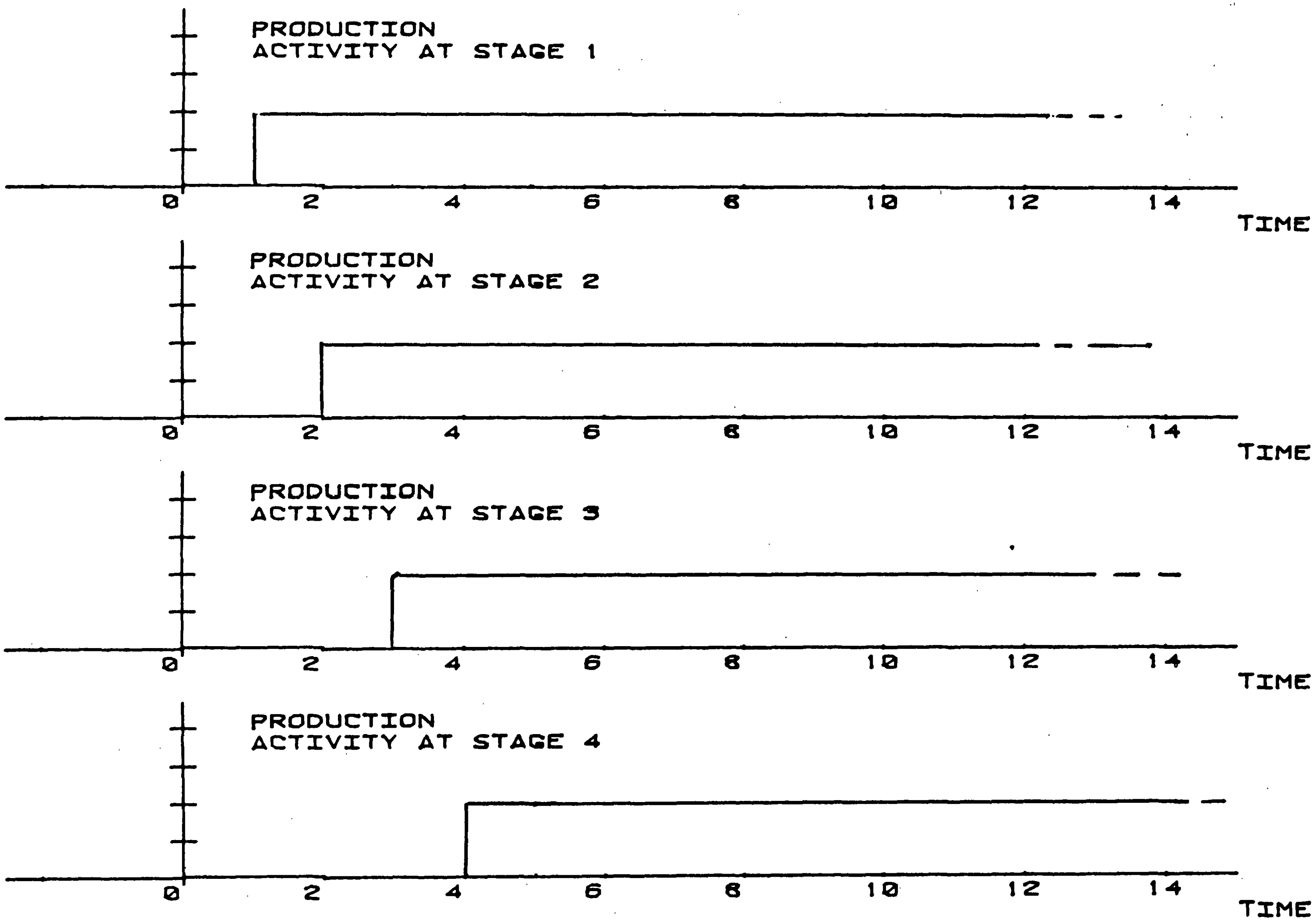


FIGURE 2.1a : CASCADED PRODUCTION START-UP

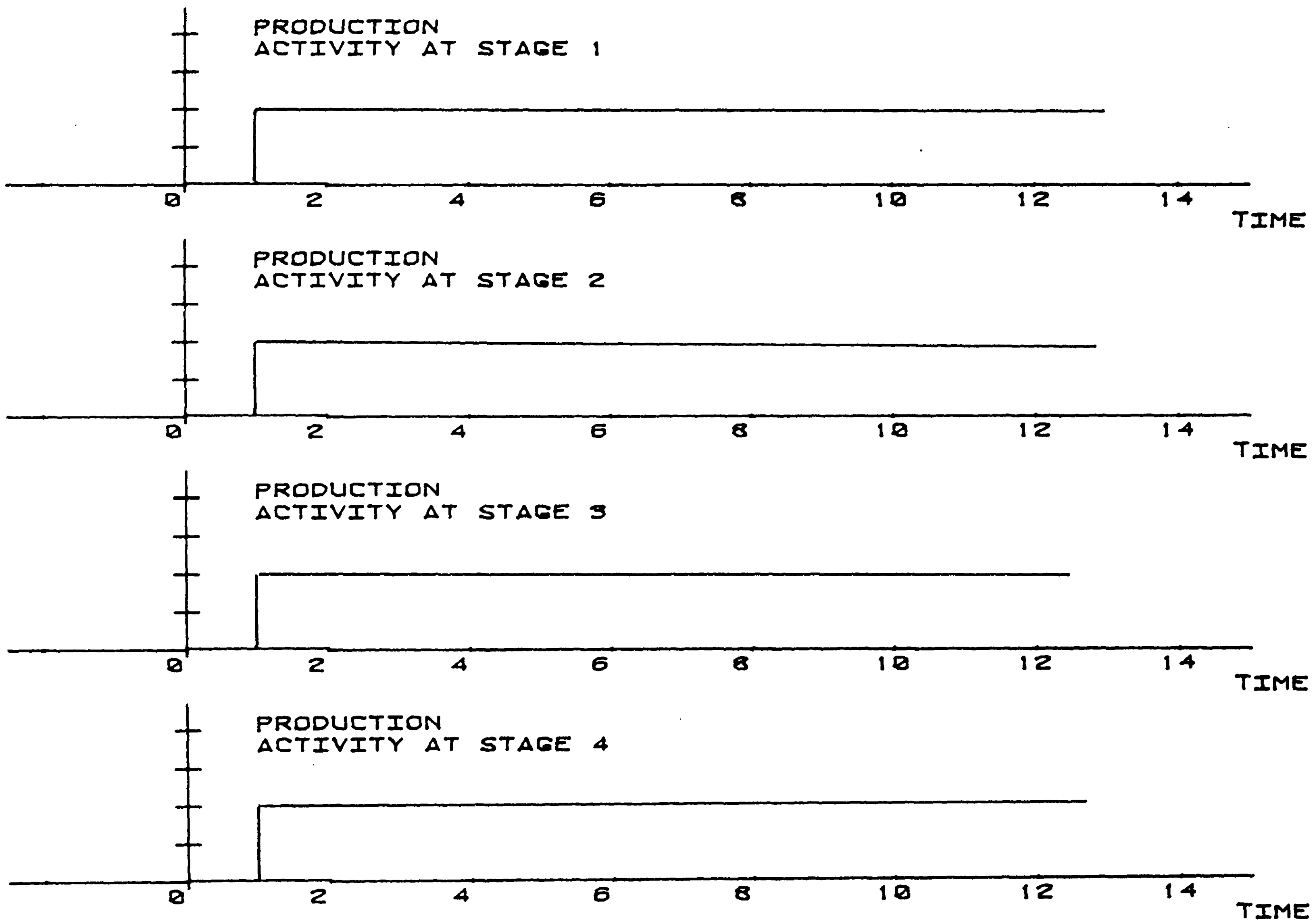


FIGURE 2.1b : SIMULTANEOUS PRODUCTION START-UP

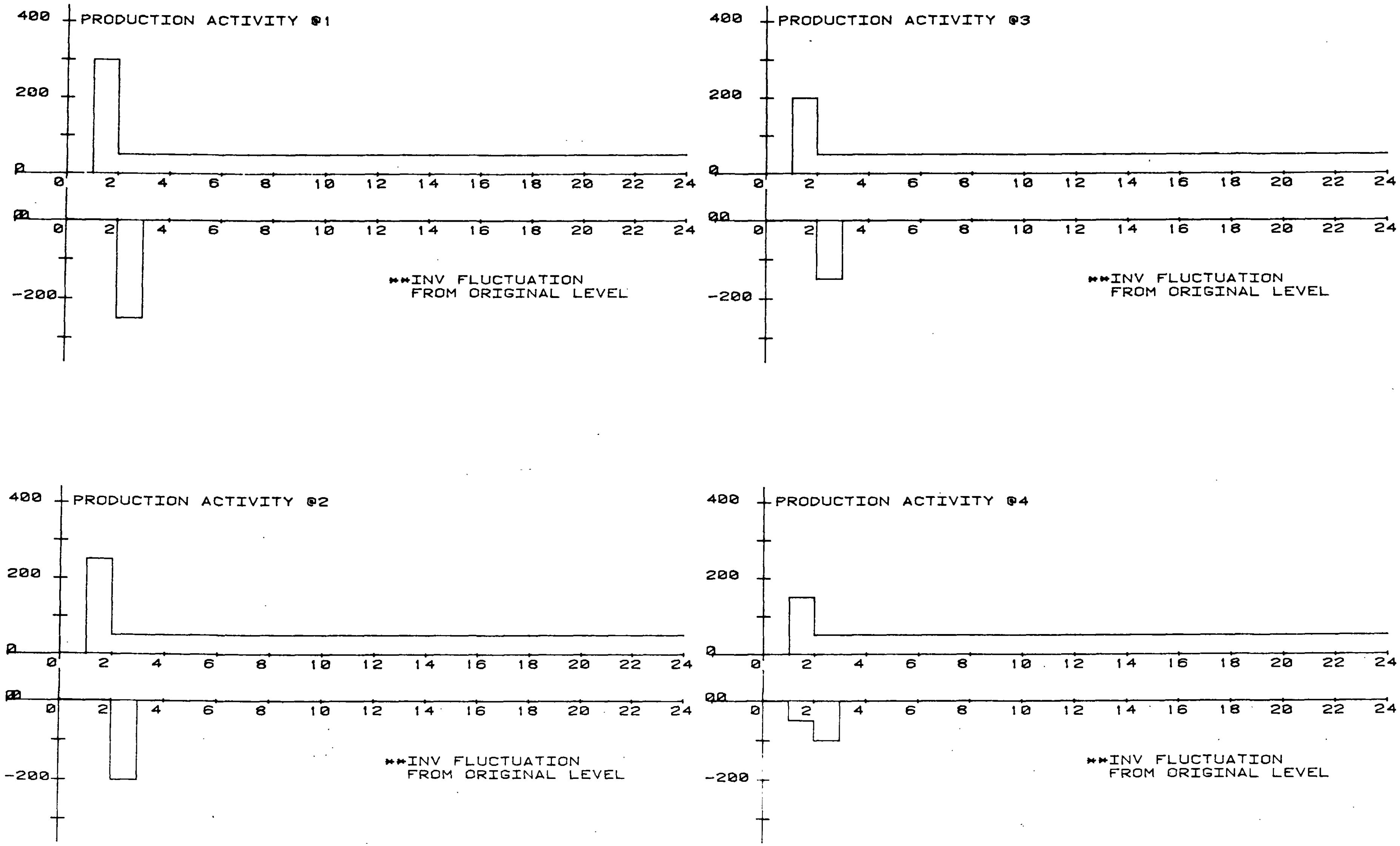
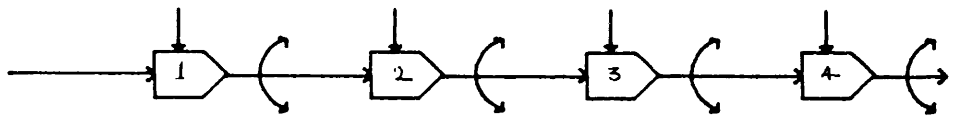


FIGURE 2.1c : CONTROLLED PRODUCTION START UP



WHERE

PRODUCTION STAGE — RESOURCE INPUT

SEPARATE DEMAND OR TO INVENTORY — FEED INTO NEXT PRODUCTION STAGE

FIGURE 2.2a : FOUR-STAGE PRODUCTION-INVENTORY SYSTEM

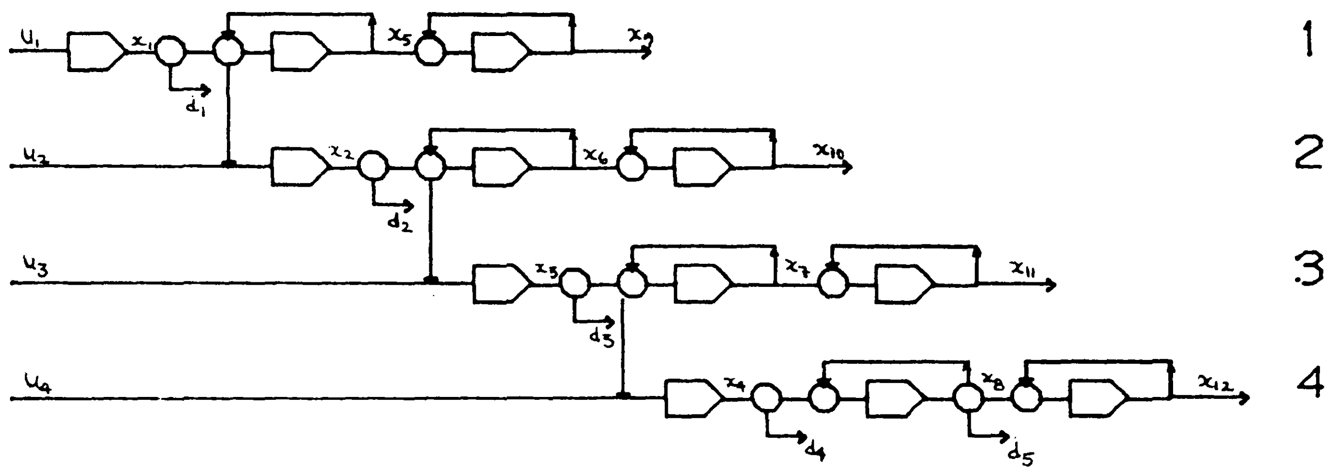


FIGURE 2.2b . MATHEMATICAL REPRESENTATION OF FOUR-STAGE PRODUCTION-INVENTORY SYSTEM.

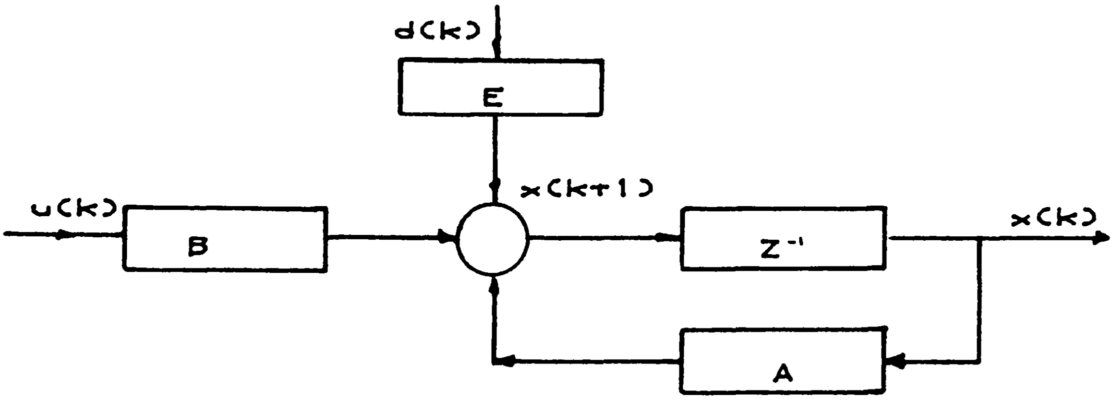


FIGURE 2.3 : REPRESENTATION OF $x(k+1)=Ax(k)+Bu(k)+Ed(k)$

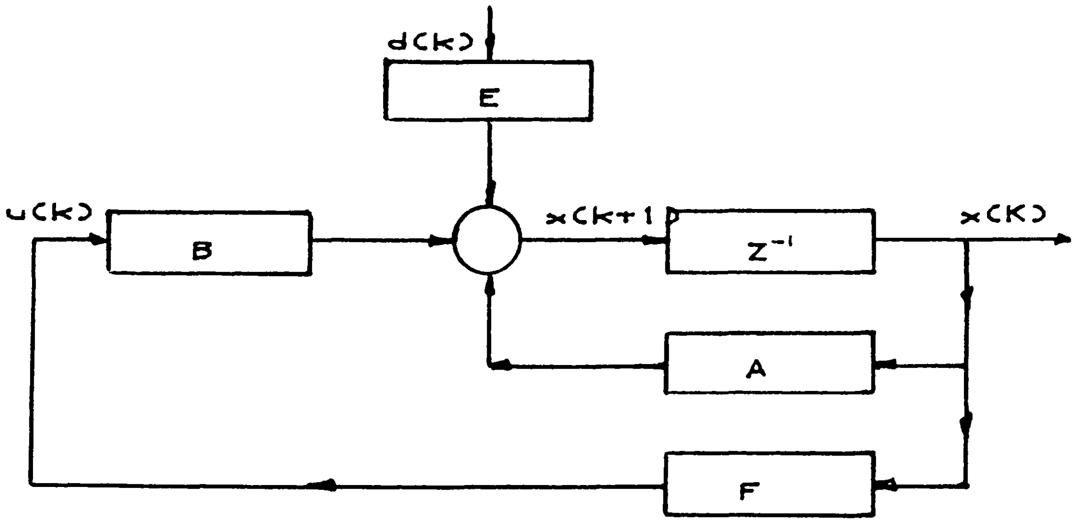


FIGURE 2.4 : ADDITION OF FEEDBACK CONTROL .

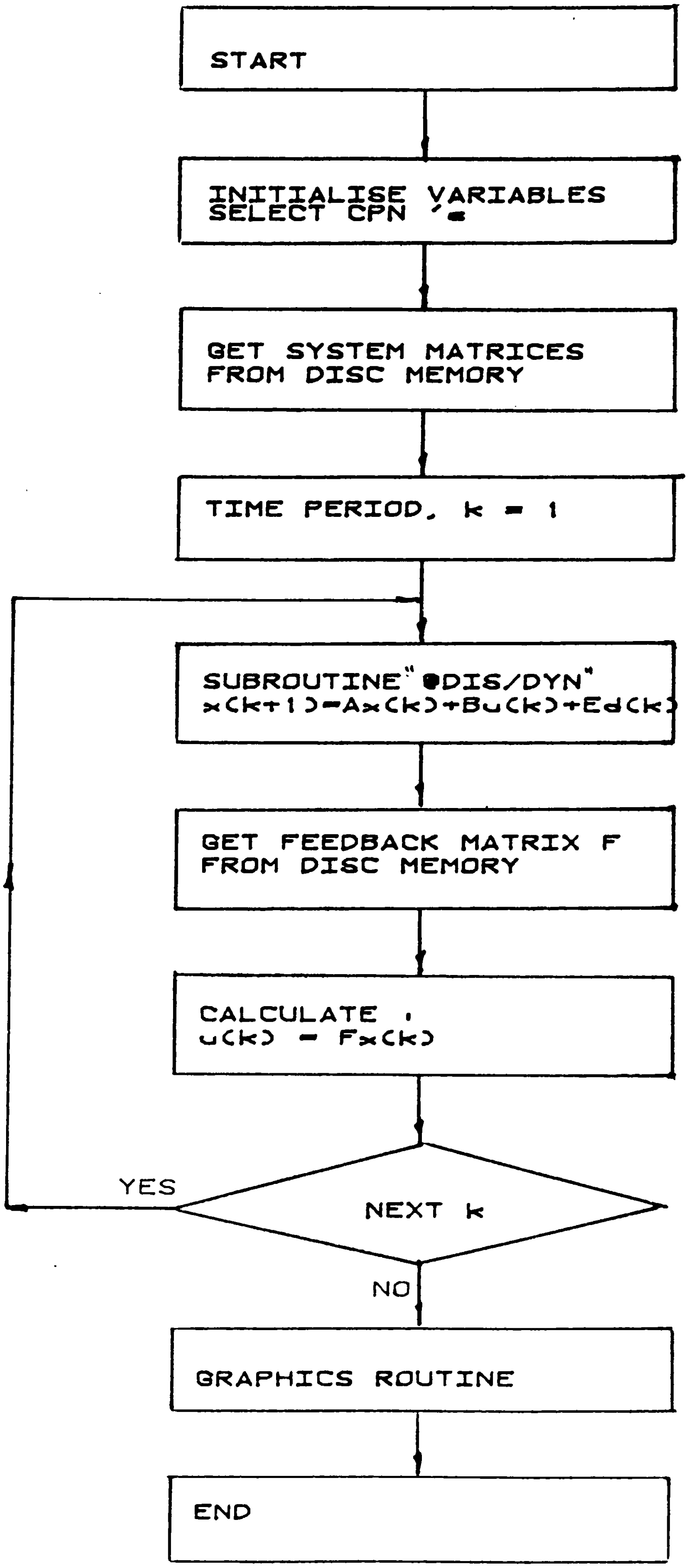


FIGURE 2.5 : OVERALL SIMULATION (A)

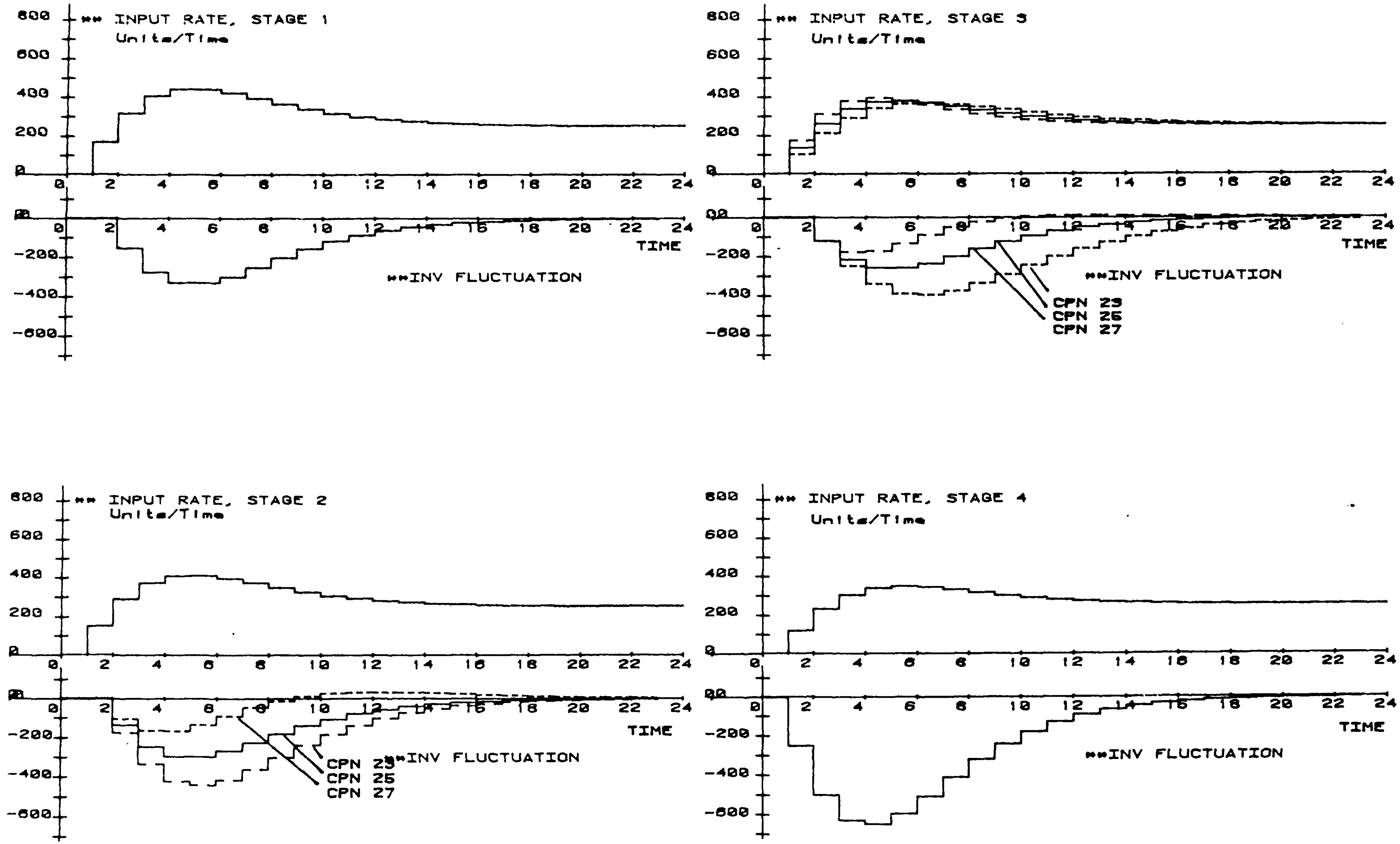


FIGURE 2.6 : EFFECT OF CPN 23, 25 & 27 AT STAGE 3

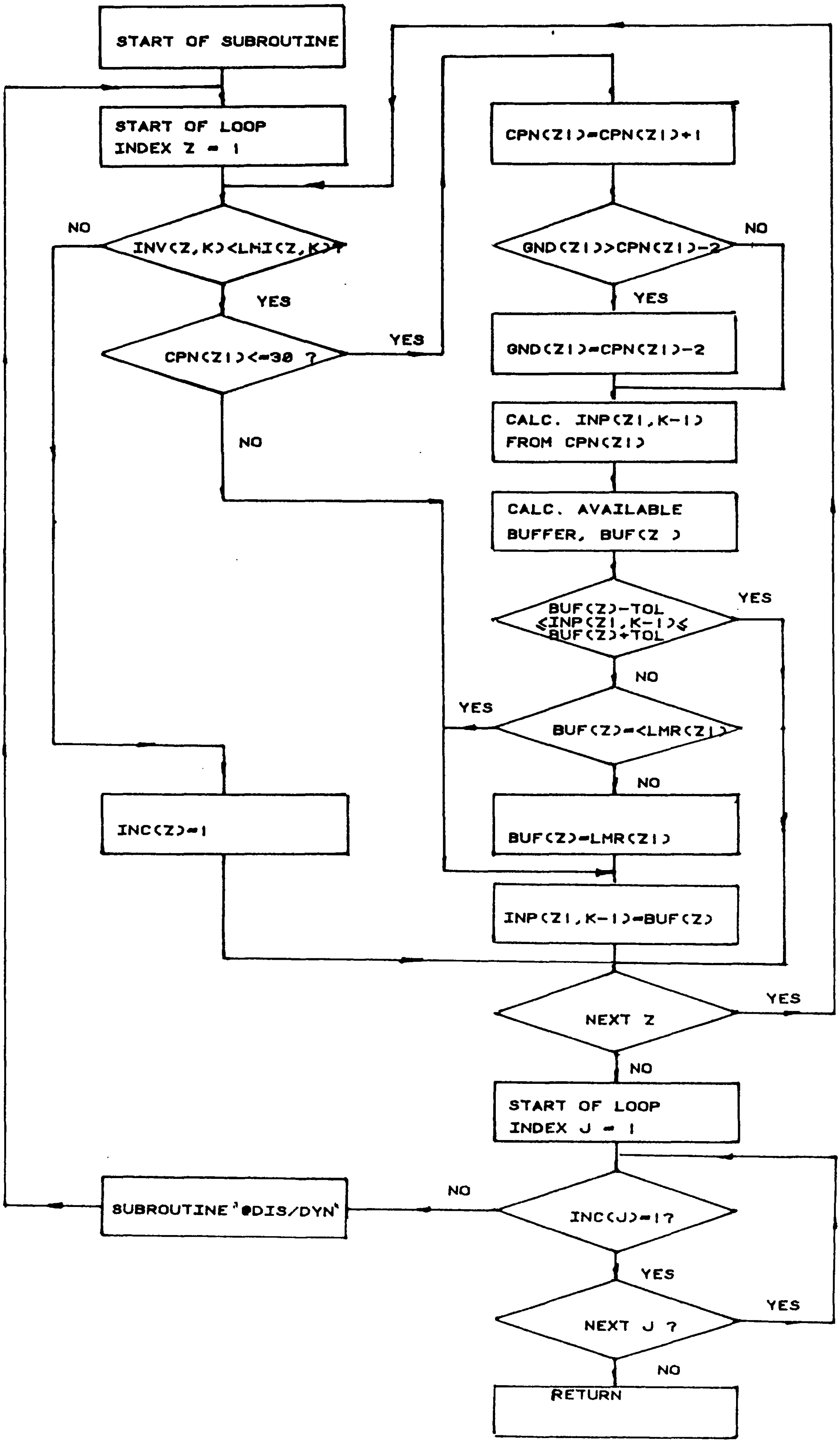


FIGURE 2.7 : SUBROUTINE "@DIS/ICCR"

Key to Abbrevations of previous subroutine.

k	:	Index for time-period.
Z	:	Index for Production-Inventory stage.
Z1	:	Index for next immediate downstream Production-Inventory stage, Z1 = Z +1
BUF(Z)	:	Available buffer at stage Z.
CPN(Z)	:	Control Policy Number at srage Z.
GND(Z)	:	Lowest value CPN is allowed to take at stage Z.
INC(J)	:	Indicator at J, =1 if inventory constraint is respected.
INP(Z1,k)	:	Input capacity at stage Z1, time = k.
INV(Z,k)	:	Current inventory at stage Z, at time k.
LMI(Z,k)	:	Inventory limit at stage Z, time k.
LMR(Z1)	:	Resource constraint at stage Z1

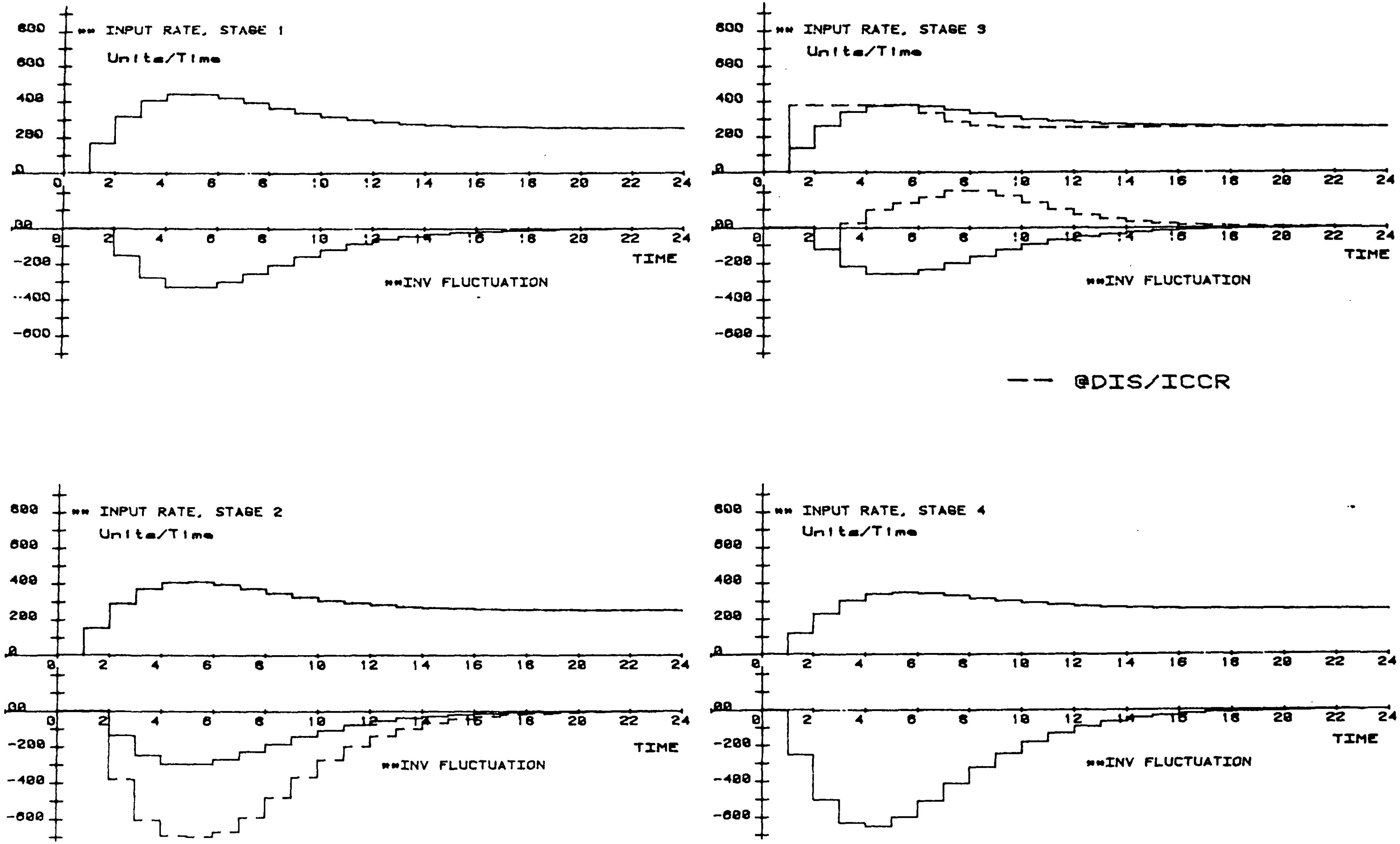


FIGURE 2.6 . RESULTS OF "@DIS/ICCR" AT STAGE 3

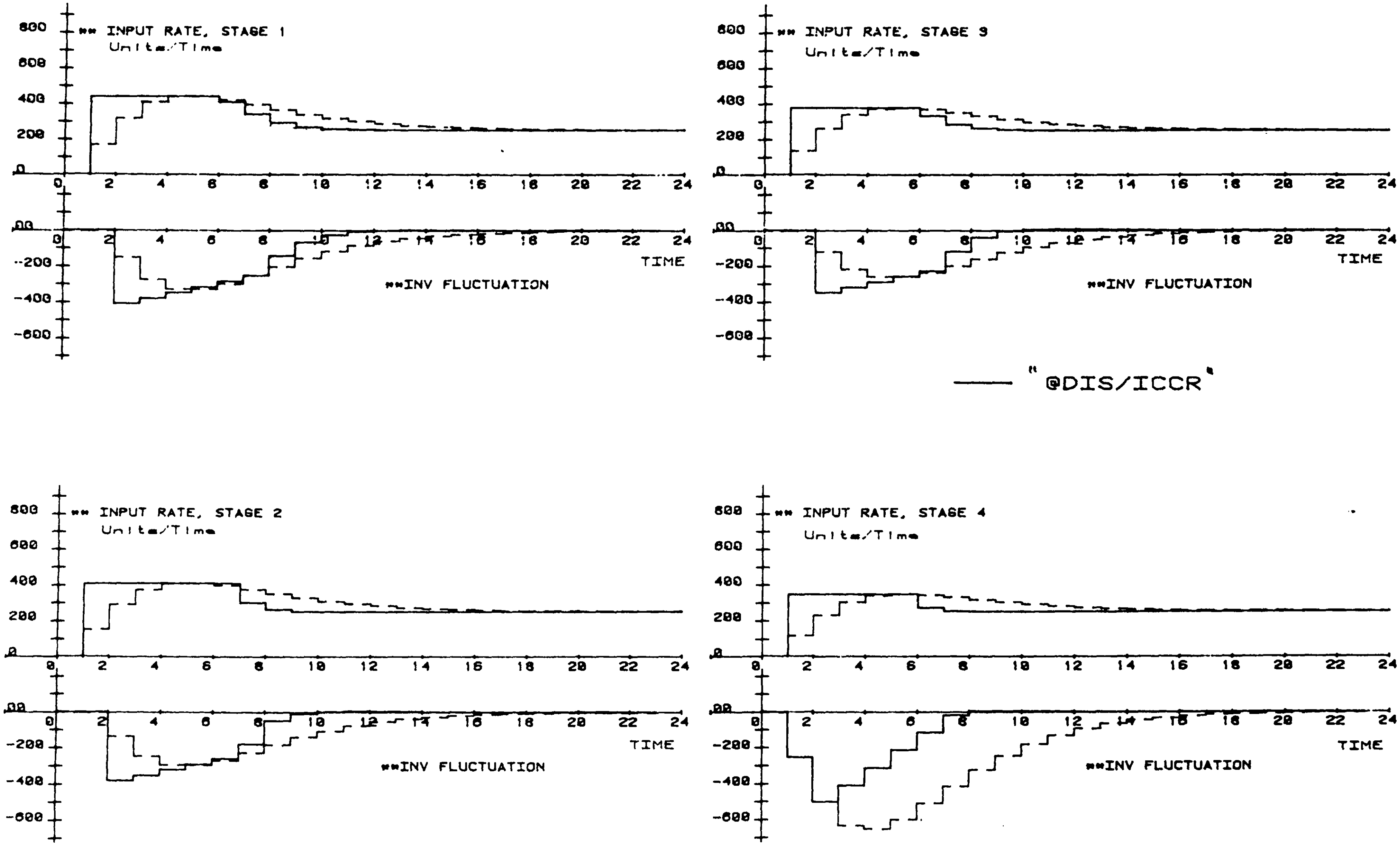


FIGURE 2.9 . RESULTS OF "@DIS/ICCR" AT ALL 4 STAGES

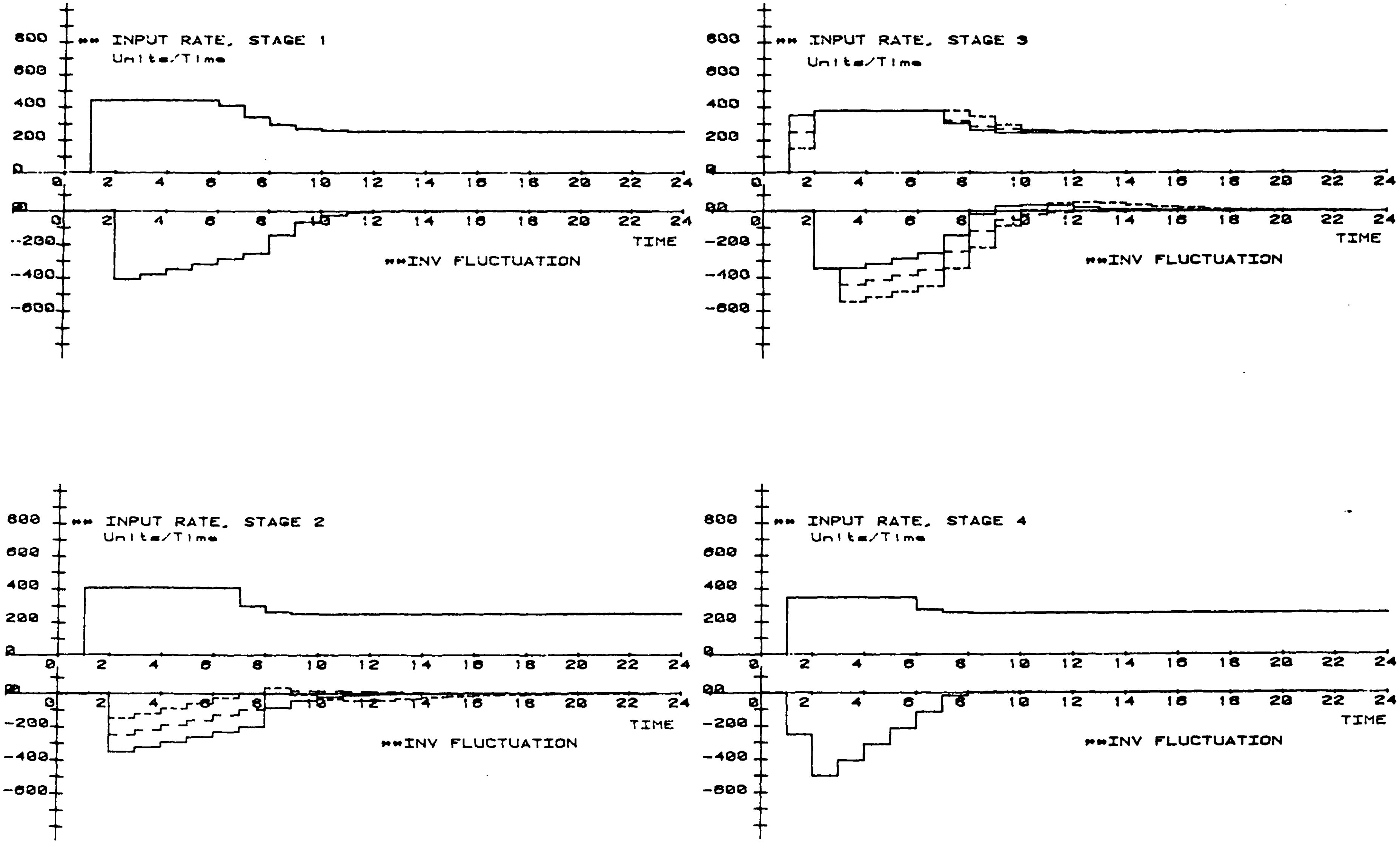


FIGURE 2.11 : RESULTS OF "DIS/ILCT", INVENTORY CHECK

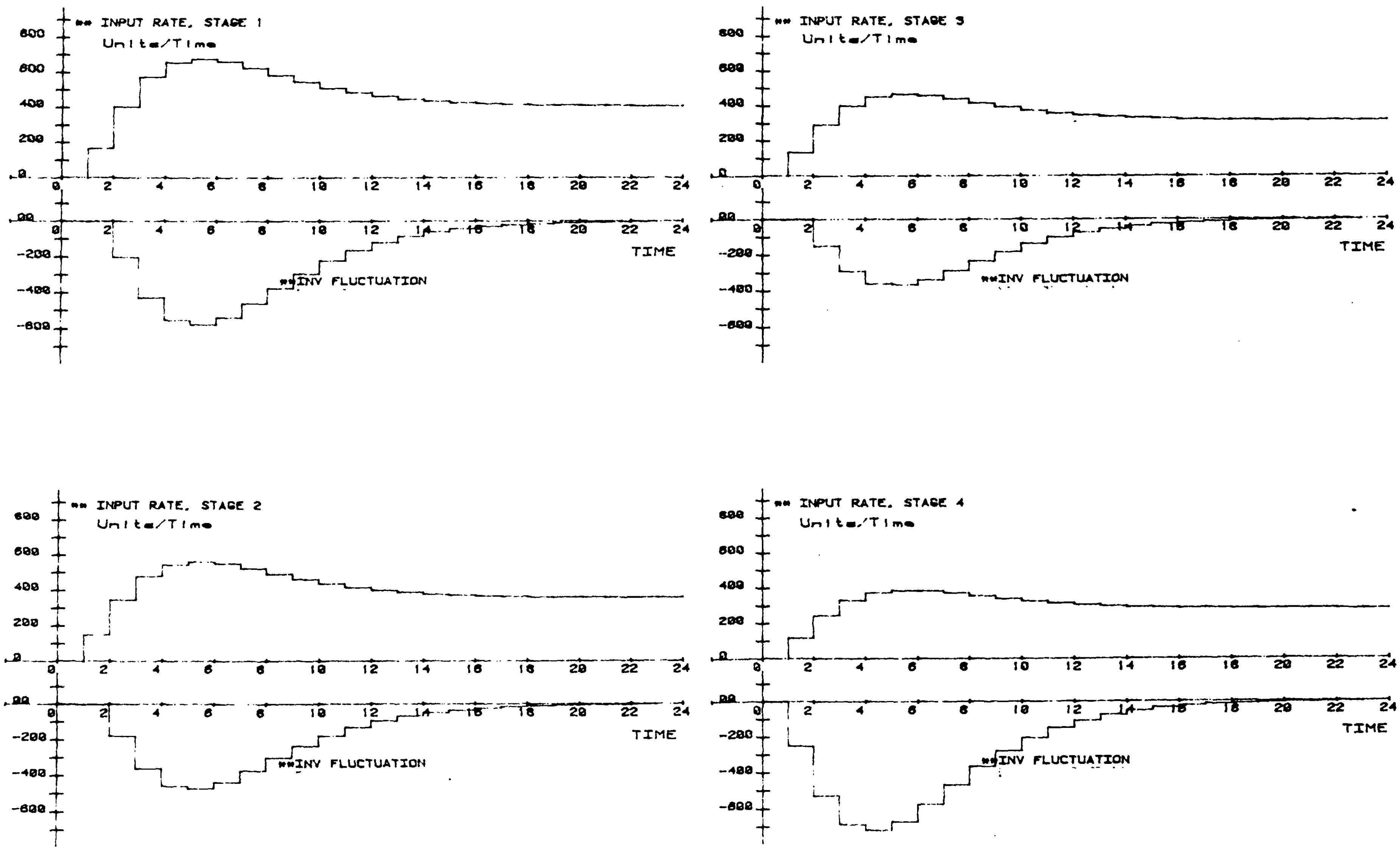


FIGURE 2.13 . RESULTS OF CONTROL SIMULATION WITH CPN 25,RUN A1

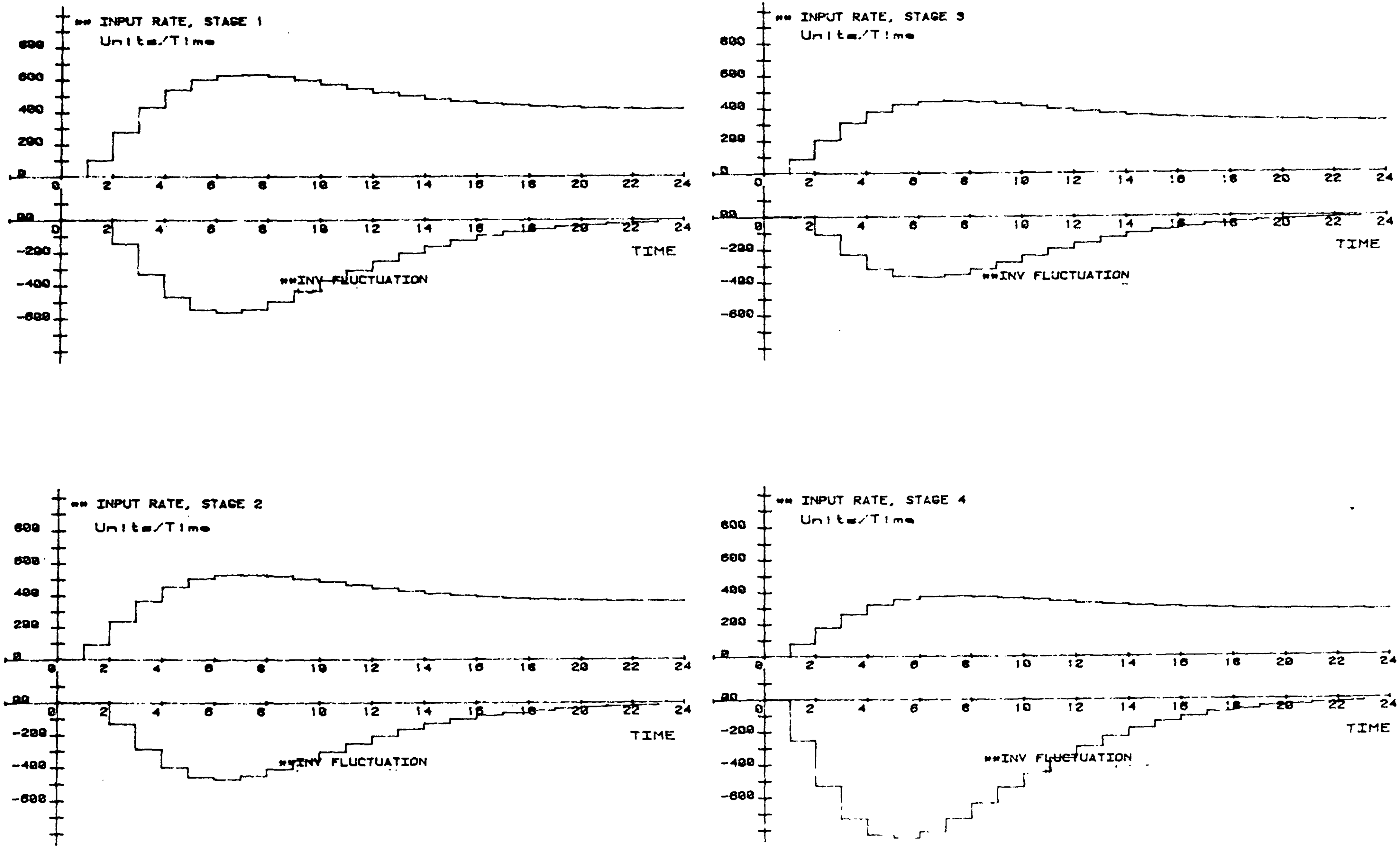


FIGURE 2.14 : RESULTS OF CONTROL SIMULATION WITH CPM 25, RUN A2

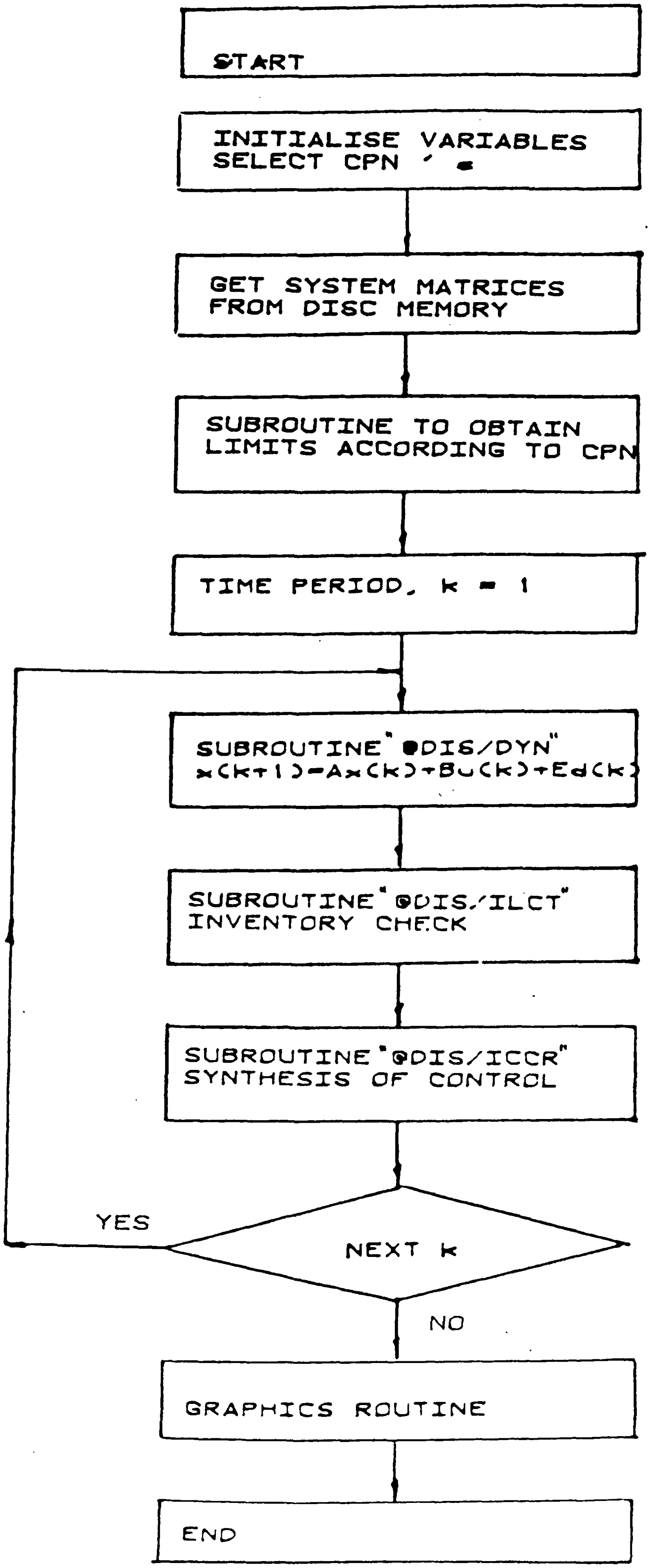


FIGURE 2.16 : OVERALL SIMULATION (C)

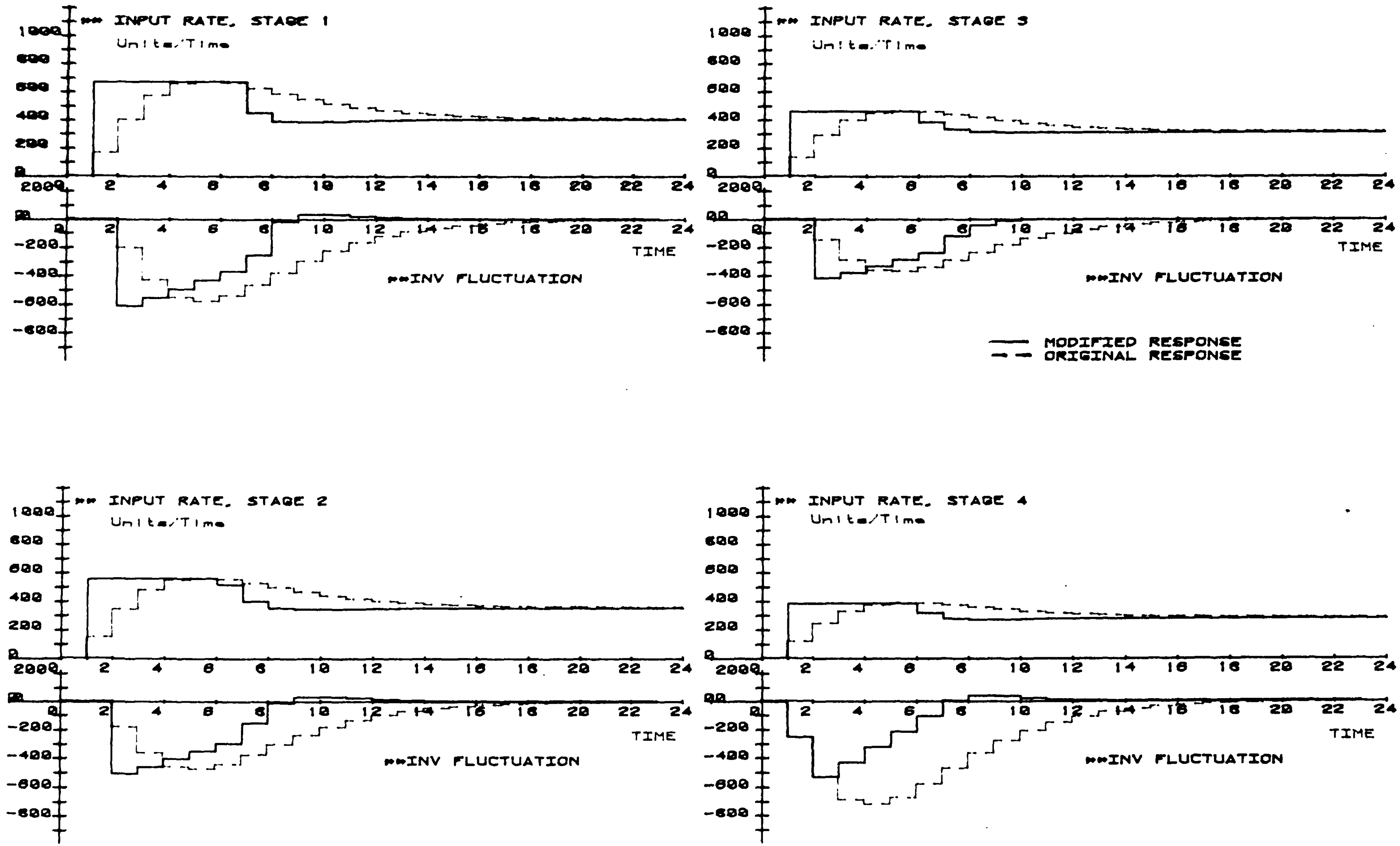


FIGURE 2.10 : RESULTS OF MODIFIED CONTROL MODEL WITH CPN 25, RUN A3

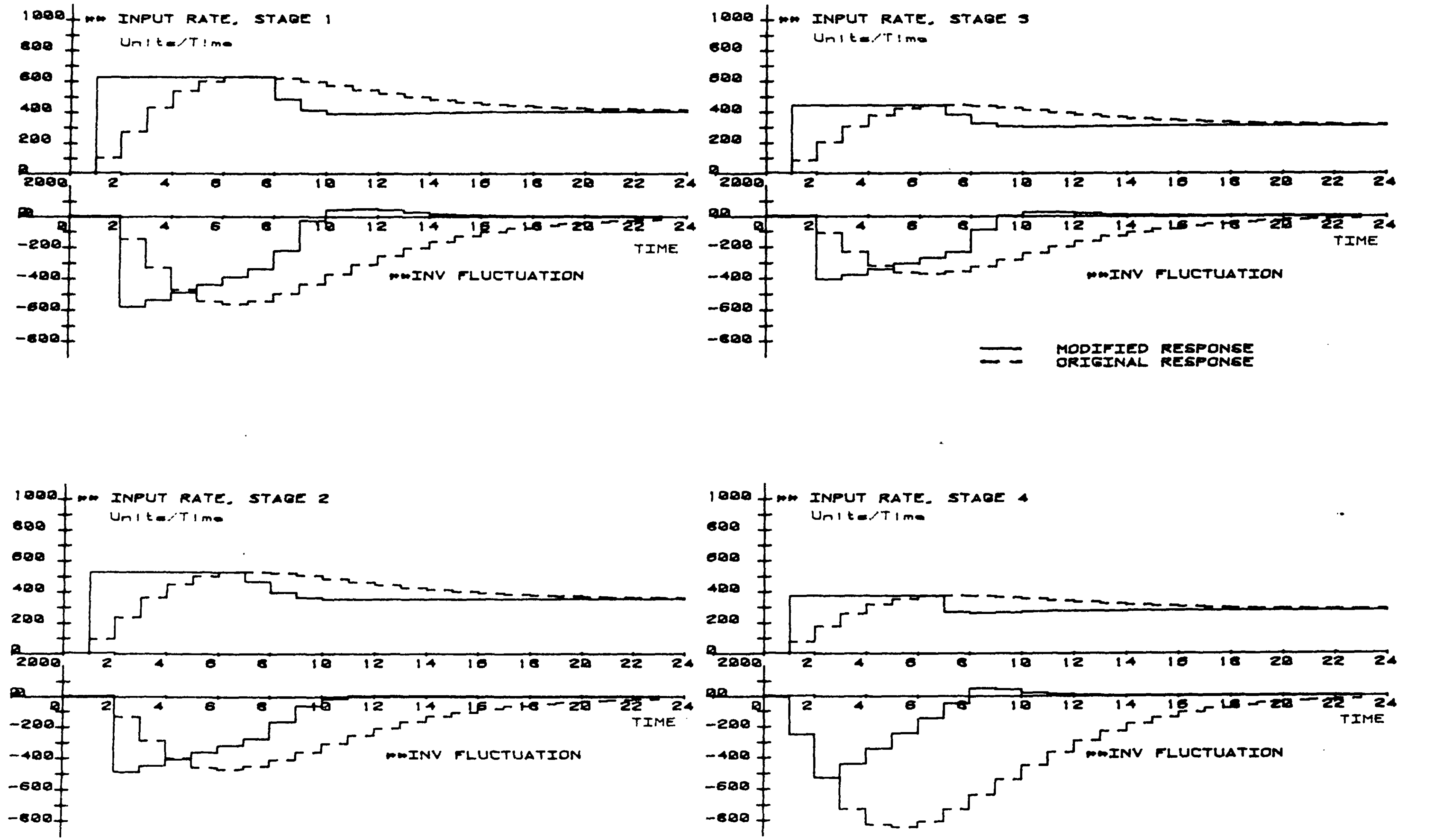


FIGURE 2.17 : RESULTS OF MODIFIED CONTROL MODEL WITH CPN 28, RUN A4

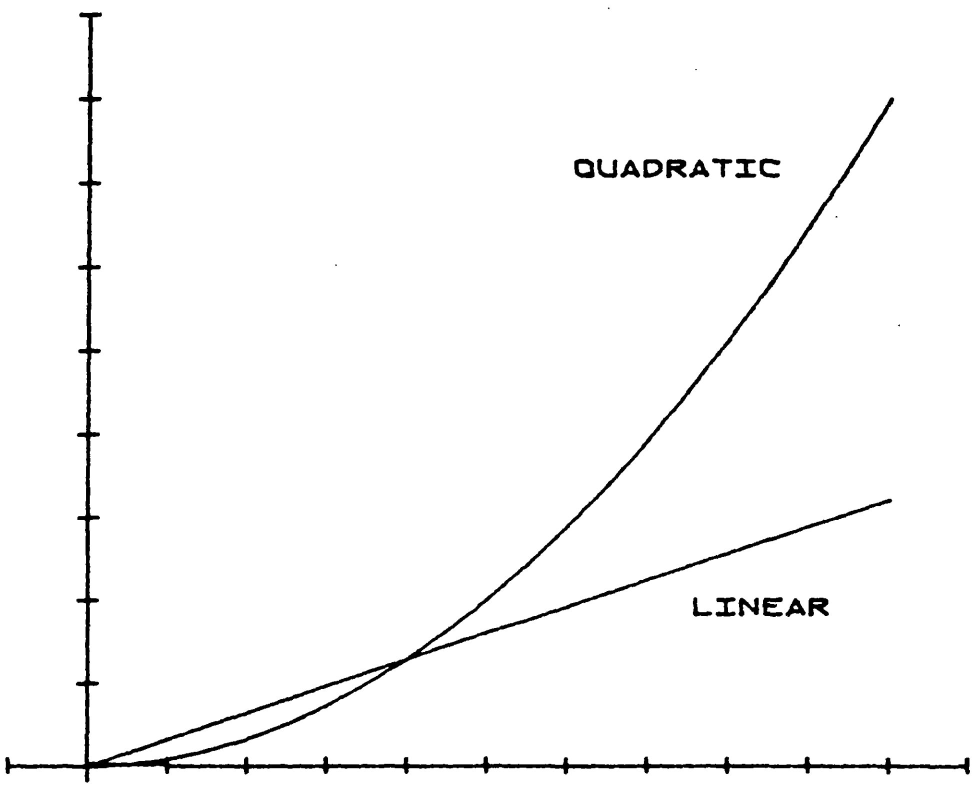


FIGURE 2.16 : LINEAR V/S QUADRATIC FUNCTIONS.

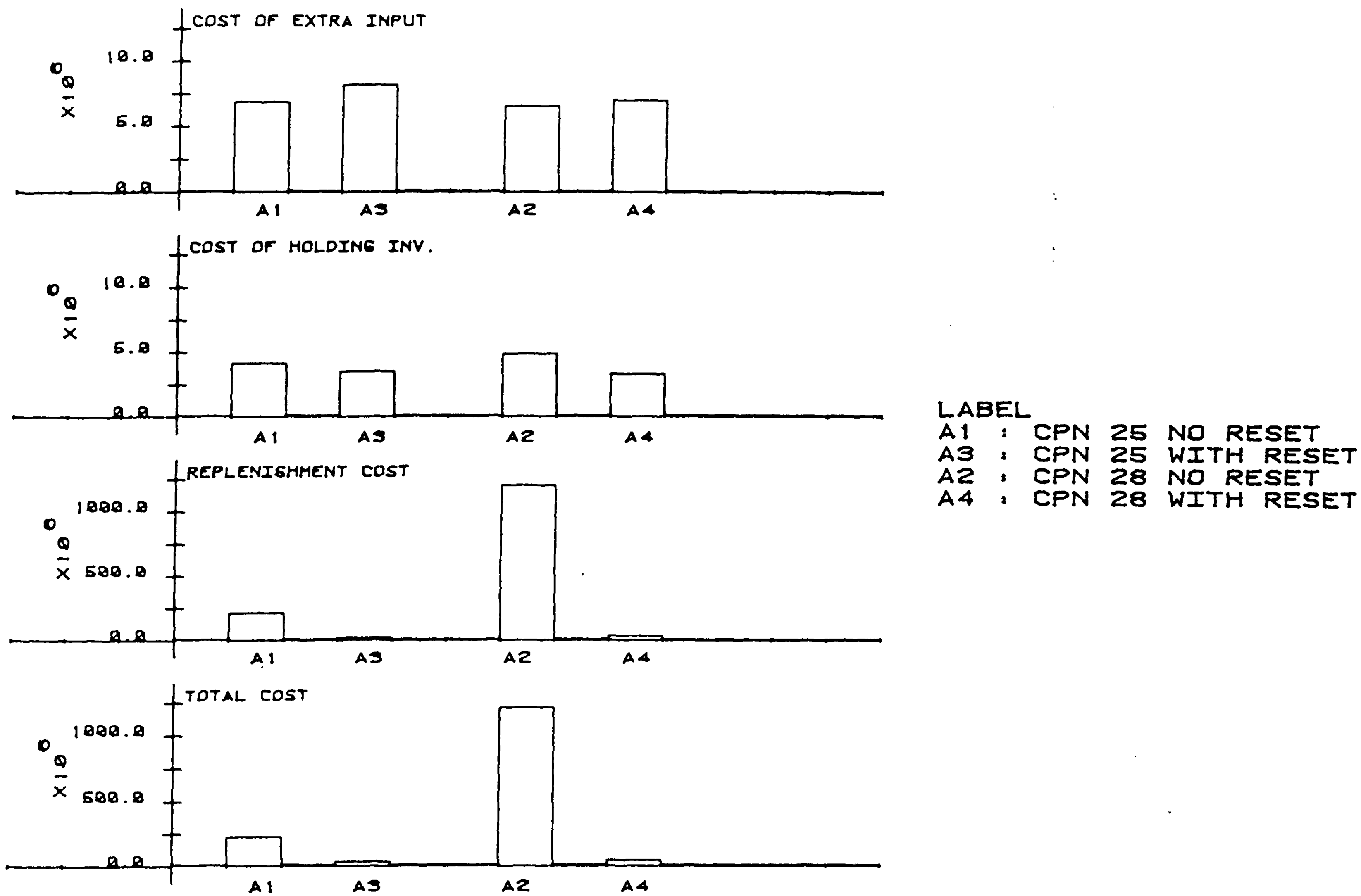


FIGURE 2.10 a-d : RESULTS OF COSTS FUNCTIONS

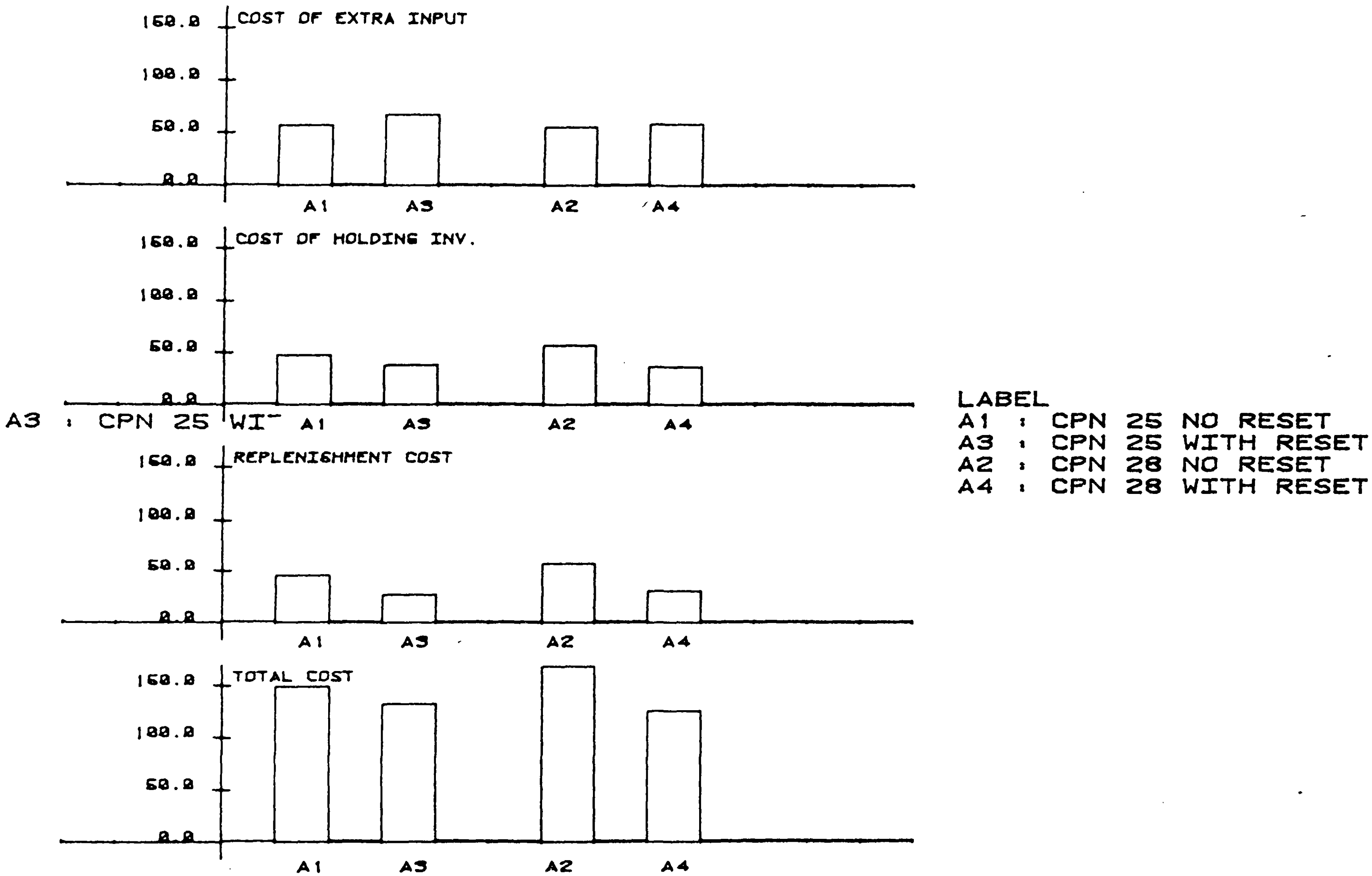


FIGURE 2.20 a-d : RESULTS OF MODIFIED COSTS FUNCTIONS

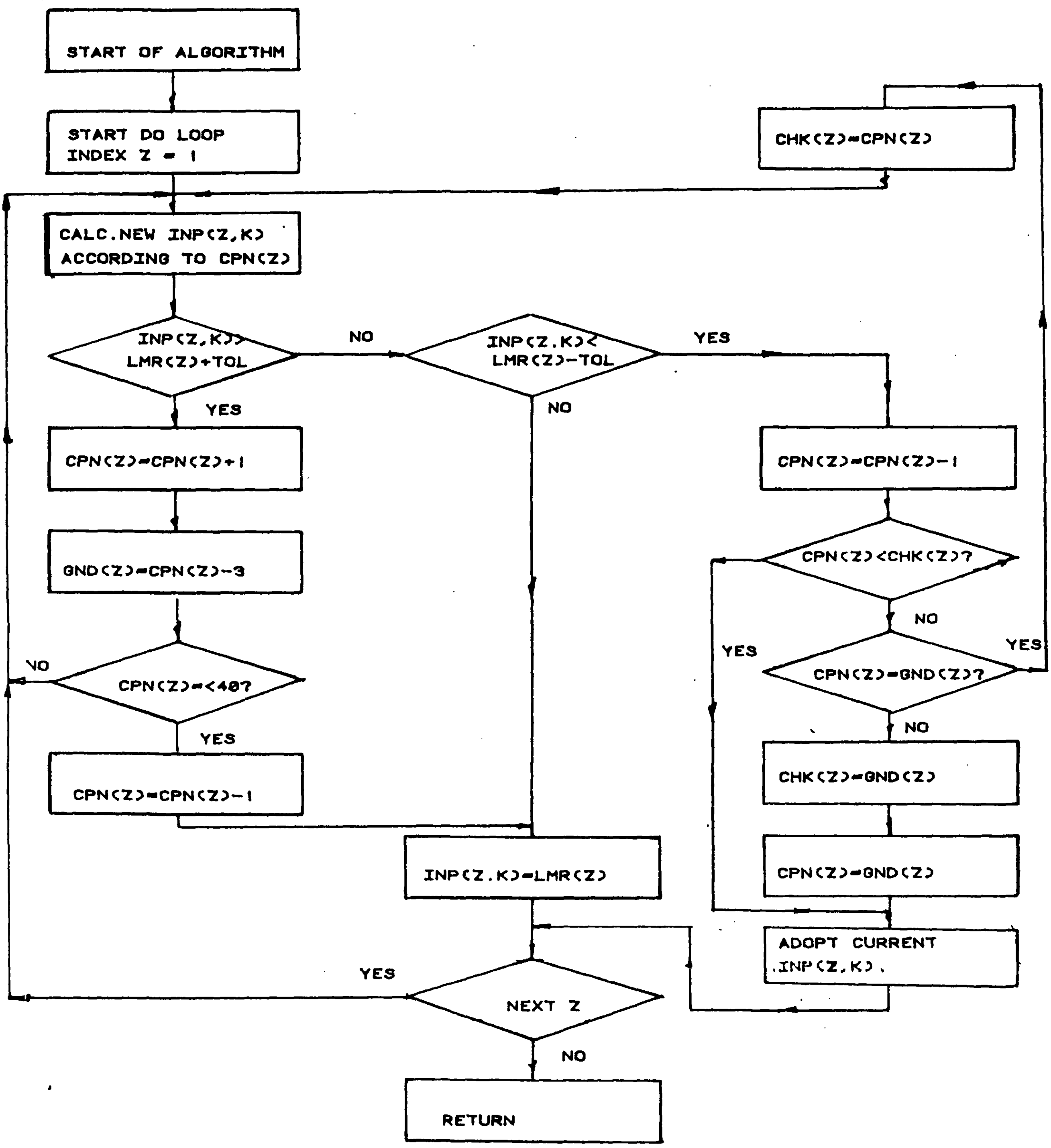


FIGURE 2.10 : SUBROUTINE "@DIS/ILCT"

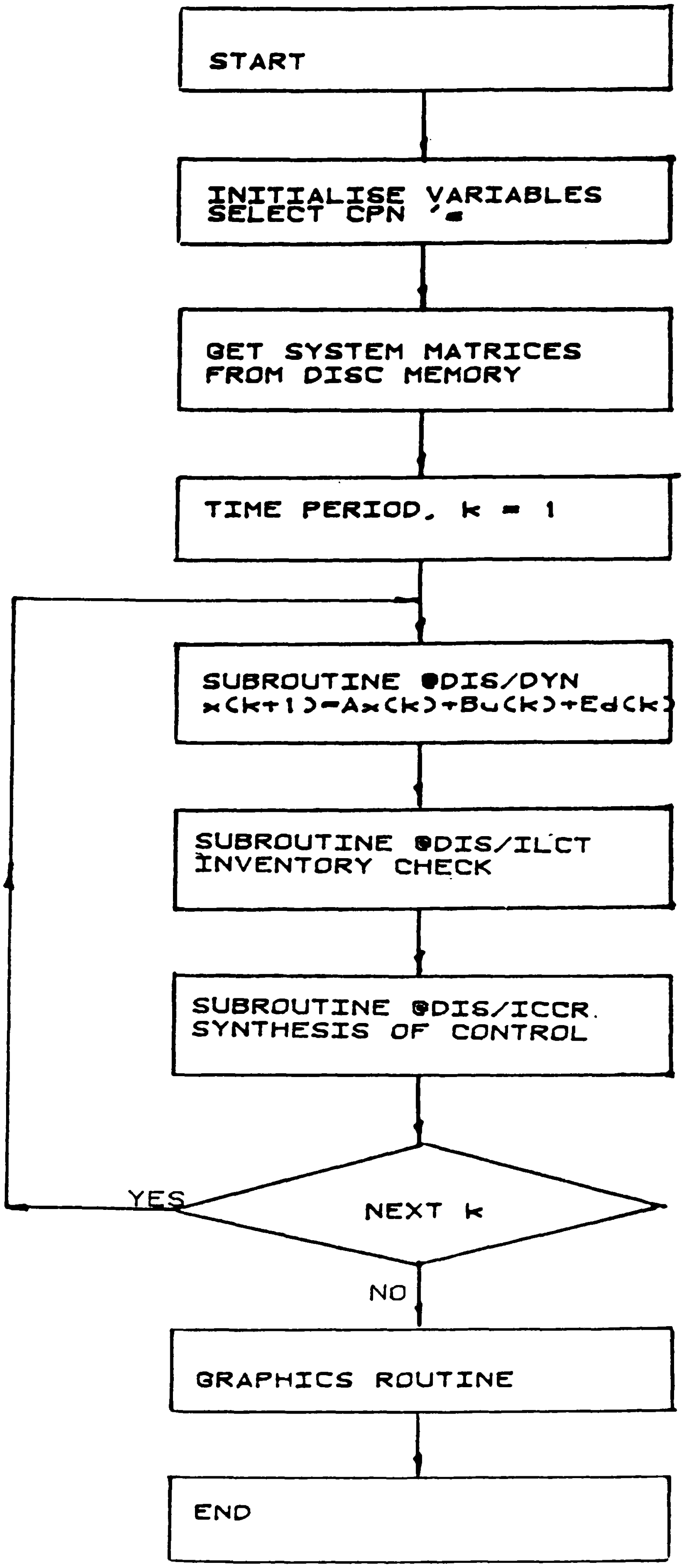


FIGURE 2.11: OVERALL SIMULATION (B)

RUNS	STAGES	1	2	3	4
A1	PROD. RATE	674	561	462	385
	BUFFER	577	473	368	719
A3	PROD. RATE	674	561	462	385
	BUFFER	611	506	416	526
A2	PROD. RATE	630	530	442	375
	BUFFER	560	468	368	843
A4	PROD. RATE	630	530	442	375
	BUFFER	580	486	406	526

Units:

Rate : Units/Time period

Buffer : Units

TABLE 2.1 : SUMMARY OF RESULTS.

PART II

PART II

APPLICATION OF MULTIVARIABLE CONTROL THEORY TO PRODUCTION CONTROL OF MULTI-STAGE PRODUCTION-INVENTORY SYSTEMS.

In this second part of the thesis, the algorithms developed in Part I are illustrated with practical manufacturing cases. These algorithms have resulted from the improved use of some of the properties of control theory in its canonical form. The development of these algorithms has been illustrated with synthetic cases in chapter 2, where acquisition of capacity, production rates and inter-stage buffers are the three parameters that have to be controlled in a co-ordinated manner throughout the whole system. Part II will now show how these algorithms may be applied to real manufacturing cases, in particular in the automotive industry. The practical relevance of the new approach is fully demonstrated in that it produces effective control policies leading to an improved usage of resources and reduced fluctuation of the inter-stage buffers.

It is noted that the problem considered here has been obtained from a study of an actual car manufacturing company in the U.K., although some generalisations have been deliberately introduced in order to maintain the anonymity of the company. The cost equations developed in Part I are also used in the actual example involving the analysis of the dynamic behaviours in car manufacturing. Various other features are also developed so that the multi-variable control theory can increasingly deal with the manufacturing control problem. These include algorithms for the practical situation where inventory levels are not sufficient to cater for the demands of the production stages.

Since in actual discrete manufacturing environment, single-product manufacturing is less frequent than the multi-product situation, the previous algorithms are either extended or new ones developed to meet the new control requirements. A multi-product manufacture is one where common resources have to be shared out accordingly to the separate demands and situations of each individual product.

Dynamic control analyses of multi-product manufacturing systems have not been extensively pursued by earlier workers. The very few cases include Drew (1975,/2/) and Hitomi and Nakamura (1976,/59/). The reason behind the scarcity of studies for this very practical problem has been mainly due to the lack of formulation techniques and the complexity of controlling the resource requirements individually and in co-ordination with the rest of the system. Chapter 4 provides an approach for such a problem where the manufacture of two car models requiring some common assemblies is considered.

CHAPTER 3

CHAPTER 3.

Control Simulation of a Single Product MULTI STAGE PRODUCTION - INVENTORY SYSTEM.

3.1. Introduction.

In production control of Multi-Stage Production-Inventory (MSPI) systems, it is often necessary to estimate the amount of extra resources and/or the dynamic reallocation of resources needed so as to be able to respond effectively to a sudden changes in operating conditions. In addition, comprehensive control policies also need to decide on the safe inventories of both finished and semi-finished products, and the constant control of such variables on a dynamic basis. Changes in operating conditions may be produced by changes in demand for the final product and/or sub - assemblies required as individual products in their own right. Thus, in a motor car manufacturing environment, the demand for the final product would be a particular car model, and the demand for power units to fit other car model variants may be considered as a separate demand at the assembly stage. To a large extent these disturbances are uncontrollable, since they are results of socio-economic events. Such disturbances would result from foreign competition, new market strategy, value of the pound sterling, world oil prices, and petrol consumption of particular car models.

In addition to responding to such external changes that affect the operating conditions, new production control decisions need also to be made in cases of re-starting the production lines subsequent to a major stoppage, e.g. maintenance shutdown and strikes. Another example that leads to such a situation is the introduction of new car models. A study of the control features of such multi-stage production-inventory systems will reveal innumerable policies based on the levels of safe stocks and the capacity rates at various time periods. The selection of the appropriate policies needs also to consider the numerous dimensions of the control problem. These include:

(i) The analysis of the individual production-inventory stages that make up the overall system. This necessitates the control of both the capacity rate and the buffer bank at each individual stage.

(ii) The consideration of local constraints such as limits on capacity rates and buffer banks.

(iii) The co-ordination of control responses so as to cater effectively the actual demands on the system.

(iv) The dynamic nature to the problem: at each time-period, control decisions are needed based on the past information on the various states of the system and the required performance of the system.

Such control problem is made more complex, considering the numerous variants of engines, automatic and manual transmission, and other additional options. The different car models that may share the same basic requirements such as engines and gearboxes, add another dimension to the problem.

The approach presented in this chapter, is to formulate the problem as a mathematical control problem, where it is required to track desired levels of inventories by controlling the input variables which are expressed in the capacity rates of the production stages. This has been originally adopted by Porter et al, (1976,/3/) and developed extensively in a previous chapter. The synthesis of a feedback control policy is carried out using the Brunovsky canonical form which introduces the controllable companion matrices (Brunovsky (1966,/67/)) as described in Section 2.2.

It is demonstrated in this chapter how some properties of the new structured formulation in a linear discrete-time model provides a basis for considering only a small number of relevant alternative policies. These policies are simulated for a car manufacturing production - inventory system on a TEKTRONIX 4052 desktop computer in order to analyse the transient and steady state behaviour of the system. From the results of the control simulation, it is demonstrated how these policies are identified as local suboptimum solutions and may then be considered as short-listed options for subsequent selection of a practical control policy by the introduction of a weighted cost function. This selection exercise is further extended to include explicitly practical physical constraints. This practical aspect of production control has been the major area into which the research efforts have been concentrated. This has been achieved by exploiting some of the properties of the controllable companion matrices adopted in the formulation of the problem. A probabilistic study in the behaviour of the system is subsequently considered in this chapter with the use of the Gamma distribution.

3.2. Problem Formulation.

3.2.1 Linearisation of Production - Inventory systems.

Adopting the systematic approach developed in Chapter 2, the automotive industry is viewed as follows:

At the strategic level, senior management issues target values for various car models to cater for the sales demands and forecast for both home and export markets. This decision is then passed down to the next level of management to produce the required values. In order to achieve these objctives this new level of control monitors the actual production of the various sub-systems and co-ordinates the flow of parts and assemblies between them. In so doing the levels of the inter-stage buffers are also controlled.

Figure 3.1a shows such typical schema of multi-stage production inventory system, where the manufacture of cars consists of both parallel and serially connected production stages. Parts produced at the various stages may be fed directly into the immediate production requirements, may be required as stand - alone products or may be put into inventory. Whilst it is possible to control and co-ordinate all the sub-systems simultaneously in the same model, the present formulation will consider six main production - inventory stages from Figure 3.1a. These are :

Stage 1 : Gear box assembly.

Stage 2 : Engine assembly.

Stage 3 : Power unit assembly.

Stage 4 : Body in white welding.

Stage 5 : Painted body production.

Stage 6 : Trim and final assembly.

and as shown schematically in Figure 3.1b and 3.1c. The aggregation of the operations into major production-inventory stages is to benefit from the hierarchical control approach as described in chapter 2.

In order to devise appropriate control policies, information is collected on the states of the relevant variables forming the feedback loop. The present development of state variable feedback control theory whereby control decisions are synthesised with feedback information is therefore most relevant in this production control context. The feedback information system and the development of multivariable control theory have been discussed in Chapters 1 and 2 respectively, in this chapter it is shown how they complement each other.

The car manufacturing system is formulated as a linear discrete-time multivariable control problem. Such a technique is obviously only applicable for linear systems with parallel and serially connected production-inventory systems. The flow-line nature of the automobile manufacture is therefore an ideal example. Nevertheless, not all the production processes can be accommodated in one simultaneous analysis. These have been grouped into major "production-inventory" stages in the present study. Each "production" stage, as viewed in this analysis still consists of a whole host of operations varying from fifteen to twenty five in number. Some of these operations are performed on transfer lines in a continuous linear mode such as the machining of engines blocks. Others are treated in discrete small batches such as in the case of car-body painting, heat treatment for the engine blocks. Therefore, the choice of the operation time is based on the output frequency of the last operation of the particular "production stage". Moreover, a judicious choice of the decision time interval is also necessary for the simulation exercise. At this control level, decision policies will determine how gear-boxes, engines and other assemblies are required to meet the final target of finished cars on a dynamic basis. In the following simulation, the unit decision time interval is chosen to be one working shift.

3.2.2 Features of Problem.

Having formulated the manufacturing model for the present case study, the main features of the production control problem are now specified. These include:

(i) A step demand for a particular car model.

(ii) The need to reach a new steady state using an (sub) optimality criterion derived from both cost and time considerations of the transient response of the system

(iii) The necessary production capacity for the various production stages. In addition to minimising such increase with respect to existing constraints,it is required to allocate them in a dynamically near optimum fashion.

(iv) The need to replenish the inventory levels for both finished products and semi finished part products.

With the above conditions there are obviously various combinations and permutations for resource levels at each particular time period. Similarly, the holding of stock for finished product (cars) and assemblies (gear boxes, engines, body in white ,etc.) present the same problem in that there is an objective to respond "optimally" for a sudden increase in demand.

It is noted that the problem is one of control as opposed to planning or scheduling. The scheduling problem (Rinnoy Kan, 1976,/83/) as such is more relevant in batch manufacturing as opposed to the present study which attempts to control the multi-stage manufacture of low-mix high volume products at the first level of production control. This problem is also different from aggregate scheduling because the latter cannot explicitly co-ordinate the multi-stage nature of production control. Some of the references /84/-/94/ indicate where various mathematical programming techniques have been applied. This distinction is believed to be necessary since, during the course of the research, the author had some initial difficulty in explaining the control problem to industrial practitioners who treated it as yet another scheduling technique, which it is not.

3.2.3 Mathematical Representation of the Automotive Manufacturing System.

In an earlier chapter, the technique of multivariable control theory introduced in Porter et al, (1976,/3/), Bradshaw and Daintith (1976,/65/), and Daintith (1977,/66/), has been applied in control problems where there is a restriction on the availability of resource inputs. This has been achieved by exploiting some of the properties of the recent developments of multivariable control theory. A similar formulation methodology is used in the following analysis and some further properties of the formulation are illustrated in their practical relevances in production control. At each stage, there are three parameters that have to be monitored, these are :

(i) Input capacity rates or desired target values, expressed in number of units produceable in one time-period. These are considered as the "control variables" of the problem.

(ii) Production rates, the effective number of units produced/time period is the "state variables".

(iii) Inventory levels which are also "state variables" of the problem.

The interrelationships of these variables are given in the equivalence equations as described in Section 2.2 . Figure 3.1d illustrates the mathematical formulation of the system and the individual state equations are given as follows:

$$x_1(k+1) = u_1(k)$$

$$x_2(k+1) = u_2(k)$$

$$x_3(k+1) = u_3(k)$$

$$
\begin{aligned}
x_4(k+1) &= u_4(k) \\
x_5(k+1) &= u_5(k) \\
x_6(k+1) &= u_6(k) \\
x_7(k+1) &= x_7(k) + x_1(k) - d_1(k) - u_3(k) \\
x_8(k+1) &= x_8(k) + x_2(k) - d_2(k) - u_3(k) \\
x_9(k+1) &= x_9(k) + x_3(k) - d_3(k) - u_6(k) \\
x_{10}(k+1) &= x_{10}(k) + x_4(k) - d_4(k) - u_5(k) \\
x_{11}(k+1) &= x_{11}(k) + x_5(k) - d_5(k) - u_6(k) \\
x_{12}(k+1) &= x_{12}(k) + x_6(k) - d_6(k) - d_7(k) \\
x_{13}(k+1) &= x_{13}(k) + x_7(k) \\
x_{14}(k+1) &= x_{14}(k) + x_8(k) \\
x_{15}(k+1) &= x_{15}(k) + x_9(k) \\
x_{16}(k+1) &= x_{16}(k) + x_{10}(k) \\
x_{17}(k+1) &= x_{17}(k) + x_{11}(k) \\
x_{18}(k+1) &= x_{18}(k) + x_{12}(k)
\end{aligned}
$$

These equations can be made up into the matrix equation as given 3.1

$$x(k+1) = A\,x(k) + B\,u(k) + E\,d(k) \qquad \text{----------3.1}$$

$$u(k) = F\,x(k) \qquad \text{----------3.2}$$

m = No of inputs in system = 6

n = m X 3

A = plant system matrix. (n X n)

B = input matrix. (n X m)

E = disturbance matrix. (n X m+1)

x = state vector. (n X 1)

(production rate, No of parts / unit time)

(inventory level, No of parts).

u = input vector. (n X 1)

(extra capacity rate, No of parts / unit time).

d = disturbance vector (n X 1)

(change in demand rate, No of parts / unit time)

F = feedback matrix (m X n)

k = argument denoting time period at k.

Controllable matrices $\bar{A}$, $\bar{B}$ and $\bar{E}$ are as follows:

Matrix $\bar{A}$.

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccccccccc} 1&2&3&4&5&6&7&8&9&10&11&12&13&14&15&16&17&18 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} &
\begin{bmatrix}
&&&&&&&&&&&&&&&&& \\
&&&&&&&&&&&&&&&&& \\
&&&&&&&&&&&&&&&&& \\
&&&&&&&&&&&0&&&&&& \\
&&&&&&&&&&&&&&&&& \\
&&&&&&&&&&&&&&&&& \\
1&&&&&&1&&&&&&&&&&& \\
&1&&&&&&1&&&&&&&&&& \\
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&&&&&&1&&&&&&1&&&&& \\
&&&&&&&1&&&&&&1&&&& \\
&&&&&&&&1&&&&&&1&&& \\
&&&&&&&&&1&&&&&&1&& \\
&&&0&&&&&&&1&&&&&&1& \\
&&&&&&&&&&&1&&&&&&1
\end{bmatrix}
\end{array}
$$

Matrix $\bar{B}$

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 1&2&3&4&5&6 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} &
\begin{bmatrix}
1&&&&& \\
&1&&&& \\
&&1&&& \\
&&&1&& \\
&&&&1& \\
&&&&&1 \\
&&-1&&& \\
&&-1&&& \\
&&&&&-1 \\
&&&&-1& \\
&&&&&-1 \\
&&&&& \\
&&&&& \\
&&&&& \\
&&0&&& \\
&&&&& \\
&&&&& \\
&&&&&
\end{bmatrix}
\end{array}
$$

Matrix $\bar{E}$

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} 1&2&3&4&5&6&7 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} &
\begin{bmatrix}
&&&&&& \\
&&&&&& \\
&&&&&& \\
&&&&&& \\
&&&0&&& \\
&&&&&& \\
-1&&&&&& \\
&-1&&&&& \\
&&-1&&&& \\
&&&-1&&& \\
&&&&-1&& \\
&&&&&-1&-1 \\
&&&&&& \\
&&&&&& \\
&&&&&& \\
&0&&&&& \\
&&&&&& \\
&&&&&&
\end{bmatrix}
\end{array}
$$

The change in operating conditions for the system is taken as 1500 units per week for a particular car model. This can be considered as either of the following conditions:

(i) An actual step-up of production output of 1500 units/ week. This situation usually arises when the company wants to increase its market share, or attempts to penetrate new markets, e.g. through exporting.

(ii) A start-up situation arising from a prior shut-down This may arise more often than expected. One major car manufacturing company in U.K. is known to have 64 and 128 stoppages (wildcat strikes) at two of their plants in the year 1980 alone. It is noted that this situation differs from a week-end stoppage and the resumption of working on the following Monday, because in such a case, all the various plants (production stages) stop at the same time. The situation currently considered is when the various production stages have stopped independently in their own time.

(iii) Introduction of a new car model.

Such an event is of a frequent occurrence in view of the severe and fast moving nature of the competition in this particular industry, both in home and foreign markets. Introduction of new models arises because of the increasingly higher requirements in the technological content in the car, more stringent legislation in safety measures a nd higher expectation of performance from customers. New methods of manufacturing, e.g. robotics, new materials allowing new designs, new design methods such as CAD (Computer - Aided - Design) have also accelerated the frequency of new car models.

Since the unit of the time period is taken as the duration of one shift, and a 10-shift week is assumed, the desired production rate is 150 cars / shift at the steady state so as to meet the demand of 1500 cars/week. Reject rates are arbitrarily chosen for the present illustration as follows:

Stage 3 : Power unit assembly = 15 %

Stage 5 : Painting stage = 10 %

Stage 6 : Final assembly = 5 %

Since rework of reject is dealt with by a separate department, any incidence of scrap will mean an initiation of production from the appropriate sources. The inclusion of the above reject rates mean an additional production of 28, 18, and 8 at stages 3, 5, and 6 respectively on top of the 150 units/shift at the steady states. These are also the entries at the 3rd, 5th and 6th positions of the vector d. The calculation of these reject values is given in Appendix 6. The final demand 150 is at the 7th position. The steady state values of the production capacities for this problem will thus be: 186, 186, 186, 176, 176 and 158 at sta ges 1 to 6 , if a constant output of 150 good units/shift is required. It is appreciated that the disturbances are assumed to be constant throughout the time horizon, a fact which may not be necessarily true in a real situation. However, it provides definite information as to how the system responds to the control policies effected under the stated average conditions. The analysis when the disturbances are "random" is discussed in a further section.

3.3 Description of Approach.

3.3.1 Structure of the Control Simulation.

The analysis of the problem can be considered in two phases. The first one is the formulation stage which creates the various matrices needed, and the second makes use of the matrices in the dynamic simulation. As mentioned in Chapter 2 (2.2.5), the simulation exercise has been carried out with a TEKTRONIX 4052 desktop computer.

At the formulating stage, equations (3.1) and (3.2) are transformed into their canonical forms with the use of the Prepelita algorithm (1971,/68/) as described in Section 2.2. and is given in Appendix 1, where the actual programs are also included. The transformation is as follows:

$$x(k) = C\,\bar{x}(k)$$

$$\bar{x}(k) = C^{-1}x(k)$$

where C, the transformation matrix is given as follows:

$$\begin{bmatrix}
1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\
-1 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}$$

The resulting equations (3.3) and (3.4) are in their canonical forms :

$$\bar{x}(k+1) = \bar{A}\,\bar{x}(k) + \bar{B}\,u(k) + \bar{E}\,d(k) \qquad \text{--- } 3.3$$

$$u(k) = \bar{F}\,x(k) \qquad \text{--- } 3.4$$

where
$$\bar{A} = C^{-1}AC$$
$$\bar{B} = C^{-1}B$$
$$\bar{E} = C^{-1}E$$
$$\bar{F} = F^{-1}C$$

These new companion matrices $\bar{A}$, $\bar{B}$, $\bar{E}$ are stored in external disk memory and are only brought into the main memory during the actual run, i.e. in the dynamic stage. The synthesis of control policies through the computation of feedback matrix F is implemented by the assignment of closed loop eigenvalues to the plant matrix. The mathematical treatment of the current approach has been dealt with in detail in an earlier chapter and in Appendix 3. Here again, for the ease of computation, feedback (sub)matrices corresponding to the 6 production stages are pre-synthesised for the range of values of 0.000 to 1.000 with increment of 0.025 . This library of feedback matrices is stored in a disc file that may be accessed directly during computer simulation runs. For ease of discussion, control policies synthesised with eigen values of submatrices set at 0.000, 0.025, 0.050, 0.075 .... are referred to as CPN (Control Policy Number) 1, 2, 3, 4, ... respectively for each individual production-inventory stage.

The dynamic nature of the second phase of the simulation is given in Figure 3.2 and is of the same nature as that of Figure 2.5 described in section 2.2.5 . It basically consists of calculating the equations 3.1 and 3.2 for a required number of time-periods. While the calculation of equation 3.1 is relatively straightforward with subrotine "@DIS/DYN" (Appendix 4.), that of equation 3.2 is more complicated since this is the one synthesising control policies according to the required CPN. These computations are actually calculated in their control canonical forms.(Equations 3.3 and 3.4) A graphics subroutine is then used to present the results of the simulation. This subroutine is similar in nature to that given in Appendix 5. Numerous commands for graphics purposes that are inbuilt in the computer system have been extensively made use of.

A time optimal approach with all the modes of control compositely set at CPN 1 will give a response as shown in Figure 3.3 . It is clearly seen that such time optimal response calls for excessive requirements of both resources and inventories. Such a policy may not be desirable or even possible in actual practice due to physical and other constraints.

3.3.2 Concept of Structured Control Policies.

Since non-time optimal solutions are innumerable, the choice of an appropriate set of policies is obviously very difficult in the attempt to strike a balance between optimality and practicality.
The approach presented in this work makes use of some of the structured properties of the particular type of multivariable control theory adopted. These properties allow the consideration of "relatively proportional" values of input rates and inventory levels. "Relatively proportional" or "structured values" being defined as those relating to a policy structured such that each

inter-stage buffer is dynamically controlled to a desired value while still catering simultaneously for the demand in input of subsequent production stages. This is achieved by setting all the CPN ' s of the various modes to a similar value in the selection of control policies. As a rule, policies with low values of CPN will give sharper response than those of higher values. Some typical results are shown in Figure's 3.4 and 3.5 for CPN's 20 and 24 respectively.

From these Figures, it is seen how the capacity rates, at all production stages dynamically control and smoothly restore the inventories with very small amounts of overshoot and undershoot. The upper limits of CPN 20 and 24 and their corresponding maximum amounts of inventory depletion are given in Table 3.1. This method therefore provides the following control solutions:

- Dynamic allocation of resources that are structured relative to each other.
- A knowledge of the minimum amount of safe stocks required for the scenario considered.

It is noticed that control responses with CPN's 20 call for higher requirements of capacity than CPN's 24. This is due to the fact that CPN 20 has its eigen values assigned closer to zero than CPN 24 giving a sharper response as explained in Chapter 2. The resource utilisation is also maximised to the limit of available capacity. This is due to the inclusions of a "reset" alogorithm in the control search subroutine as explained in Section 2.3.3. Were it not for this algorithm, the response for a CPN 24 would have been as shown in Figure 3.6. In section 2.3.3, the benefits of the "resetting" technique has already been demonstrated analytically.

3.3.3 Practical and Economic Relevance of Structured limits.

In this section it is shown how the use of the structured limits gives rise to some practical and economic advantages. In fact they are identified as local suboptimum solutions in the innumerable solutions that exist.

Structured capacity, that is, the maximum values to which the capacity rates of the production stages can be set, are determined to be :

273, 273, 250, 261, 240 and 200 respectively

for production stages 1 - 6 using a control based on CPN 25. The response of this structured setting is given in Figure 3.7 showing smooth control at all the production stages.

One logical analysis will be therefore to examine the response of the system when one of the limit cannot be adjusted to structuredly match the rest, and to assess the cost-benefits/penalty associated with this response as opposed to one that is fully structured. In order to carry this analysis, the control simulation given in Figure 3.2 is performed again with varying situations where the maximum capacity rates are not balanced with respect to each other. The cost function developed in Chapter 2 is used to assess the respective responses. The maximum capacity rate at production stage 3 (assembly of power units) is varied from 220 to 280 in steps of 10 in each separate run (referred to as runs B1 - B7 respectively) while the limits at the other production stages are set relatively proportional to each other as they would have been, were the limit at stage 3 still equal to 250. The details of the runs are given in Table 3.2 and the responses for the runs are given in Figures 3.7 - 3.13 for runs B1 to B7 individually. They are redrawn superimposed in Figure 3.14A and 3.14B where B1 to B4 are in the former figure and B4 to B7 are in the latter.

It is noticed that from Figure 3.14A the responses for stages 4, 5, and 6 (welding of car body, painting of car body and final assembly) are identical in all runs. They are to a large extent isolated from the imbalance injected, which is to be expected since they are actually in a separate line and being operated with a separate control. In Runs B5-B7, where the capacity rate at stage 3 is set higher than 250 units/time period, it is seen that there is a sharper restoration of the inventories of power units. On the other hand, it also means that more parts need to be drawn than can be produced at the engine and gearbox production. This results in the continuously severe depleted states of inventory at these two production stages. This feature may of course be a disadvantage in the event of further disruptions leading to a high likelihood of stockout. Decreasing the capacity rates at stage 3 causes a more sluggish restoration of the power units buffer. As seen in Figure 3.8, the state of depletion is worse for 8 time periods before finally slowly recovering to the original level. This results from the difficulty in producing the necessary power units to meet the demands of the final car assembly and the inherent reject assumed at these two production stages. The inventories at levels 1 and 2 overshoot drastically because of the lack of operating capacity at the power unit assembly (stage 3) to use them fully as they would have, if the maximum capacity were 250 units / time period.

3.3.4 Quantitative Analysis.

In order to assess the various responses in a sensitivity analysis approach, a 3-component cost function is used. This was developed in Chapter 2 and used for a synthetic case. In this Chapter, the explanation of the costs is re-introduced and is discussed further in this context of an actual manufacturing concern.

Total cost J = J1, Costs of Extra Inputs
+ J2, Costs of Inventory Held
+ J3, Replenishment Delay Penalty Cost.

$$J1 = \sum_{k=1}^{T_s} P \left\{ (U(k) - Us)_n^{[2]} \right\} \quad \text{----3.5}$$

$$J2 = Q \left\{ (M)_n^{[2]} \right\} \quad \text{----3.6}$$

$$J3 = \sum_{k=t_r}^{T_s} R \left\{ I(k)_n^{[a + .1(k-t_r)]} \right\} \quad \text{----3.7}$$

where

$[\ \ ]$ upper square brackets contain the exponent to which all elememts are raised.

$(\ \ )_\eta$ - The elements of the vector are normalised, divided by their steady state values.

U(k) - m X 1 column vector for control input at time (k).

Us - steady state value of U(k).

Ts - Time at which steady state is reached.

P - weighting matrix for U(k) (1 X m)

M - m X 1 column vector for inventory held.

Q - weighting matrix for M. (1 X m)

I(k) - m X 1 column vector that need to be replenished at the inventories.

t_r - time period in which replenishing starts.

R - weighting matrix for I(i) (1 X m)

a - arbitrary constant.

The absolute magnitude of the values of the weighting matrices P, Q and R are chosen such that each respective resulting cost, namely J1, J2 and J3 is on a similar scale, i.e. the costs have the same units. This is done so as to make inter-comparison possible. The choice of these values is effected from a combination of both their financial values where appropriate and their relative importance as viewed subjectively by the management concerned.

3.3.4.a Costs of Extra Inputs.

At the initial start-up state, extra capacities are required at the production stages to cater for the necessary demands at the various points of the system. Moreover a controlled start - up, as explained in Chapter 2.2 is a control approach that attempts to replenish the inter-stage buffers. J1 as given in equation 3.5, is the cost associated with these extra requirements and they are expressed as a fraction of their steady state values. This "normalisation" technique allows a more realistic comparison for different cost of variables. This comparison would have been otherwise very difficult to assess. The difficulty of deciding on the weights assigned to the heterogeneous variables as absolute extra capacity and absolute safety buffers is alleviated by considering the fluctuations as percentages of their respective steady state values. As explained in Section 2.2 a quadratic cost function instead of a linear one is used so as to penalise high excess values of inputs. This is to emphasise the higher costs involved at high values. Whilst this cost function may not be based on real absolute financial values, the practical implication is important: it highlights the intention of avoiding high addition of manufacturing resources as would be expressed in excessive overtime, speeding up production lines drastically, subcontracting.

In addition to the economic disadvantage of excessive overtime, measures to increase or fluctuate the production rates will find very little favour from the workforce, especially one as heavily unionised as that of car manufacture.
The values of the weighting matrix P are chosen as

$$\begin{bmatrix} 8 & 8 & 8 & 4 & 4 & 12 \end{bmatrix}$$

so as to arbitrarily represent the relative importance of the jobs involved. The first three values are twice as much as the fourth and fifth one, because they consist of the manufacture of power units that involves a relatively higher amount of labour than the more mechanised and/or automated operations for the car-bodies. Finally there is a higher amount of labour at the final stage of trimming and assembly. Therefore policies leading to higher requirements of capacity involving more manual labour will be penalised to a larger extent than those operations that can be speeded up mechanically. When this cost function J1 is applied to the seven Runs B1-B7, the results are shown in Figure 3.15a where there is a slight but gradual increase in costs.

3.3.4.b Costs Of Inventory held.

This is the cost J2 associated with the holding of inter-stage buffers and is given in equation 3.6. These values are obtained by performing the control simulation runs without any constraint on the inter-stage buffers, the maximum amounts of the depletion giving the minimum buffers required. A quadratic cost function is again used to represent the intention of avoiding excessive holding of inventories. The individual values of the buffers are expressed as new ratios of the steady state requirements of the subsequent production stages. This ratio analysis facilitates the inter comparison exercise. The weighting matrix Q is given as :

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 5 \end{bmatrix}$$

The choice of these weights has been based on the real financial values of the sub-assemblies, together with the relative importance given to them by the particular management.

The resulting costs are shown in Figure 3.15b, a similar trend to 3.15a is noticeable.

3.3.4.c Replenishment Delay Penalty Costs.

This is the cost attributed to the delay for the inter-stage buffers in being restored to their original levels. The depleted states of the buffers are increasingly penalised so as to reflect the necessity of replenishment. Timely replenishments are required to face the next change in operating conditions or the re-starting of production lines. Such need to replenish the buffers is also vital in view of other stochastic disturbances that perturb the manufacturing system. Therefore the cost function J3 as given in equation 3.7 is structured in such a way that it varies with two parameters.

(i) The relative amount left to be replenished, i.e. the ratio of the absolute value to the desired value. The larger this ratio, the higher the penalty cost will be.

(ii) The time elapsed in a depleted state. This is achieved by the structure of the exponent $(a + 0.1(k - t_v))$ increases with k, the time period, i.e. it continuously increases the penalty related to the delay in replenishment.

It is noted that cost J3 favours fast replenishment, which will mean extra high demands of resources at the initial start-up. This fact will be penalised by J1 and J2. This situation will then create an opportunity to find a local sub-optimum in the various strategies of control.

The weighting matrix, R used is :

$$\begin{bmatrix} 2 & 2 & 4 & 2 & 2 & 15 \end{bmatrix}$$

The values are chosen so as to give relatively higher penalty against slow replenishment of final car and power units. Here again the choice of the weights is one that is designed to reflect the practical situation as much as possible and has therefore been one arrived at as a result of various discussion with the management involved. The costs for the seven runs are given in Figure 3.15c where Run No B4, i.e. the structured one is the most cost-efficient. The wide fluctuations of Run B1 (Figure 3.8) are shown to be 20% less cost-effective (Figure 3.15c) as compared to Run B4. This cost penalty has arisen when the conditions of the run B4 are only 12% less in the maximum capacity rate at stage 3 only (i.e. 220 instead of 250 units/time period). Further variation from the structured values at the other stages will obviously lead to higher instability and penalty costs.

The costs results for Runs B3 and B5 are very comparable to Run B4 since their conditions are only 4% from the structured one. Nevertheless, it is evident that Run B4 with structured values is indeed the most cost-effective.

Costs associated with extra capacity requirements (e.g. overtime, float labour, etc) and buffer inventories are concepts realistically compatible with production control management. In the present cost structure, these two costs are not explicitly considered in their financial values. They are augmented by other non - financial factors, which led to the implementation of quadratic terms. Costs J3, (equation 3.7) associated with the replenishing of inventory, have received the least attention from industrial management compared to the costs of extra capacity and inventory holding, J1 and J2 respectively. This is due to their usually intangible nature arising from:

(i) Idle labour when the preceding stages are unable to feed the necessary units creating the situation usually known as starving.

(ii) Lost sales and decline in goodwill in the inability to produce on time. These are penalties that are difficult to assess directly, and have therefore been largely ignored by industrial management.

The cost structure developed here by the author for this replenishment delay is believed to be original and to contribute to the gap mentioned. This cost J3 has been considered to be representative of practical circumstances by industrial management of the host company.

3.3.4.d Local Suboptimum Solution.

The total cost is given as a summation of the three costs J1, J2 and J3:

$$J = J1 + J2 + J3$$

It has already been mentioned that the values of the weighting matrices have been chosen such that the resulting costs are on the same denominator. The results of this total cost are shown in Figure 3.15d. The structured policy, i.e. run B4 shown in Figure 3.7, does indeed provide the most economic policy on the basis of the cost functions used. This is because all the control variables are synthesised in such a way so as to control the state variables in balance with each other, therefore with little or no overshoot and/or undershoot. Run B4 has a 10% cost advantage compared to Run B1, and 4% improvement over Run B7. A minimum in the total cost is noted. Any positive increase in the capacity rate will favour replenishment at that particular stage, but will also deplete more severely the two feeding buffers. The cost benefits of replenishing one inventory is balanced against the slower replenishment at two preceding stages. The converse also applies, i.e. the penalty associated with a slow replenishment as a result of a moderate input of capacity, is made up by the benefits obtained from a milder depletion at the two feeding stages.

Runs B3 and B5 are very close to Run B4, and their costs can be considered to be the same for practical purposes. When stage 3 operates at an original maximum of 240 units/shift (Run B3, Figure 3.14B),i.e. 10 units below the structured one, the inventory at stage 3 takes a longer while to replenish. But the penalty associated with the delay is made up by the fact that buffers of engines and gearboxes at stages 1 and 2 feeding into the assembly of power units are depleted less severely and are replenished faster than in Run B4. In Run B5, where stage 3 operates at an original maximum of 260 units/shift, i.e. 10 units/shift above the structured value of 250/shift, the inventories of the engines, gearboxes stay in more severe states of depletion. However, this disadvantage is alleviated by the fact that the inventory of the assembled power units is replenished faster.

Therefore the local sub-optimum obtained from the cost analysis has a certain range of tolerance within which very practical and near-optimum solutions are obtainable. For this particular problem scenario, the tolerance margin is an order of 5%. Run B1, which is only 12% off the structured value gives very wide fluctuations as witnessed in Figure 3.8. These inventory fluctuations were shown to be in the order of 20% more for Run B1 than Run B4. The cost benefits obtained with inventory responses in Run B4, are to some extent lost by the extra capacity required and the states of depletions of the feeding buffers. The overall costs show an improvement of 10% from Run B1 to Run B4.

It should be realised that the total cost is a combination of implicit and explicit values. In actual practice, such costs may be very difficult to assess, therefore they are structured so as to reflect the desired control requirements rather than actual explicit financial ones, hence the use of a quadratic function. The choice of the weights is again a combination of implicit and explicit values. It is implicit to the effect that it may be derived from the experience of management as to which parameter more importance ought to be given. It is explicit in so far that the weights may derived from the actual values as labour content, added value. Obviously the use of the weights may be altered interactively so as to explore other possible scenarios where different relative importances are given to the variables.

The same analysis is now carried out for a hypothetical manufacturing situation where there is no reject at all and the resulting cost analysis is given in Figure 3.16. (RUNS B1 - B7). Run B4 is shown to be the local suboptimum with a marked cost-benefit of 25% over run B1

If a non-normalised cost structure were adopted as had been in some of the previous work, the weights would have been applied directly to the absolute values of the variables as opposed to their relative changes. The new cost structure is given as:

J1, costs of extra inputs :

$$\sum_{k=1}^{T_s} P\left\{(U(k) - Us)_n^{[2]}\right\}$$

J2, costs of inventory held:

$$Q\left\{(m)_n^{[2]}\right\}$$

J3, replenishment delay cost:

$$\sum_{k=t_r}^{T_s} R\left\{I(k)_n^{[a + .1(k-t_r)]}\right\}$$

The results of which are shown in Figure 3.17. The local suboptimum at Run B4 is much more obvious. Nevertheless, throughout the rest of this thesis, the original normalised cost is to be used because of

its greater practical value as explained earlier .
The analysis is repeated by introducing an "imbalance" at the final car assembly (stage 6) instead of doing so at the power unit assembly. Seven new runs are carried out with the production varying from 170 - 230 in steps of 10 in each run (Table 3.3). The structured values being:

273 273 250 261 240 200

The dynamic responses of the 7 runs (Runs C1 to C7) are given in Figure 3.18 - 3.19. From the series of responses it is seen that varying the input rate at the final car assembly affects the replenishment rates at 3 stages:

(i) The final inventory of finished cars.

(ii) The inventory of power units.

(iii) The inventory of car bodies.

As would be expected, increasing the capacity rates at stage 6 will cause the inventory of finished cars to replenish faster, but to the expense of having the inventories of power units and car bodies at severely states for a long while. Conversely decreasing the capacity rate will cause the inventory of finished cars to replenish in a sluggish manner. The inventories of power units and car bodies overshoot by a substantial amount since they cannot be used up in time at the final assembly stage.
The same normalised cost function previously described is used for the Runs C1 to C7. The results of the costs equations 3.5-3.7, i.e. J1, J2 and J3 are given in Figures 3.20a to 3.20c, and the total cost in Figure 3.20.d. From these figures, it is still demonstrated that the run with structured limits, i.e. Run C4 gives the most cost efficient policy.

Changing one of the variables in the exponent of equation (5), namely changing variable "a" to 1.4 from the original value of 2 give a similar costing result as shown in Figure 3.21: Run C4 is still marginally better than the rest.

It has been shown from the costing analyses that it is better to allocate excess capacity than insufficient capacity. In cases where there is a relatively higher importance in the replenishment of power units assemblies, as in the event of a separate demand of power unit into another car body or for KD (knock-down) purposes, a different weighting matrix R is used.

$$R = \begin{bmatrix} 2 & 2 & 6 & 2 & 2 & 15 \end{bmatrix}$$

The change made is at the third entry with a new value of 6 instead of 4 as previouly. The costing results in Figure 3.22 show that run C3 will be most cost - effective. A slight decrease from the structured value at stage 6 (final car assembly) will give the opportunity to refill the power units faster, while not imposing any serious penalty onto the current demands for the finished car.

3.4 Selection of Structured Control Policies.

3.4.1 Shortlisting Approach.

In the previous chapter, it has been demonstrated how the use of structured policy provides cost effective sets of control limits for the various capacity rates. Theoretically, there are an infinite number of such sets of structured limits since they are obtained by setting the eigenvalues of the system matrix compositely from zero to one in whatever incremental amount considered desirable (e.g..001 or .00001 etc). In this particular case we are considering eigenvalues ranging from zero to unity in increment of 0.025, i.e. CPN 1 to CPN 40. Control responses obtained from all CPN's set collectively to the same magnitude provide a discrete solution that has been shown to be a local sub-optimum.

The approach now to be described can in fact be considered as a second selection procedure among shortlisted options. These shortlisted options are the structured ones which have been shown to be cost-effctive among a cluster of neighbouring solutions, i.e. local suboptimum. Therefore this special feature makes the search for an appropriate policy much easier starting from the original innumerable solutions. The control simulation model described in detail in the earlier section 3.3 of this chapter is made use of agian. A number of separate runs has been performed with CPN 20, CPN 21, CPN22 .... CPN 32 respectively. The results of these separate runs have been stored on disk files for subsequent analysis with the cost functions J1, J2 and J3 (equations 3.5-3.7). The latter analysis provides the final selection procedure in examining the costs incurred for each of the runs(CPN 20 - 32). It is recalled that these costs have been developed specifically to assess the control responses so as to select the most appropriate control.

3.4.1.a Costs of Extra Inputs.

$$J1 = \sum_{k-1}^{T_s} P\{ (U(k) - Us)_n^{\{2\}} \} \qquad ----3.5$$

The rationale behind the quadratic approach as explained in Section 2.4 , is to avoid high excess capacity inputs. The values of the matrix P are :

$$\begin{bmatrix} 8 & 8 & 8 & 4 & 4 & 12 \end{bmatrix}$$

The results of the cost function is given in Figure 3.26a. The costs of each respective runis shown to decrease progressively with increase of CPN. Such a characteristic is due to the fact that low CPN values call for sharper response with higher values of capacity rates thereby causing a relatively higher cost.

3.4.1.b Cost of Inventory held.

$$J2 = Q \quad (M)^{\{2\}} \qquad ----3.6$$

This is the cost associated in holding the minimum amount of safe stocks to cater for the problem scenario. Here again a quadratic function is chosen instead of a linear one so as to avoid excessive holding of safe stocks. The matrix Q is chosen as :

$$Q = \begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 5 \end{bmatrix}$$

The increasing weights in holding the assemblies as they are closer to the finished state is to avoid holding goods with increasing added value, since it reflects more tied up capital. The variation of costs for the CPN is shown in Figure 3.26b, where a gradual decrease in cost with increasing CPN is noticeable. The reason behind this behaviour is linked to the reason for the costs of introducing extra inputs. With low CPN, a slower response with lower demands of capacity is obtained, which obviously in turn means the holding of less inventories of assemblies.

3.4.1.c Replenishment Penalty Costs.

$$J3 = \sum_{k=t_r} R \quad I(k)_n^{(a + .1(k-t_r))} \qquad \text{----3.7}$$

This is the penalty cost attributed to the state of depletion of the inventories of assembly and final product. The cost is structured so as to penalise slow recovery to the original level. The values of matrix R are :

$$\begin{bmatrix} 2 & 2 & 4 & 2 & 2 & 15 \end{bmatrix}$$

and the variable a =2. When such a cost function is applied to the responses of CPN 20 - 32, the results are as shown in Figure 3.23 . In this case, it is seen that the costs increase with increased CPN. High CPN call for lower extra capacity rates which will replenish the inventories at a slower rate. Therefore in order to avoid the possibility of a stock out situation, slow replenishment associated with higher CPN are increasingly penalised. It is observed that the rise in such cost is fairly gradual between CPN 20 and 27, with only an increase of 50% between them. From CPN 28, the costs escalate drastically.

3.4.1.d Total Costs.

The total cost equation is : $J = J1 + J2 + J3$

When this is applied to the runs obtained from CPN 20 - 32, the results are as shown in Figure 3.23d where a shallow trough is noticeable in the range of CPN 25 - 28. Therefore adjusting the capacity limits to those as used in CPN 25 - 28 will provide more cost - effective solutions than otherwise. CPN 27 incurs the least cost of all, and the actual responses of the system under this CPN are given in Figure 3.24.
It is pointed out that this analysis has not considered any practical constraints explicitly, nevertheless some definite guidelines are presented, obtained from a cost structure that avoids high demands in capacity or slow buffer replenishment. This conclusion, of course, depends on the weighting matrices selected.

With a suitable choice of weights, the control policies thus obtained, provide management with a prior knowledge of the necessary measures that have to be taken in the very near future. This allows management to take necessary actions in time. These may involve the negotiation of extra workforce, or extra working hours on a dynamically scheduled basis. Similarly, a prior knowledge of the necessary buffers can trigger the production of these necessary floats whenever practicable. This is usually achieved through extra working during the week-ends. With the proposed approach, this practice of week-end working will be of a more organised nature as compared to an ad hoc one. The next section demonstrates how to exploit these practical features, even when further constraints such as availability of resources and inter-stage buffers exist.

3.5 Automatic Setting of Capacity limits.

3.5.1 Constraint on Manufacturing Capacities.

In Section 3.2 and 3.3, it has been shown how structured limits are cost-effective solutions within a neighbourhood of solutions. In later section, the cost function is used to assess the various shortlisted options with structured limits. So far the actual physical capacity constraints of the manufacturing system have not been explicitly incorporated in the analysis. These have been implicitly included in assigning weights and structuring the functions so as to drive potential solutions away from those requiring excessive demands in capacity or those with slow recovery responses. The practical significance of the previous analyses is to provide some solution guidelines when the various capacity rates may be available within a fairly wide range. Such a situation is possible in certain industries such as car manufacturing where capacities do exist that will allow the rates of production lines to be substantially increased in certain circumstances. In more practical cases, some production stages would have a certain operating limits for a particular engine model or car body, while the limits at other production stages are still flexible. Therefore it is of benefit to adjust these flexible limits so as to be proportional to the ones which are fixed. The benefits of structured limits have already been demonstrated in Section 3.3.2. Control policies so derived dynamically adjust the capacity rates of the production stages in such a structured way with respect to each other such that each production stage refills the buffers in a smooth way with little or no overshoot or undershoot while simultaneously providing suboptimally for the demands of the next production stages. In addition, the amount of inventory depletion gives an indication on the minimum amount of safe stocks of assemblies required for the particular problem under consideration. An algorithm referred to as "@DIS/ICCA" has been developed for such a purpose and is shown in Figure 3.25. The nature of this algorithm is based on iteration: a short simulation run of 6 time-periods is performed with an arbitrary starting CPN. At the end of this trial, the maximum value needed at the critical production stage is compared to the actual practical limit. If the comparison is satisfied (within a certain tolerance), the maximum values at the other individual production stages are chosen as the new limits for the full proper run. If the comparison is not satisfied, the CPN value is incremented by one scale and the small test run performed again. This is done until the appropriate limits are obtained for the full run.

Illustration

Using the same problem characteristics given in Section 1:

Stage 3	:	Power unit assembly	, average scrap = 15%
Stage 5	:	Body painting	, average scrap = 10%
Stage 6	:	Final assembly	, average scrap = 5%

Extra demand for car units is 150/shift.

The production capacity at stage 3, i.e. the assembly of power units is assumed to have a capacity of 250 units/shift. A practical control strategy is to adjust the capacity rates of other production stages such that they are "structuredly balanced" with respect to the fixed one and between each other. Moreover in order to be truly effective, the corresponding amounts of safe stocks need to be worked out. The algorithm "@DIS/ICCA" given in Figure 3.25 is used and the resulting response is shown in Figure 3.26 on a dynamic basis for a time horizon of 24 time periods. Table 3.4.a gives the maximum values of the capacities required and the corresponding safe stocks.

Were the limiting production stage at 5, (painting of car bodies), with a constraint of 260 units/shift, the rest of the production stages would be as given in Table 3.4.b and the dynamic response is illustrated in Figure 3.27.

The effectiveness of the algorithm is clearly demonstrated.

3.5.2 Constraint with Inter-stage buffers.

It is noticed again that the control simulation has been performed without any consideration as to whether the required inter-stage inventories are available or not. It has simply been assumed that they are sufficient. The relevance of such an approach is in the provision of the necessary control measures as requirements of manufacturing resources and buffers. In some cases, there will be the opportunity to obtain the necessary buffers in due time, therefore the control has to be carried out as effectively as possible with two joint purposes. Firstly to make use of the various practical advantages obtainable from the present multivariable control theory approach (as structured limits) and secondly making "optimum" use of the existing resources as manufacturing capacities and actual buffers. Of course, these two objectives need to be co-ordinated with respect to each other. Thus, using the same problem specification as in 3.4.3 and the structured limits thereby obtained, an additional constraint is added to the size of the available inter-stage buffers.

These structured limits are :

273, 273, 250, 261, 240 & 200 at stages 1-6,

and their associated minimum buffers are:

250, 250, 228, 240, 218 & 318 .

These have been obtained with the subroutine "@DIS/ICCA" assuming a maximum operating rate of 250 units per shift at stage 3 ,assembly of power units.

Four separate runs where the inventory of gearboxes is assumed to be 250, 200, 150 and 100 respectively at the beginning of the simulation. An algorithm structured as "@DIS/ILCT" (Figure 2.10) described in chapter 2 is included. The new overall simulation model is now given in Figure 3.28. The Runs are referred to as D1 to D4 and are given in Figure 3.29, Run D1 being the fully structured one. The effect of the availability of the inventory is clearly reflected within the immediately connected production - inventory stages. The lowest values of depletion for runs D1 to D4 are given in Table 3.4 . When the gearbox buffer is small at stage 1 (run D4) there is a more severe state of depletion at the subsequent stage number 3 (power units) if the final demand is to be satisfied. The only minor advantage in this situation will be that less cost is attributed to the holding of buffers at both stages 1 and 2, i.e. engines and gearboxes.

The same cost structure is used for the four runs and the results are illustrated in Figure 3.29a-d. The cost-benefit in using a structured control policy with its associated float values as in Run D1, is clearly better than the rest. It is noticed, however, that the various states of the system are soon driven to their steady states, irrespective of the initial buffer conditions. (Figure 3.29). Nevertheless, this does not prevent looking for some measures which may improve the overall response. The following approach has been explored :

(i) Increase the immediate downstream capacity rate with the additional option of:

(ii) A decreasing in the capacity at the stage two steps ahead.

Wherever possible, production stages immediately linked to the one controlled down are also altered accordingly. This correction is achieved by the use of the algorithm "@DIS/ICCA" considering the new value as the fixed one. The correction routine referred to as "@DIS/INVC" is given in Figure 3.30.

It is pointed out that the first alternative is not always feasible, as in situations where the stage in consideration is already operating at its maximum capacity. It is then and only then that the second option is implemented. Of course, this is dependent on the fact that such a production stage is indeed present. Thus if the magnitude of the buffer at stage 5, i.e. painted car-bodies are insufficient to meet the full operating capacity at the final assembly (stage 6), the resulting severe depletion at stage 6 may not be alleviated since there is no stage 7.

Illustration.

It is now attempted to improve run D4, where the maximum float available at stage 1 is only 100 units of gearboxes at the initial conditions. Since the subsequent stage, 3, is already operating at its maximum level, the suggested procedure (option (ii)) is to lower the capacity rate at stage 6, the one after 3. The decrement is chosen to be approximately a tenth of the difference between the actual buffer size and the recommended one. Since this difference is 150, the maximum value of the operating rate at the final assembly is decreased by 15, i.e. from 200 to 185.

This value of 185 at stage 6 is then used as a fixed limit in running the subroutine "@DIS/ICCA" again. The new suggested values are :

250, 250, 230, 239, 220 and 185.

Only stages 4, 5 and 6 will take the new values since they can be considered to be together in a separate sub-system that need to be co-ordinated among themselves. Stages 1, 2 and 3 still keep their original values. Therefore, the new maximum limits recommended for this case (where the original constraints are maximum capacity of 250 units at stage 3 and 100 in buffer 1) are :

273, 273, 250, 239, 220 and 185.

The Run is referred to as D5 and is shown in Figure 3.31 . Superimposed with the drawing are Runs D1 and D4 for comparison. D1 is the fully structured response, D4 is the run where the inventory constraint is set, and D5 is the corrected response of D4.

The following points are noted from the responses in Figure 3.32.

(i) At stage 3, it is seen that the inventory of the power units depletes less in D5 than in D4, and has a slightly better restoring response.

(ii) The buffers at stages 4 and 5 show a relatively similar restoring pattern for D4 and D5.

(iii) The inventory at the finished cars has a slightly slower restoration to steady state.

Using the same cost equation as Section 3.3, the resulting costs are shown in Figure 3.32, it is demonstrated that a slight improvement is obtained for D5 over D4 in the overall. The improvements obtained from:

(i) the more spread-out usage at stages 4, 5 and 6.

(ii) the better inventory control at stage 3.

are eroded by the slower replenishment of the final finished cars. But the corrected response D5 is still better than D4 in overall, while D1 the structured response is the best of the three.

3.6 Contingency Measures for Probabilistic Disturbances.

3.6.1 Introduction.

In an actual industrial environment, various disturbances occur which reduce the number of effective units at the end of each production-inventory stage. Numerous events may cause delay and stoppages that will lead to severe imbalance in the system, if unchecked.

Some of these events are :

- machine breakdown
- tool change
- absence of operator
- repair time delay.

Any such mishap at the production stage or at a transit stage may cause the downstream production stages to be idle when they have used up their respective feeding buffers. The upstream production stages may also be blocked, as a result of the subsequent buffer banks having a physical storage limit. Such a state of affairs becomes increasingly critical with a production situation based on very frequent shipping from one stage to another, since the control operates with very small in-process buffers. In other words, control simulation based on time-intervals whose magnitude is small, will suffer from the above-mentioned problem more frequently. Therefore, in the procedure of controlling the production stages and inter-stage buffers so as to cater for the step input change in the final and/or intermediate demands, the various stochastic disturbances have to be taken into consideration together with an appropriate shipping rate.

The approach adopted so far, has been to consider each production stage with a certain constant efficiency rate, derived from historical statistical records. A deterministic value has therefore been used for the reject rates. As has been described in the previous chapters, such an approach has provided a definite and practical usefulness to the problem in terms of allocation of necessary resources, amounts of float on a dynamic basis.

In an actual environment, disturbances are random in nature, which would at first sight reduce the practical benefits of the technique described in the thesis. In order to prevent such a situation, the reject rates which previously have been assumed to be of fixed value are presently considered in their possible range of occurrences. The Gamma distribution is used for the probabilistic analysis. The approach adopted is to perform the runs separately a number of times corresponding to varying values of reject rates. Each run is still being associated to a constant reject rate but, with a series of such runs, it is possible to have a band of responses in which the actual event has a known probability.

3.6.2 Gamma Distribution:

In order to treat the variations of the disturbances in a computationally easy manner, the Gamma distribution is chosen to represent the possible reject occurence in the system:

$$f(x) = \frac{\alpha \quad (\alpha\ x)^{r-1} e^{\alpha x}}{\Gamma(r)}, \qquad x > 0$$
$$= \quad 0 \qquad , \qquad \text{elsewhere.} \qquad \ldots 3.8$$

where r and α are two parameters.

$\alpha > 0$

$r \geq 0$

$\Gamma(r) = (r-1)!$

If X has a Gamma distribution as given by equation 3.8 , it can be shown that:

E(x), the expected value = r/α .

V(x), the variance = r/α^2

The presence of two parameters r and α allows a higher flexibility in the matching the analysis to the practical case.

Such distributions for 3 expected values 5, 10, and 15 are shown in the sets of graphs of 3.31, 3.31 A - C respectively. For each set, the distribution profile is given for a range of values of parameter r (2 - 7) and is arbitrarily chosen as one. As shown in Figures 3.33, the parameter r is very effective in controlling the probability distribution profile so as to match the actual case.

The probabilty function is given as :

$$P(X' > x) = \sum_{k=0}^{r=1} e^{-\alpha x} (\alpha\ x)^{k} / k!$$

The functions are shown in Figures 3.34 to 3.36 with r = 2 to 7 for the three expected values 5, 10, and 15. For the present analysis, r = 7 is assumed to be the case more representative of the actual case and the probability functions for r = 7 with expected values 5, 10 and 15 are shown in Figure 3.33. From this last figure, probability values may be obtained and have been tabulated in Table 3.6.

3.6.3 Probabilistic analysis.

In this section, it is shown the probability studies have been carried out while still using the same control model. Four separate scenarios with different reject rates (Table 3.7) are investigated. The control problem as:

How to adjust the various production rates when there is a limiting capacity rate of 280 units/shift at power unit assembly. ? The demand rate is 150 units of finished cars / shift.

The adjustment of production rates is obtained from the algorithm "@DIS/ICCA" descibed in the previous section and the control simulation performed in four separate runs E1, E2, E3 and E4. The results are shown dynamically in Figure 3.34. The salient features as maximum capacity inputs and the inter-stage buffers for each run with different scrap occurence are tabulated in Table 3.8.

From Figure 3.34 and Table 3.6, it is possible to perform the following analysis:

(i) The probability that the actual system respond as delineated between the two conditions in Runs E4 and E3 is

$$P1 = P(1 > x_5 > 0) = 1 - .014228 = .985772$$

$$P2 = P(15 > x_{10} > 0) = 1 - .101632 = .898368$$

$$P3 = P(20 > x_{15} > 0) = 1 - .178081 = .821919$$

where x_s = variable whose expected value is s.

Therefore it is possible to state with a probability of .72788 that the system will undergo the dynamic states as designated between the responses E4 and E3. This gives a knowledge of the range of the expected resource requirements and the range of inter-stage buffers as shown in Table 3.8

This analysis can be repeated for the various other runs. Thus, if it is needed to know the probability of system response between E3 and E2, it is as follows:

P1 = $P(10 > x_5 > 0) = .985772$

P2 = $P(15 > x_{10} > 5) = .83308$

P2 = $P(20 > x_{15} > 10) = .631042$

P1 X P2 X P3 = .518228

The probability that the system reacts between E3 and E2 is thus .5183

3.6.4 Discussion.

It is noted that the above analysis provides a very straightforward approach for the provision of contingency measures under various conditions of scrap or reject rates. The same analysis can obviously be repeated when an additional constraint is introduced as to the availability of buffers as discussed in Section 3.5.2, i.e. with the inclusion of the "@DIS/ILCT" routine.

The main feature of the approach is that each separate analysis can only consider the disturbance as constantly acting, i.e. at a constant average value. This is the result of the particular canonical form adopted. (Brunovsky,1966,/67/). This characteristic will therefore exclude an analysis involving a randomised reject situation. Nevertheless, it is strongly believed by the author and also by the management in question, that for the actual control problem considered here, the approach fulfills its purpose. It is recalled that the present control problem is to monitor and co-ordinate the production and inventory states of assemblies in multi-stage manufacturing system with the duration of a working shift as one productive unit. These values can then be set as target values for each shift. The control problem of obtaining these assemblies is at a lower level in the hierarchical problem. It is at this new level that there is the need to take into consideration the associated stochastic disturbances, for example the vagaries in the monitoring of engine block lines, crankshaft lines, piston lines and other sub-assemblies. The control of these individual lines form a separate study, and have been extensively analysed. (References /95/ - /105/).

3.7 Discussion and Conclusion .

In this chapter, it has been shown how automobile manufacture lends itself as a case study for the multivariable control model developed in Chapter 2. This has been achieved by linearising the manufacturing system into a linear discrete-time multi-stage production-inventory model and applying the control tool developed in Chapter 2.

The concept of "structured control policies" has also been introduced in such a control environment where it is necessary to co-ordinate the production capacities and levels of inter-stage buffers of assemblies. The synthesis of such structured control policies is achieved by setting all the CPN's at the same value. These structured control policies have been shown to be local sub-optimum control solutions by the use of a cost structure developed in the previous chapter.

The selection of such control policies has also been developed to cater for the situation where practical constraints have to be taken into account. These are in the availability of the manufacturing capacities and the actual floats ofassemblies.

The analysis has been repeated on a probabilistic basis with the introduction of the Gamma probability distribution. Such an exercise has provided a further insight into the range of different scenarios possible in such situations together with a knowledge of their probable occurrence.

In the various work carried out in modelling and control of manufacturing systems in a dynamic mode, the actual choice of the duration of the decision time interval has been fairly arbitrary. Forrester (1961,/28/) studied the effect of varying the time delay in the acknowledgement of a signal, synthesising a control measure, and actually implementing it. The study of the actual duration of the unit time period per se has been left untouched by subsequent workers. Christensen and Brogan(1971,/1/), Porter et al (1976,/3/) used the time period of one week for their manufacturing models. One aspect implicit in the adoption of a certain time duration is that the products only move from one production stage to another at intermittent periods of the chosen duration. The practical implication is that the number of units that has to be moved needs to be a work load sufficient to keep the subsequent production stage busy until the next shipment. Obviously, the longer this unit time period is, the larger the amount needs to be. Moreover the dynamic nature of the "push-type" analysis will amplify any values into higher fluctuations. Therefore in a practical environment such as transfer line production, such an analysis will lack practical significance if an unrealistic time period is chosen. It is considered by the author that this has been the case in some earlier work.

In this analysis, the duration of one shift has been adopted as the decision time interval. This implies movement of the products at the beginning or at the end of the shift, which is both technically feasible and actually practised for certain particular models. The technical feasibility depends on the integration of the production stages with respect to each other, i.e. how close or remote are the different production plants. Thus for some car manufacture where the engines, car bodies and final assembly are carried out in different parts of the country or even in different countries, the time period that has to be considered in the analysis will be of a very long duration. This will involve the shipping of large batches due to the uneconomic nature of sending small batches. This state of affairs has led to very high stockbuilding policies at the various production stages to cater for various contingencies. Even the transit inventory is usually very substantial since basically it has to provide a work load capable of keeping the next plant operational until the next shipment.

Of course, it is appreciated that such examples of car manufacture where the different assemblies are produced at different locations have been a result of various strong socio - economic reasons rather than logistics ones. Companies starting from a "green-field" situation like the Japanese Toyota (Sugimori et al, 1977,/106/) have managed to adopt a more logistic approach for car manufacture and have been able to reap substantial benefits . These include :

- Integrated factory for car production.
- Closeness between production stages allow the possibilty of more frequent shipment, if not a virtually continuous one. If this integrated aspect were to be modelled mathematically it will mean a dynamic analysis using a shorter basic unit time period as opposed to a longer one associated with a production system with intermediate plants very far apart.
- Very little in-process inventory. It usually amounts to only a couple of pallets as compared to warehouses or ferryloads of engines or car-bodies. This has been the result of the "Kanban" (just-in-time) production based on a "pull-type " manufacturing practice.

On the other hand, this practice of working within very tight tolerance of small inter-stage buffers has its disadvantages, namely the possibility of stopping the whole production line if the minute in-process inventory is used up faster than expected or the feeding production stage is unable to provide the required rate. The argument from such practitioners is that attention can thus be focused onto the problematic area.

In order to avoid the above situation, the present research has attempted to combine the best of the two practices, i.e. pull and push systems. It uses a short decision time interval as in the pull system which can only be achieved with an integrated manufacture. Here it has to be pointed out that the present research has benefitted from the favourable situation whereby the host company had actually an integrated manufacture for a new car model that was being launched. The closeness between the different production-inventory stages made the choice of one shift as the decision time interval very practical. These are practical rates of shipment of the assemblies for the particular model. Whilst they are not as close as to that of a pull system, the present synthesis of control based on the push system provide:

- Structured control policies with regard to the dynamic capacity requirement and float fluctuation that satisfy the prevailing demands and take into account the practical constraints.

CHAPTER 3

FIGURES

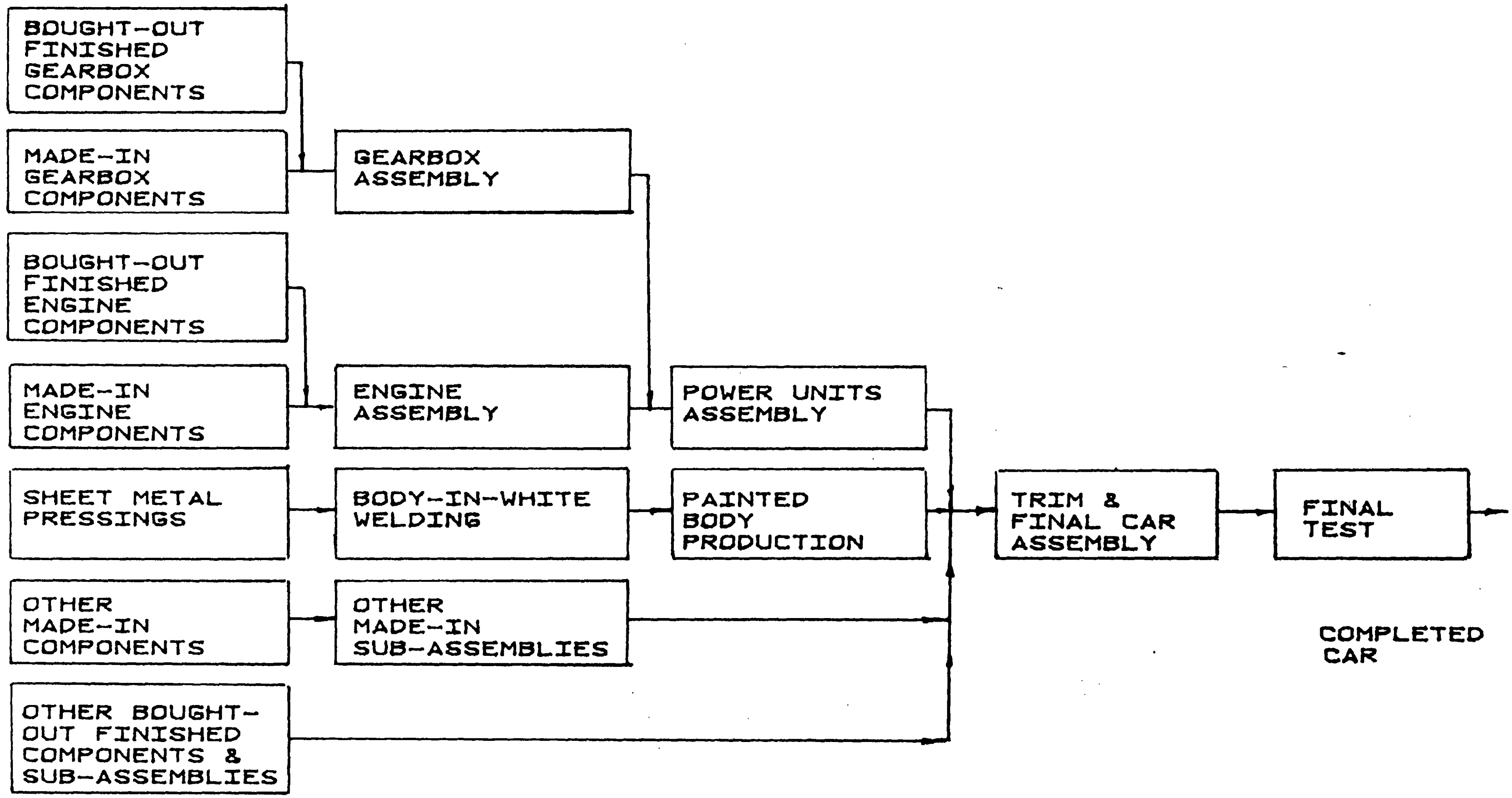


FIGURE 3.1.a : SCHEMATIC DIAGRAM FOR MAJOR STAGES IN CAR MANUFACTURE

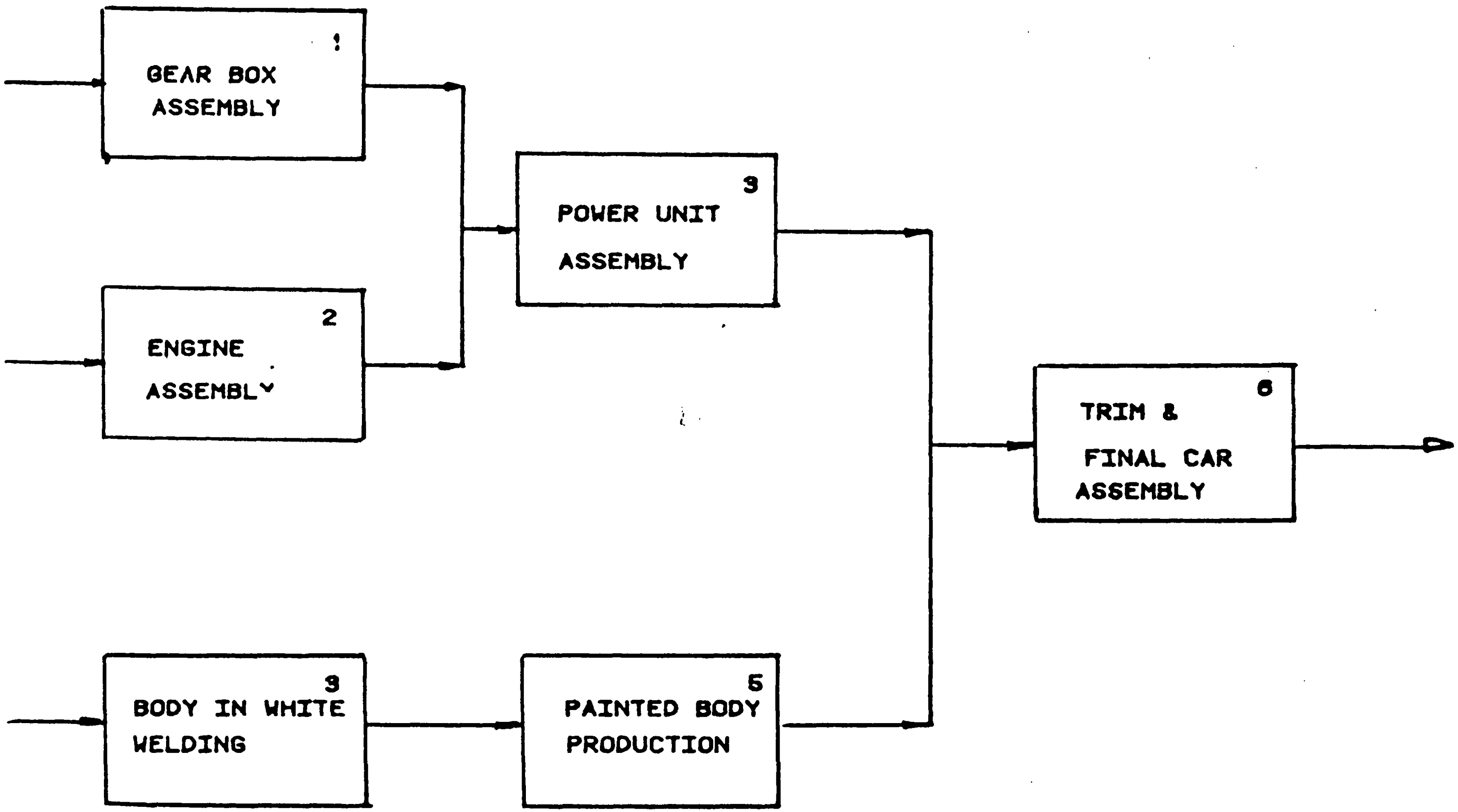


FIGURE 3.1b : REPRESENTATION OF AUTOMOTIVE MANUFACTURE IN 6 MAJOR PRODUCTION-INVENTORY STAGES.

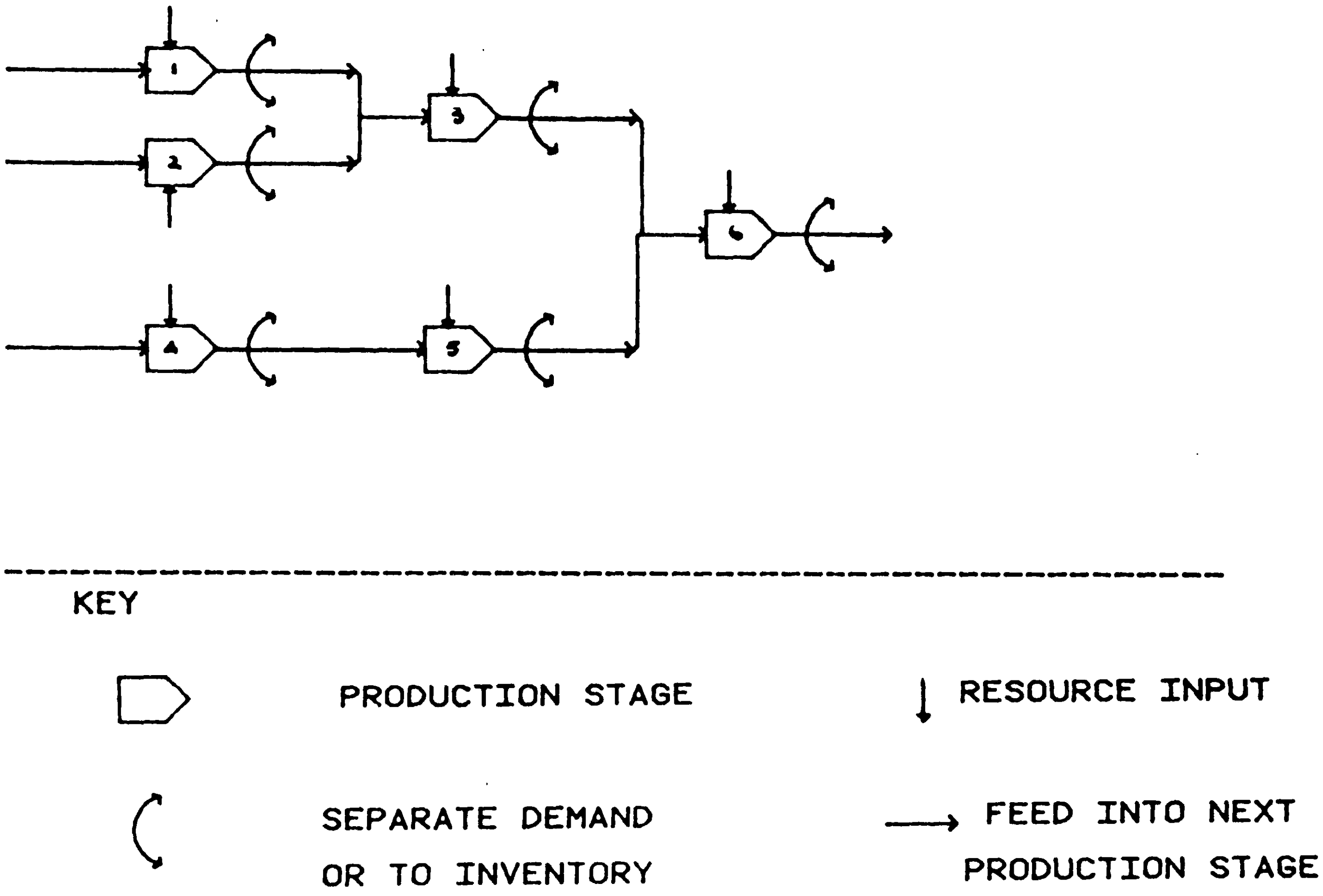


FIGURE 3.1C . SCHEMATIC MODEL OF AUTOMOTIVE MANUFACTURE

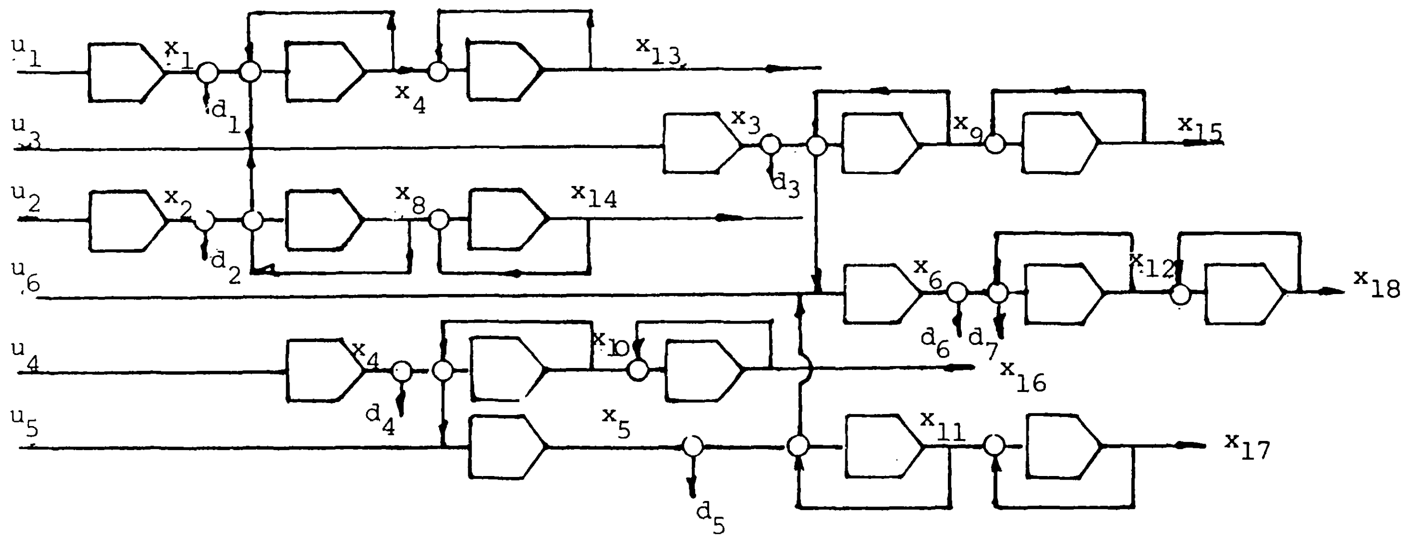


FIGURE 3.1d : MATHEMATICAL REPRESENTATION OF MODEL

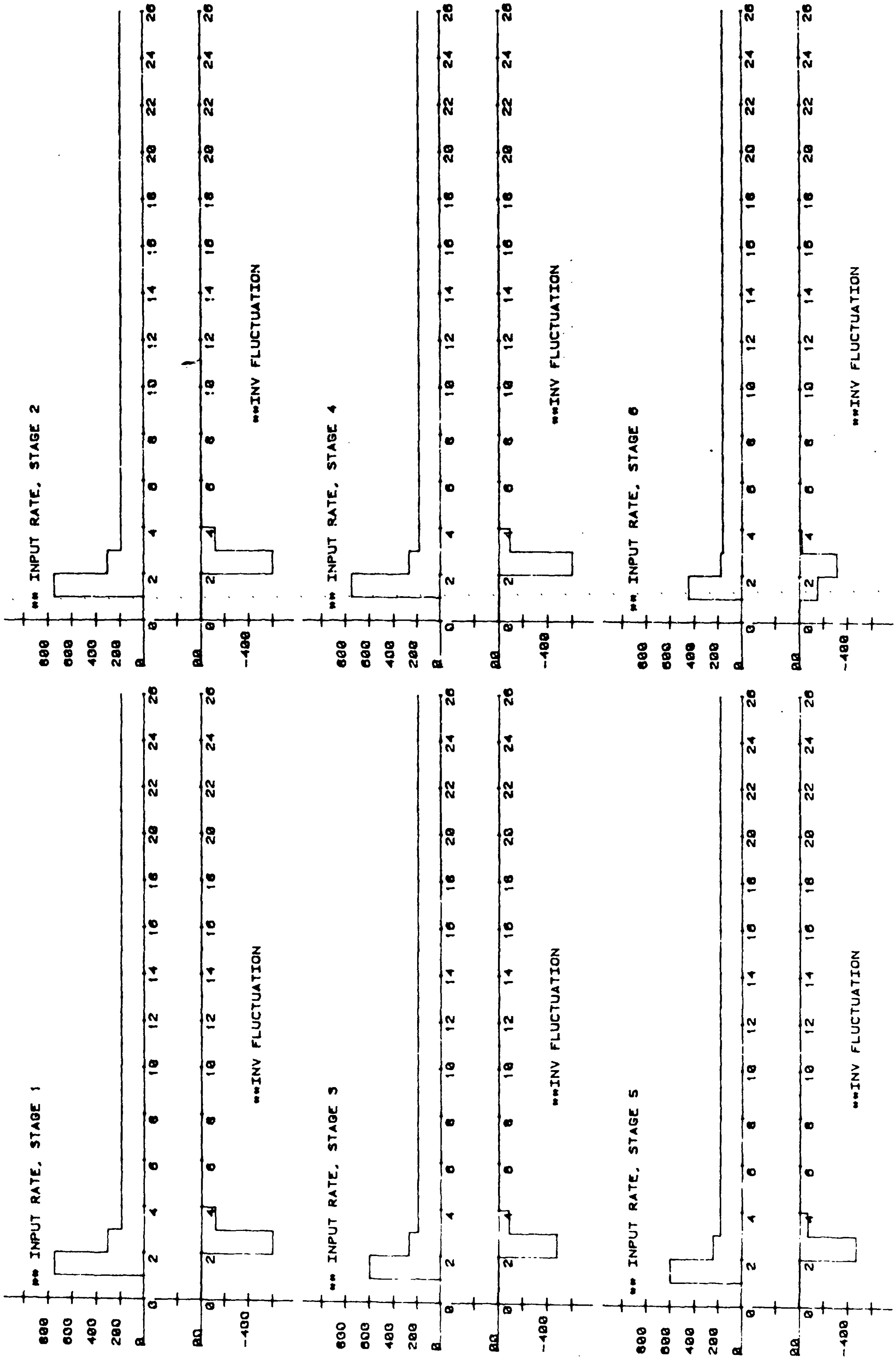


FIGURE 3.3 . RESULTS OF SIMULATION WITH CPN 1

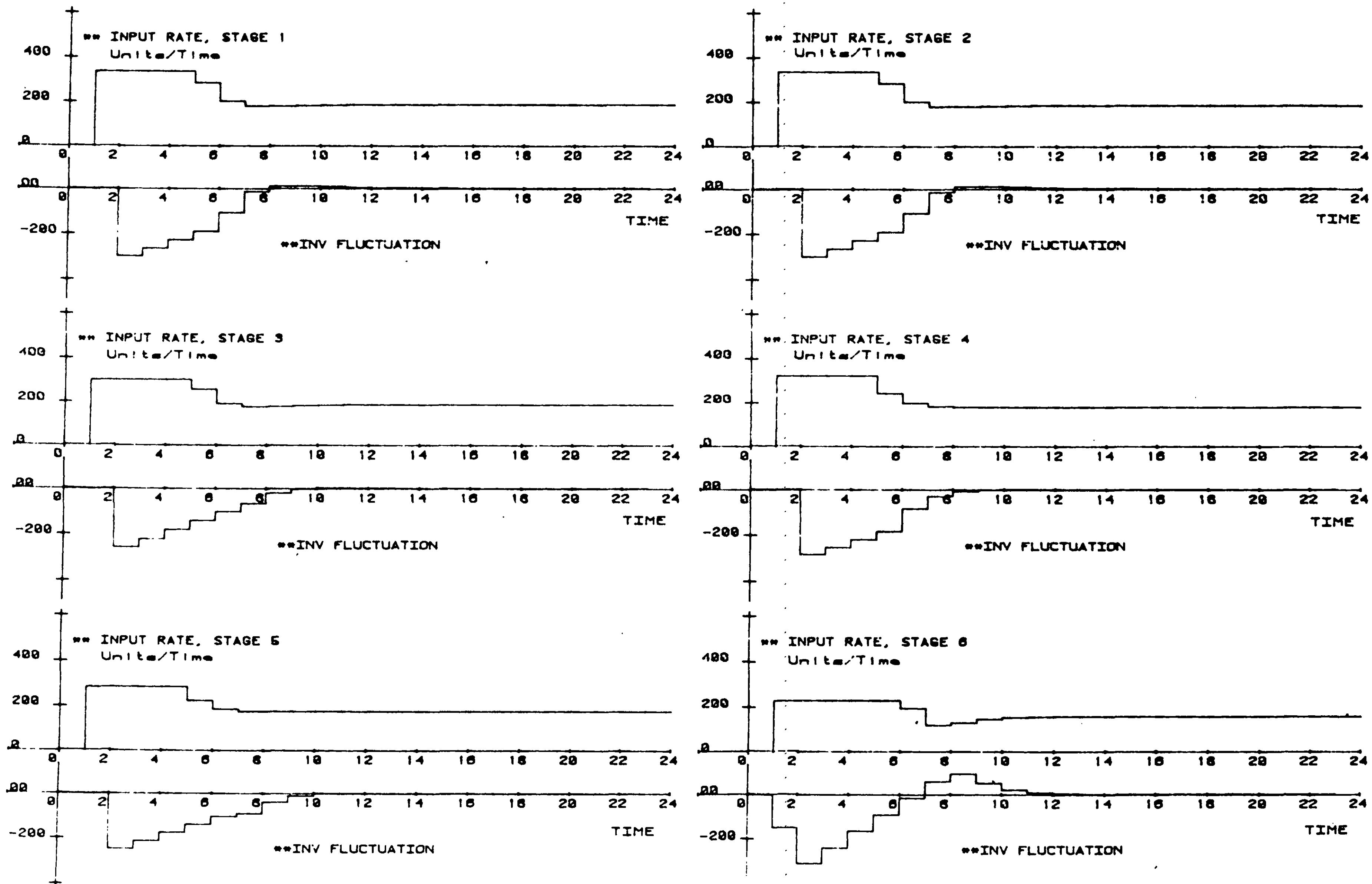


FIGURE 3.4: DYNAMIC RESPONSE OF SYSTEM WITH CPN 20

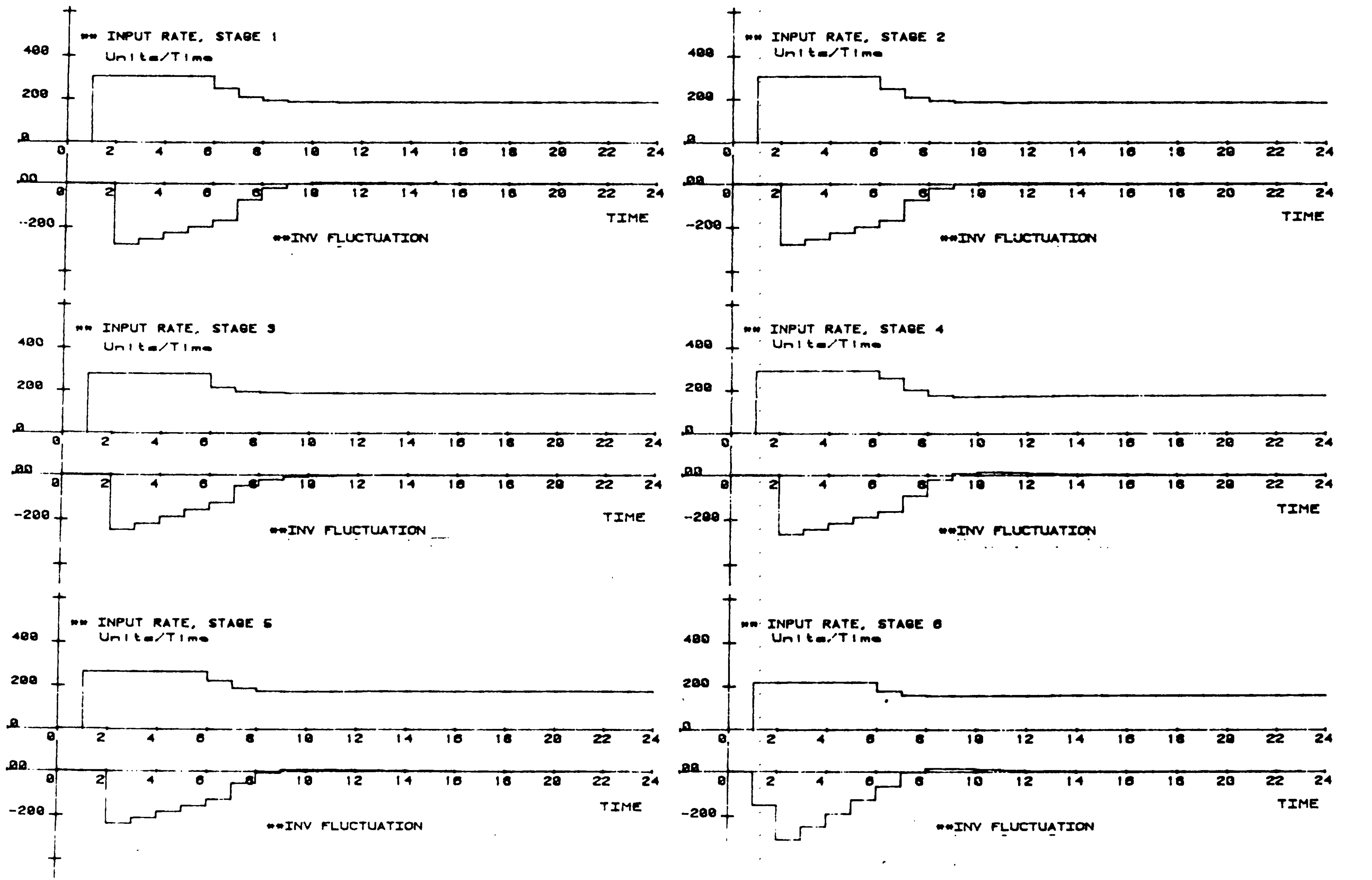


FIGURE 3.5: DYNAMIC RESPONSE OF SYSTEM WITH CPN 24

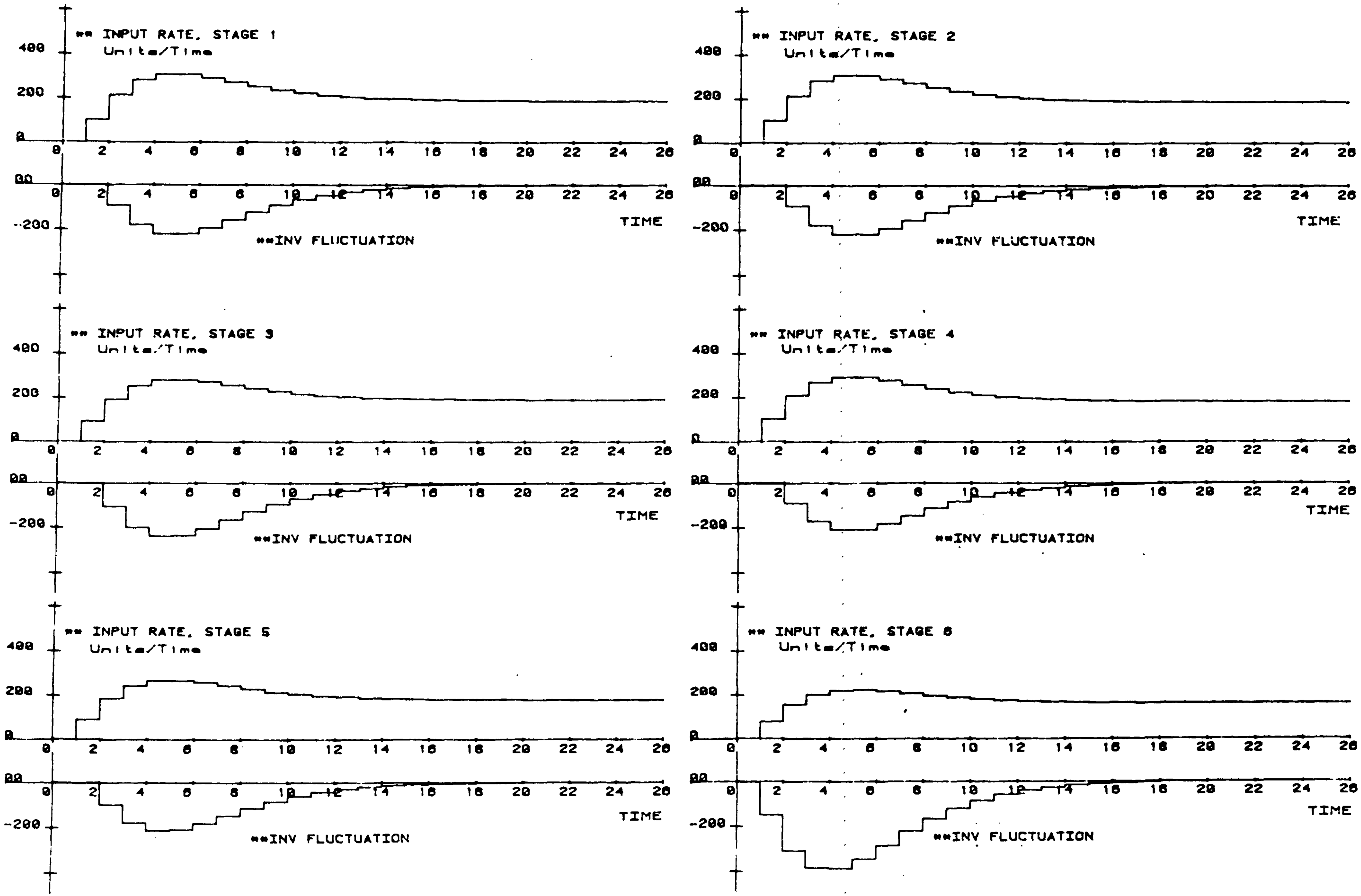


FIGURE 3.6 : DYNAMIC RESPONSE WITH CPN 24 (NO RESET)

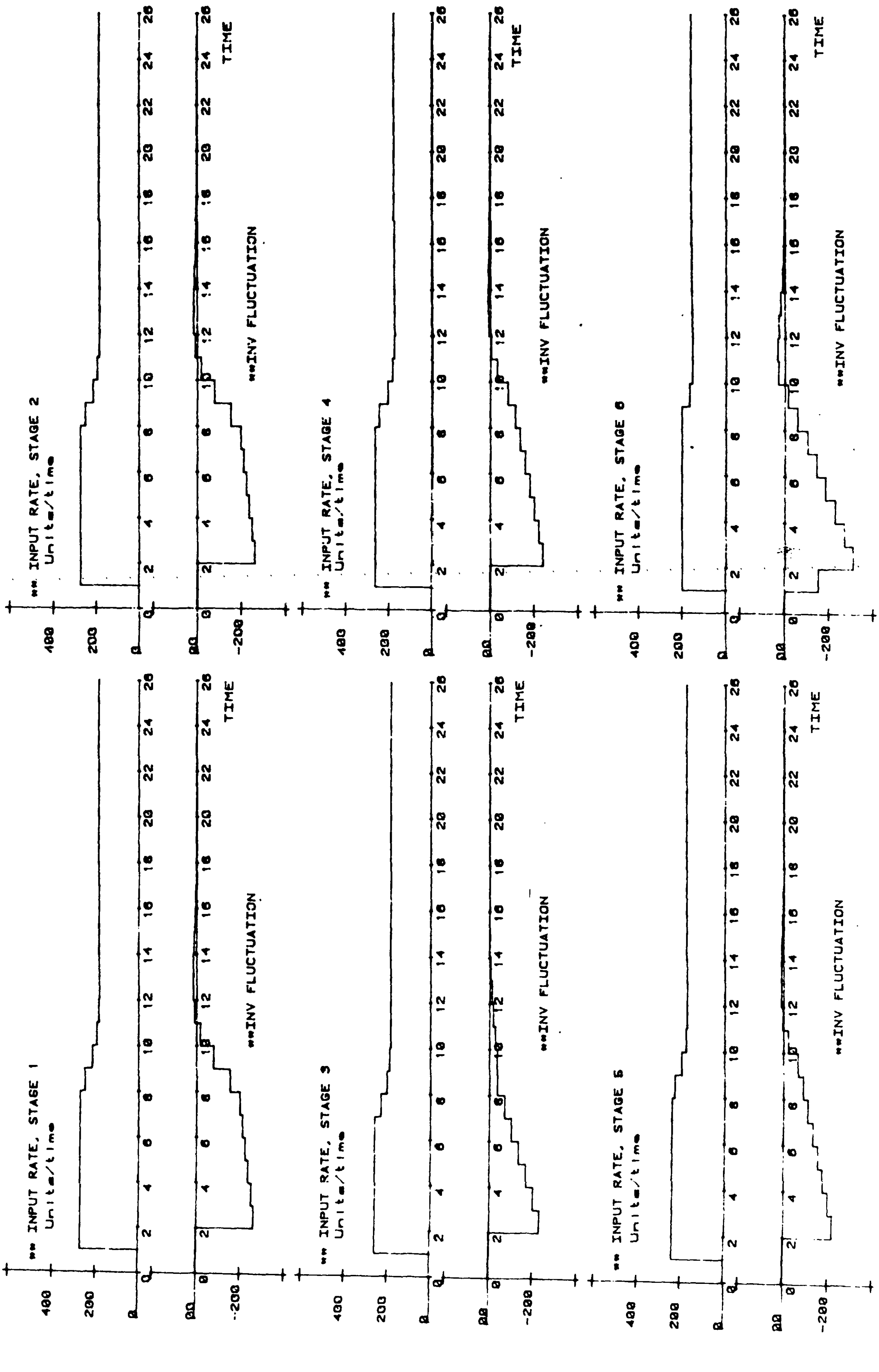


FIGURE 3.7 : STRUCTURED DYNAMIC RESPONSE OF SYSTEM, RUN B4

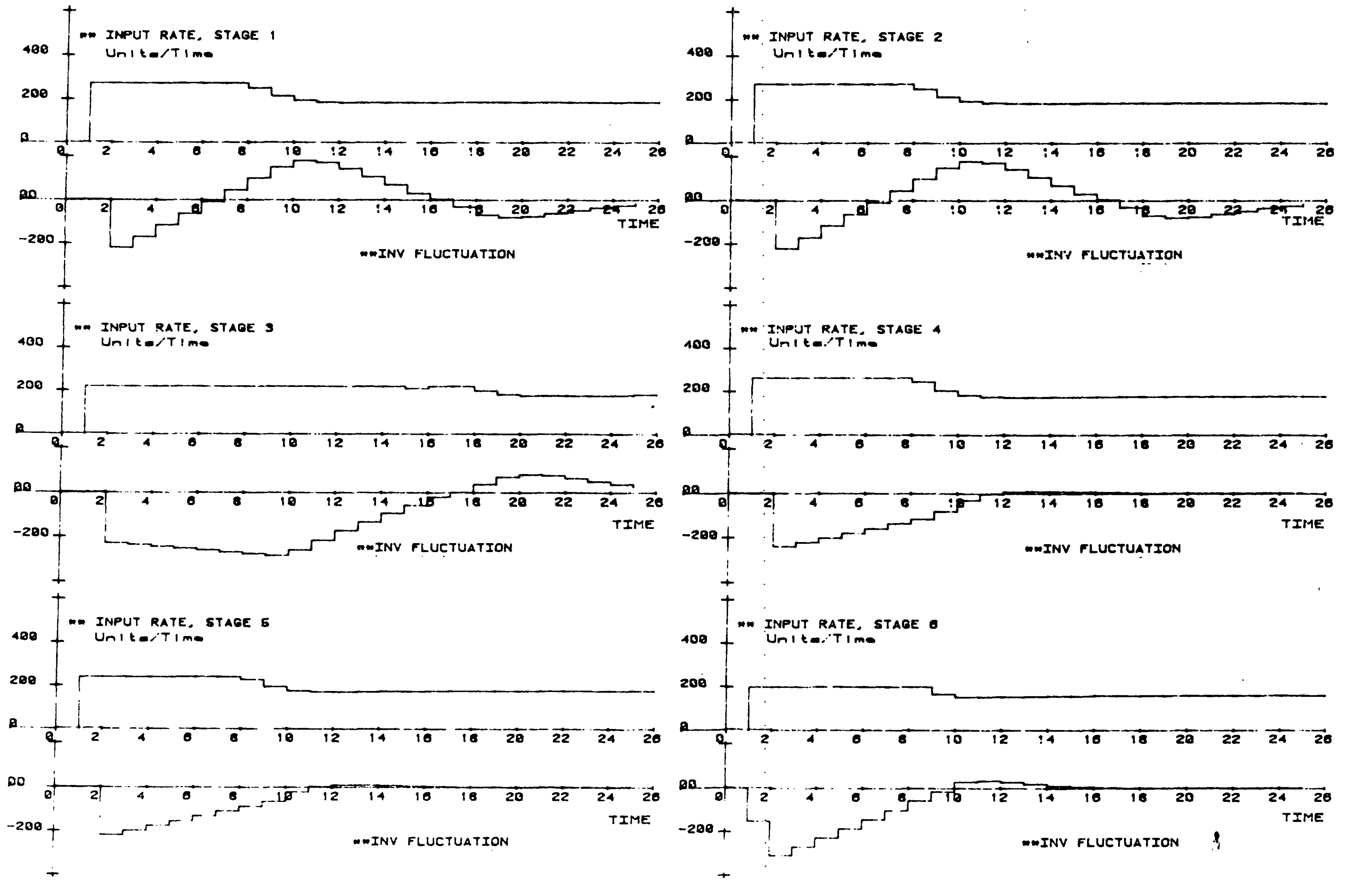


FIGURE 3.6 : DYNAMIC RESPONSE OF SYSTEM, RUN B1

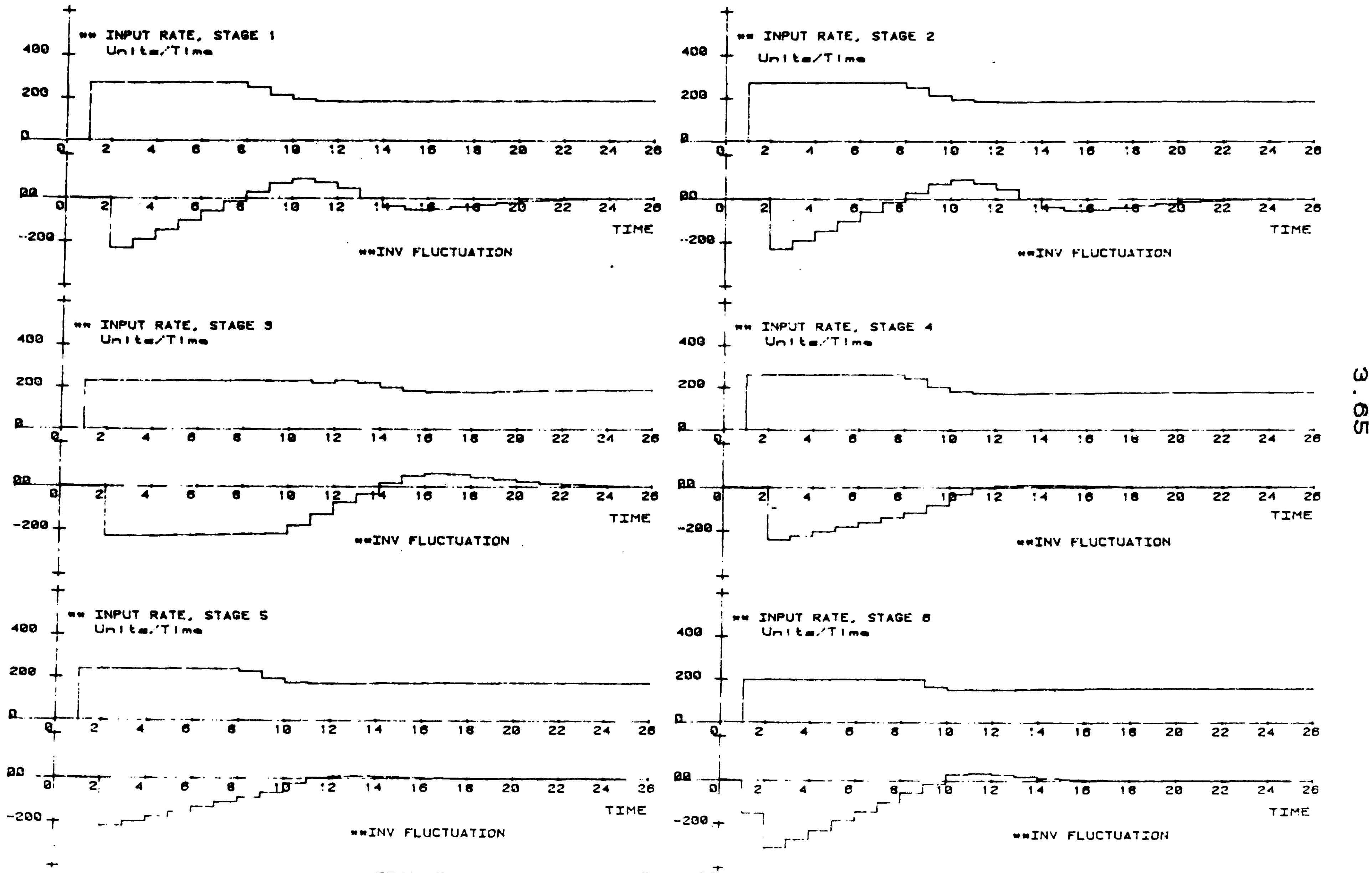


FIGURE 3.9 : DYNAMIC RESPONSE OF SYSTEM, RUN B2

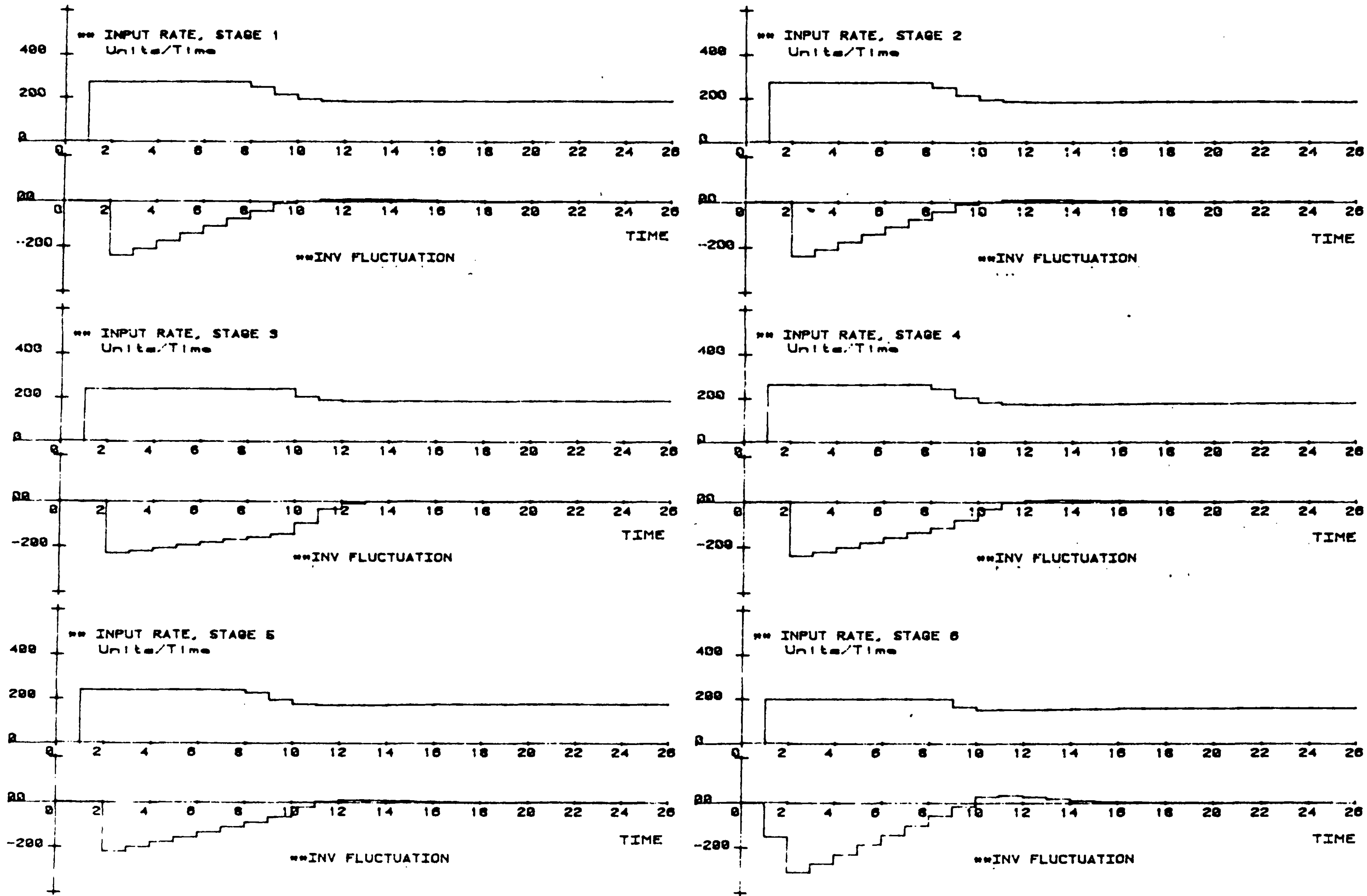


FIGURE 3.10: DYNAMIC RESPONSE OF SYSTEM, RUN B3

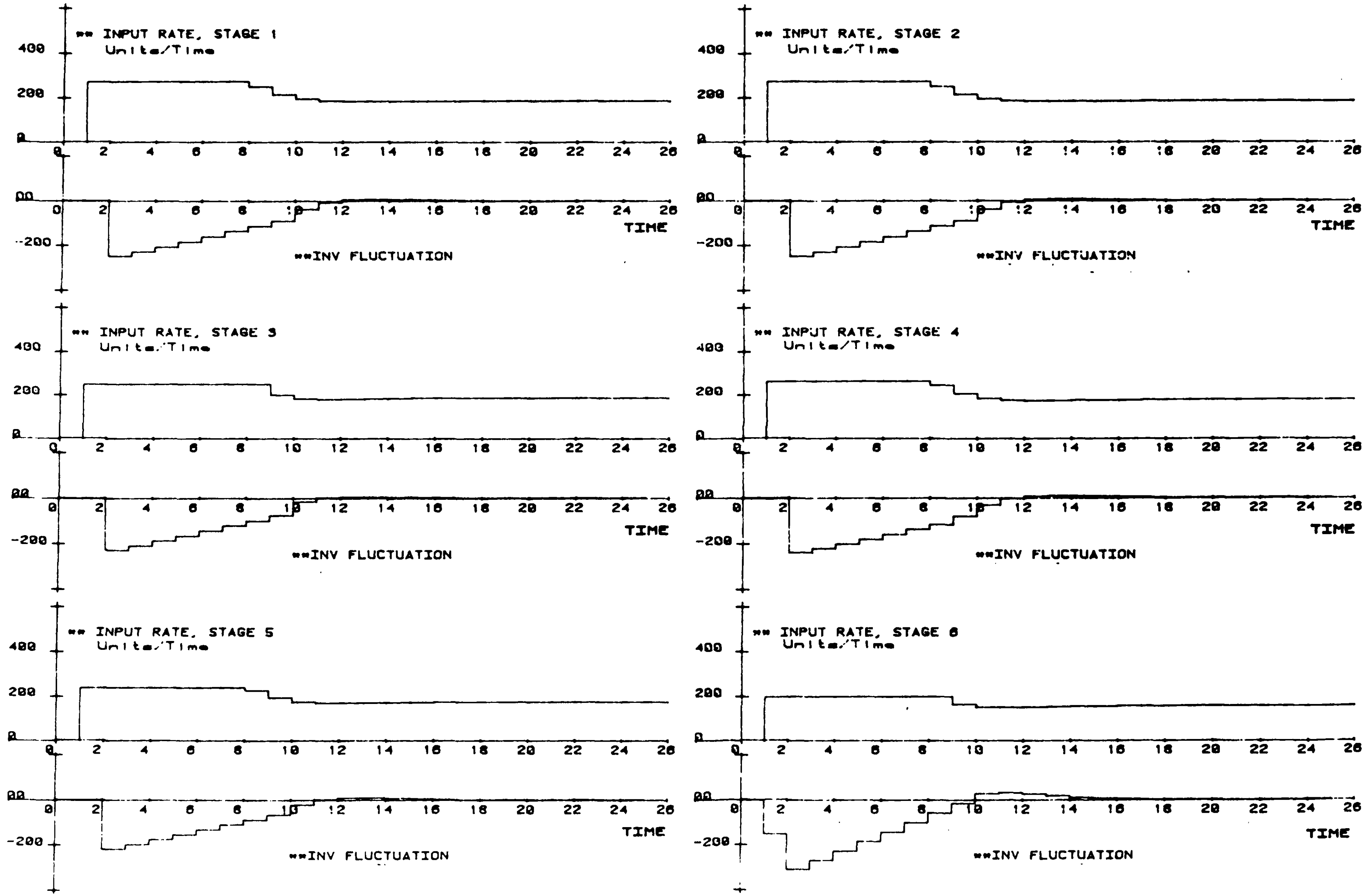


FIGURE 3.11: DYNAMIC RESPONSE OF SYSTEM, RUN BS

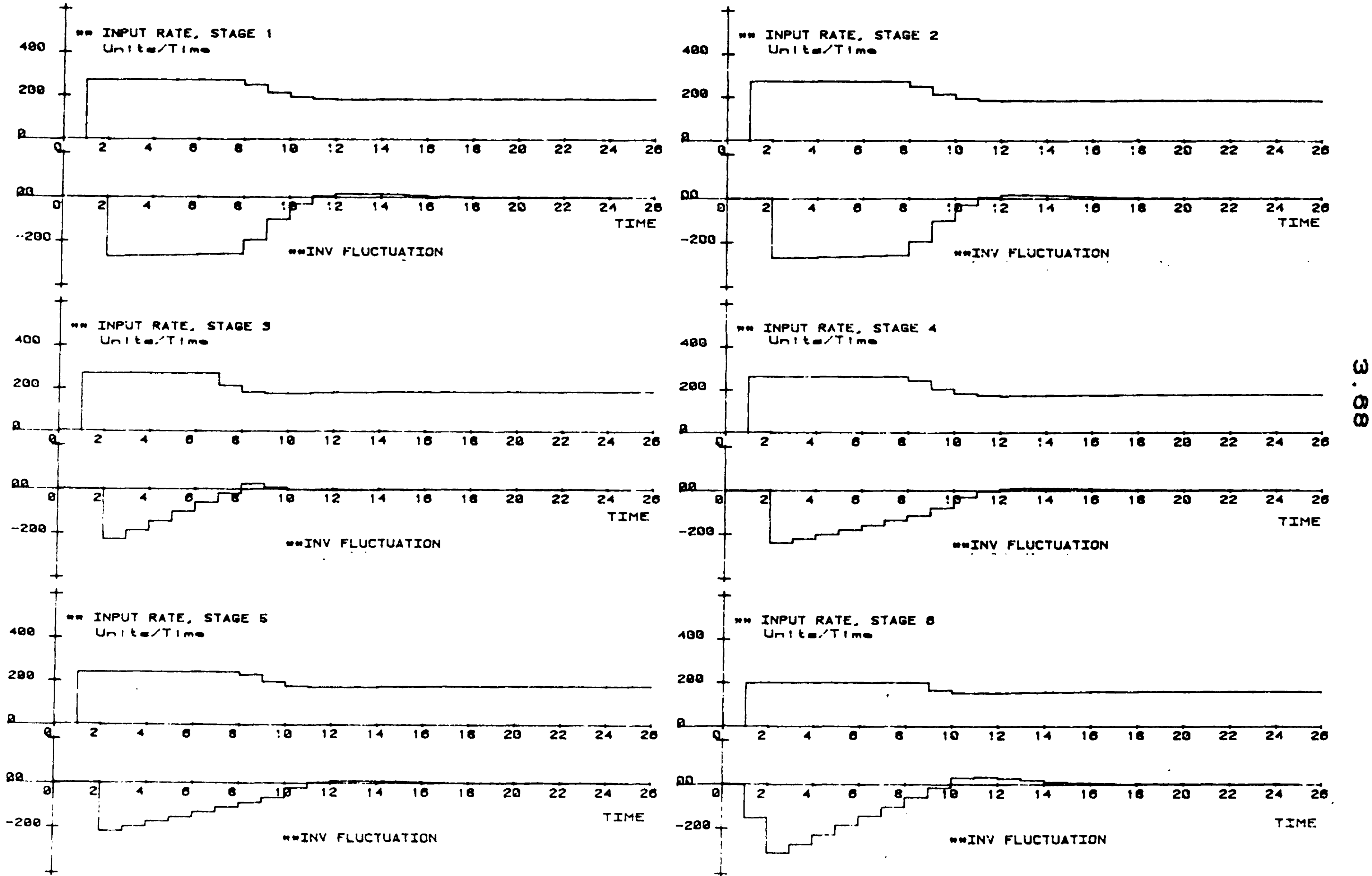


FIGURE 3.12: DYNAMIC RESPONSE OF SYSTEM, RUN B6

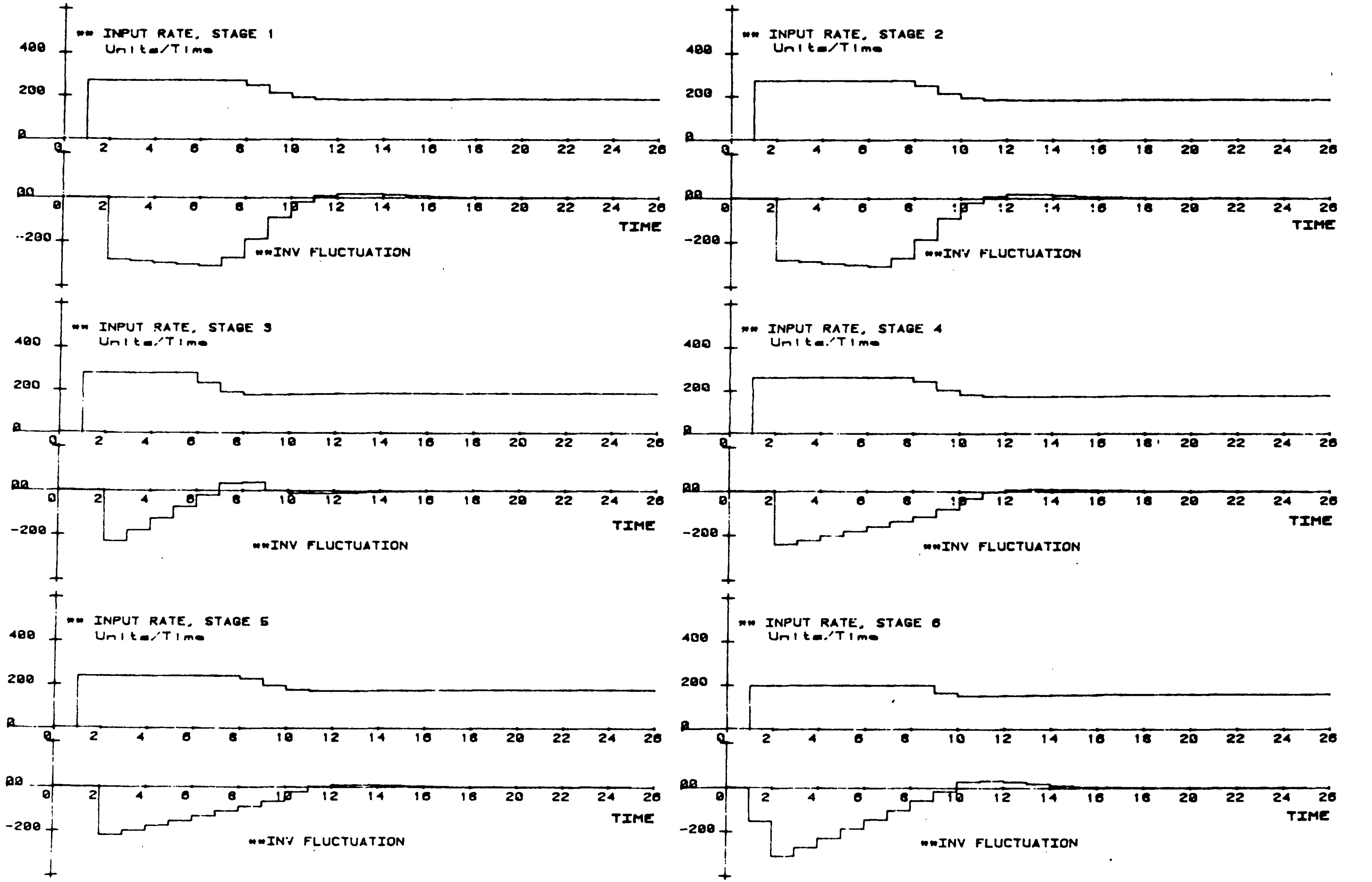


FIGURE 3.13: DYNAMIC RESPONSE OF SYSTEM, RUN B7

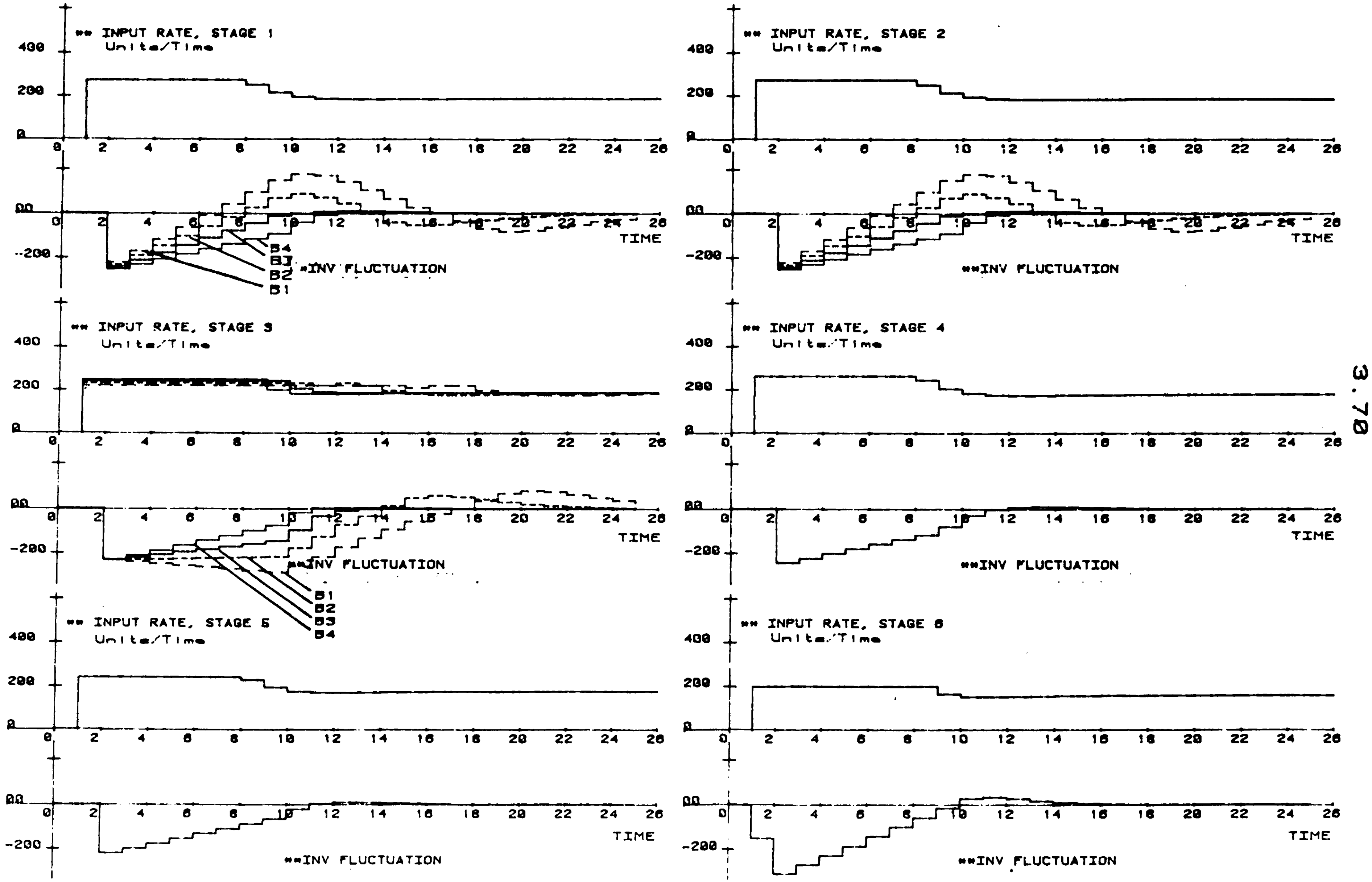


FIGURE 3.14a: DYNAMIC RESPONSES OF SYSTEM, RUN B1-B4

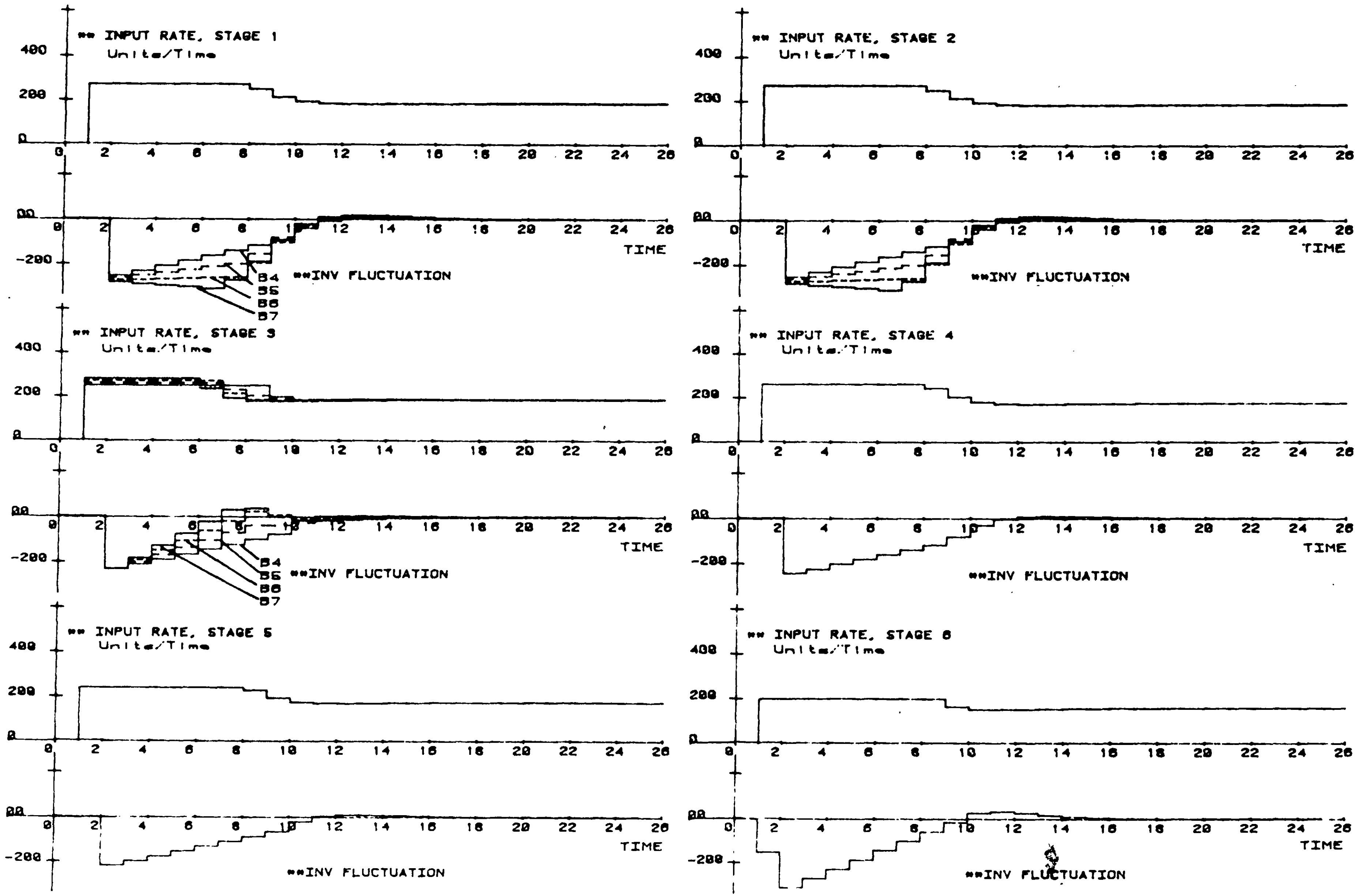


FIGURE 3.14B, DYNAMIC RESPONSES OF SYSTEM, RUN B4-B7

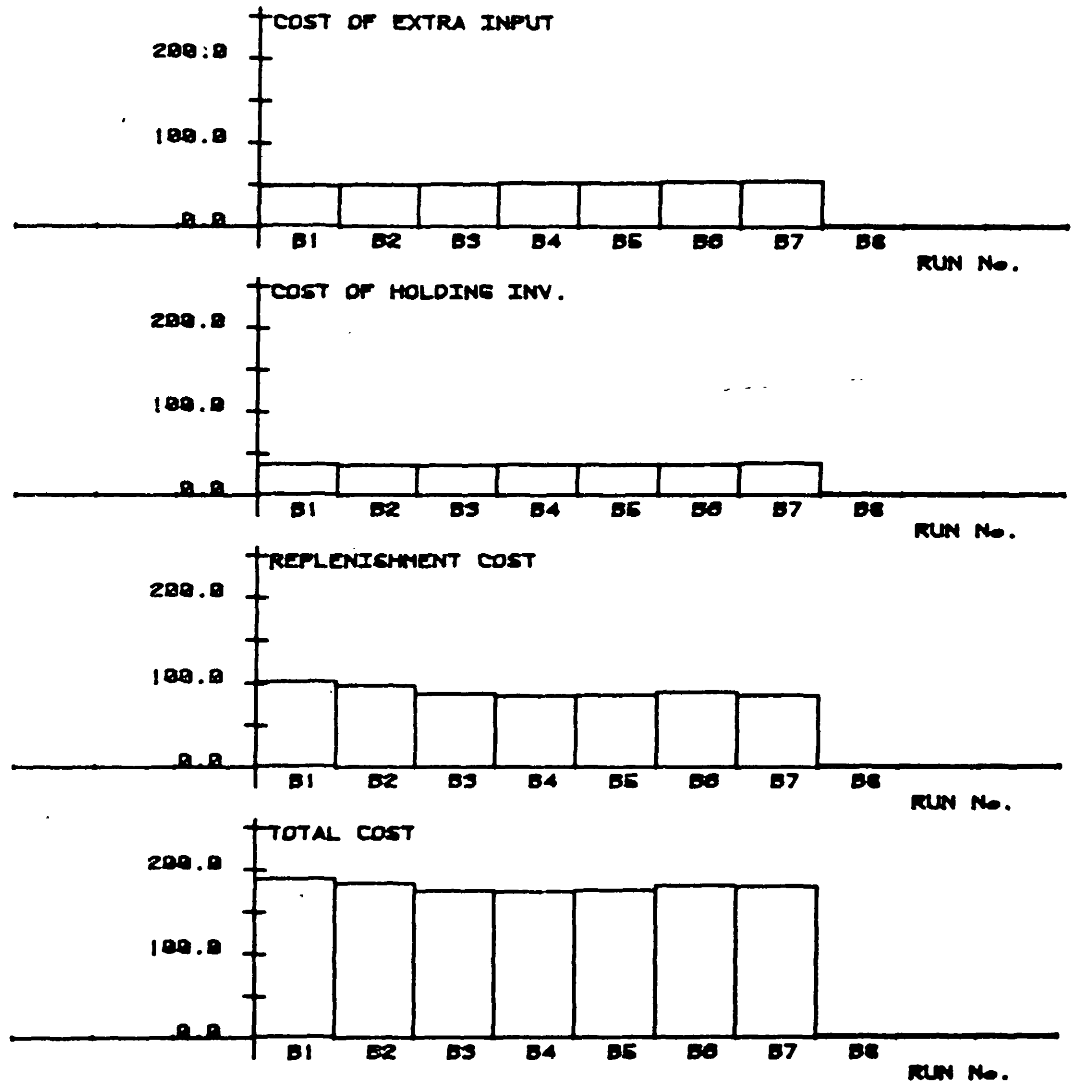


FIGURE 3.15a-d : RESULTS OF NORMALISED COST STRUCTURE

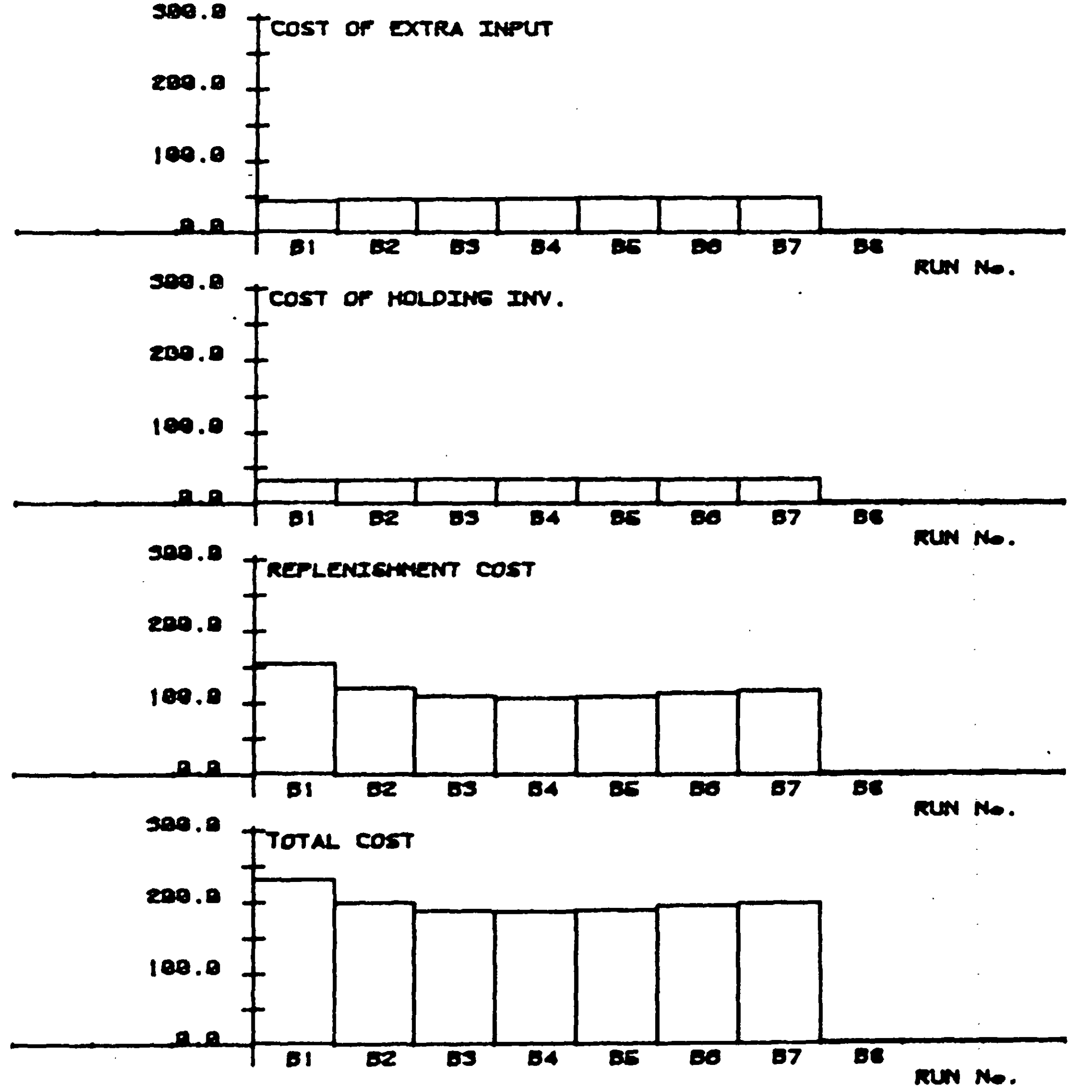


FIGURE 3.10a-d : RESULTS OF NORMALISED COST STRUCTURE (NO SCRAP).

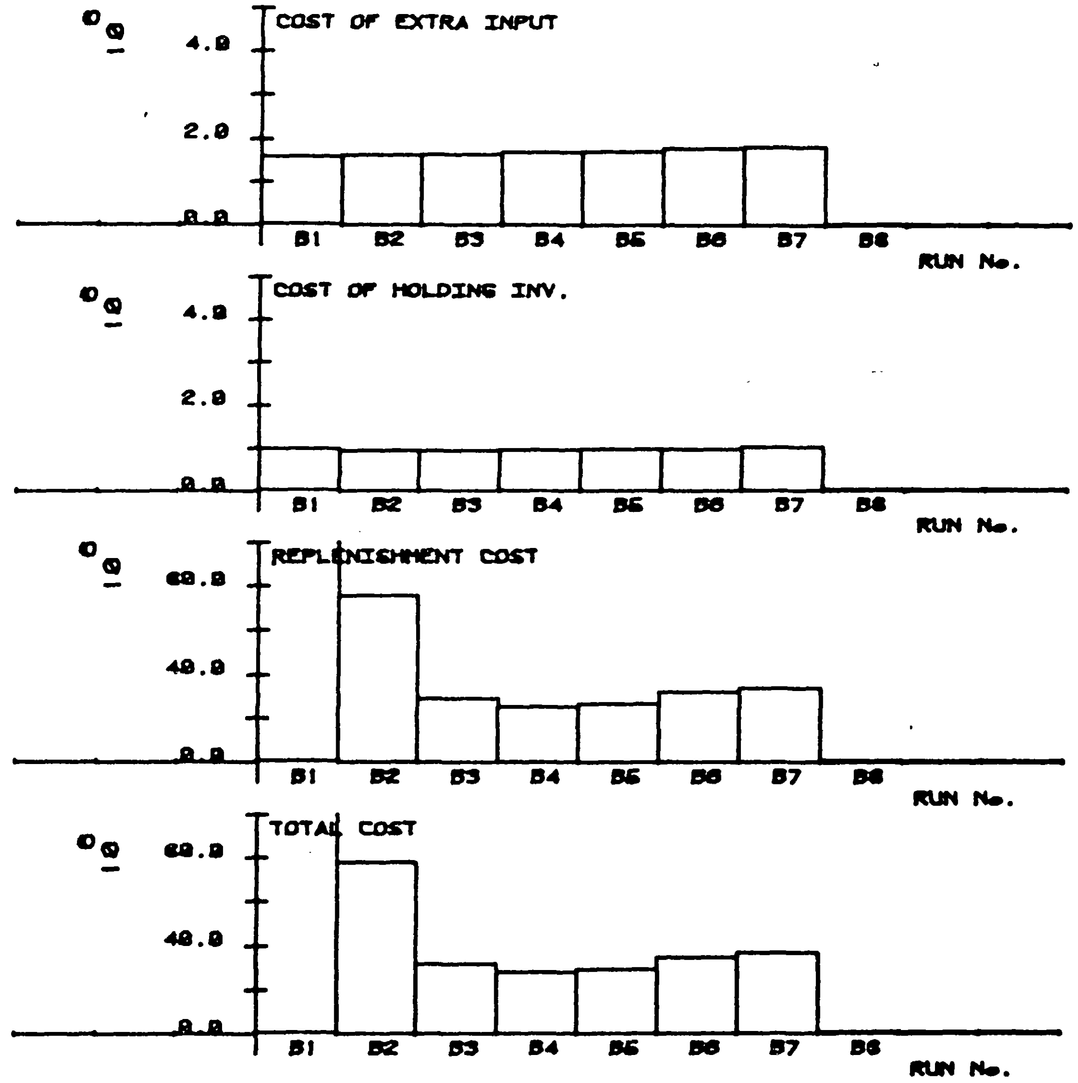


FIGURE 3.17a-d : RESULTS OF NON-NORMALISED COST STRUCTURE

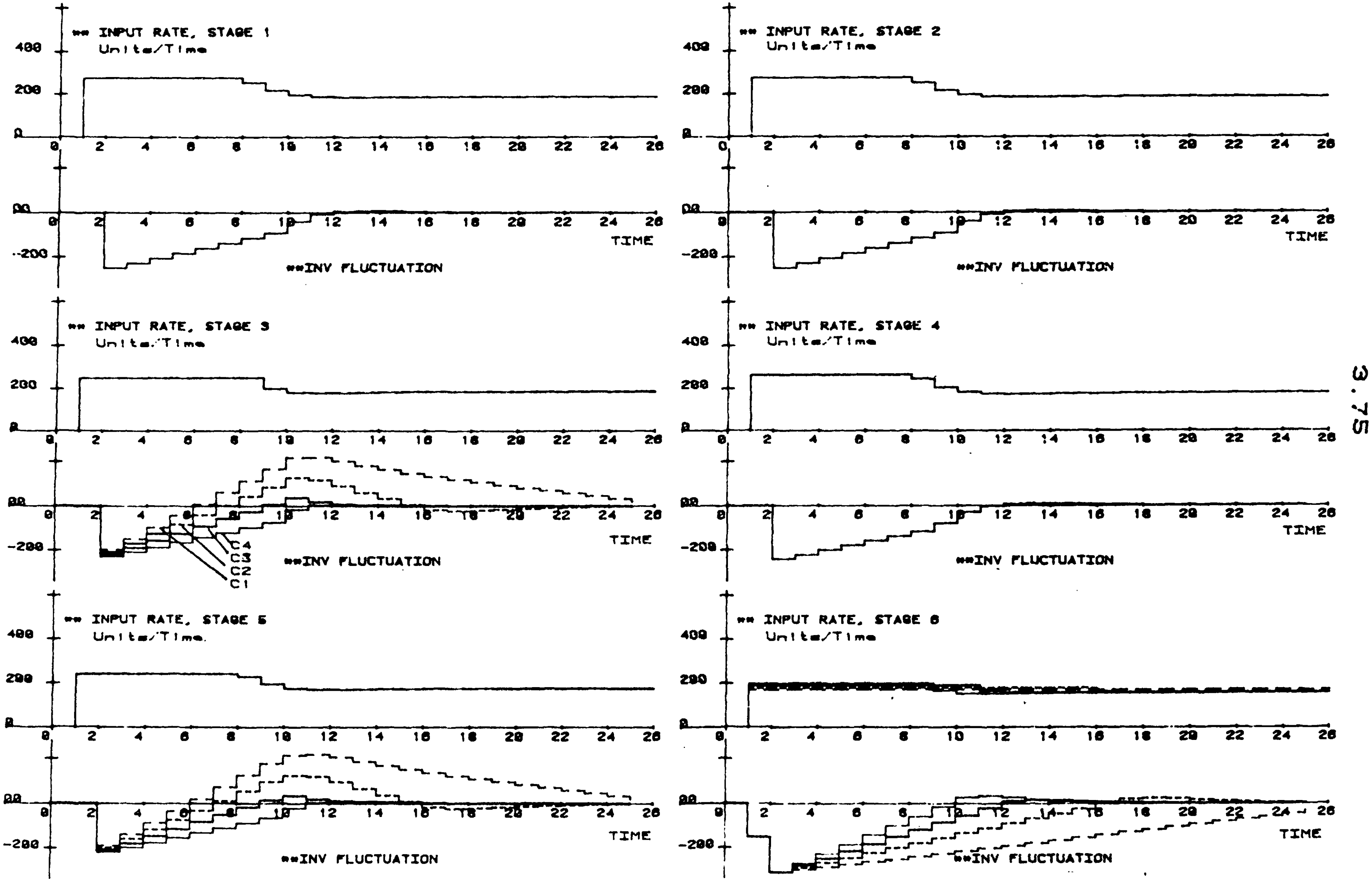


FIGURE 3.16 : DYNAMIC RESPONSE OF SYSTEM IN RUNS C1 TO C4

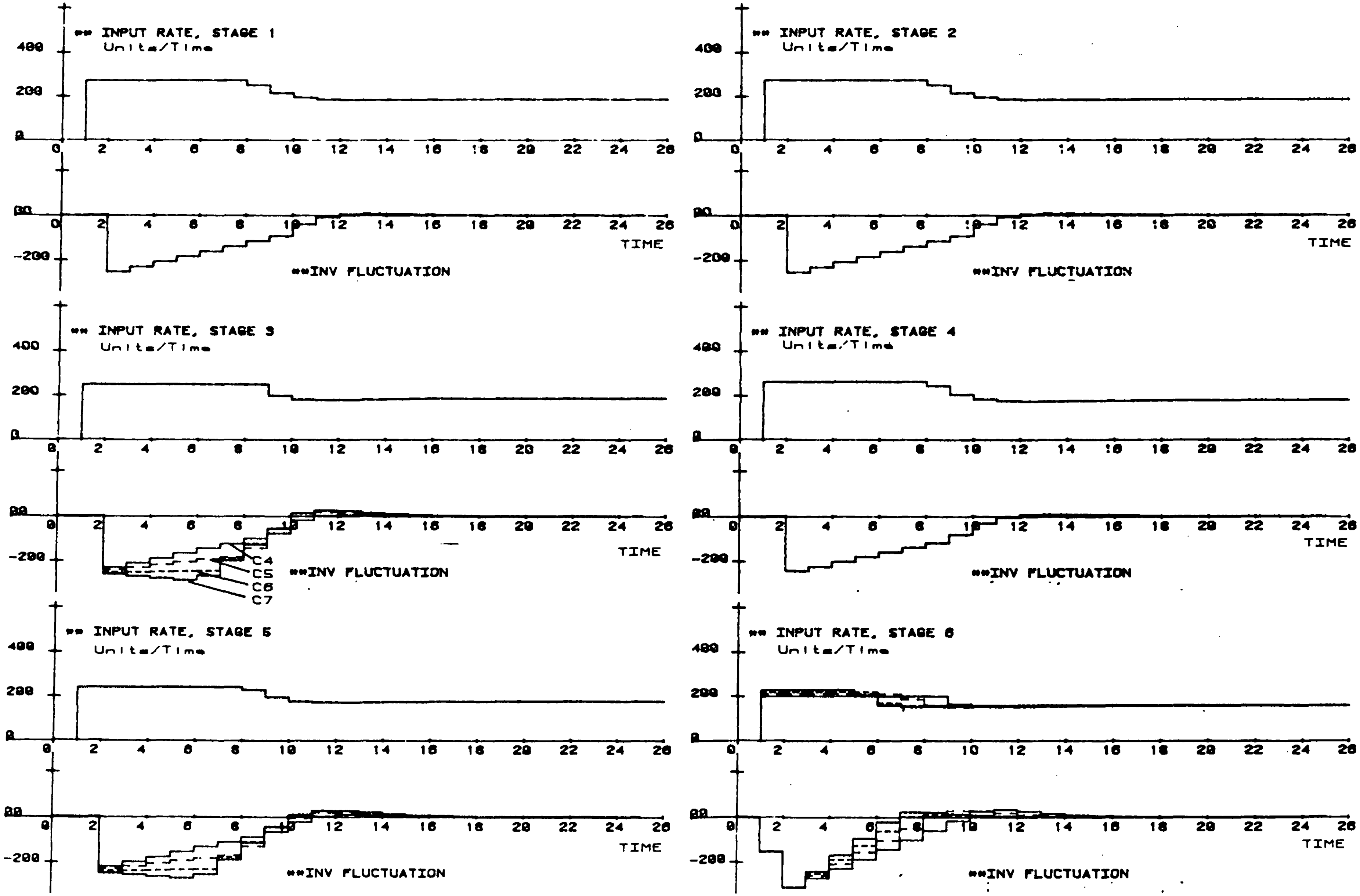


FIGURE 3.19 . DYNAMIC RESPONSE OF SYSTEM IN RUNS C4 TO C7

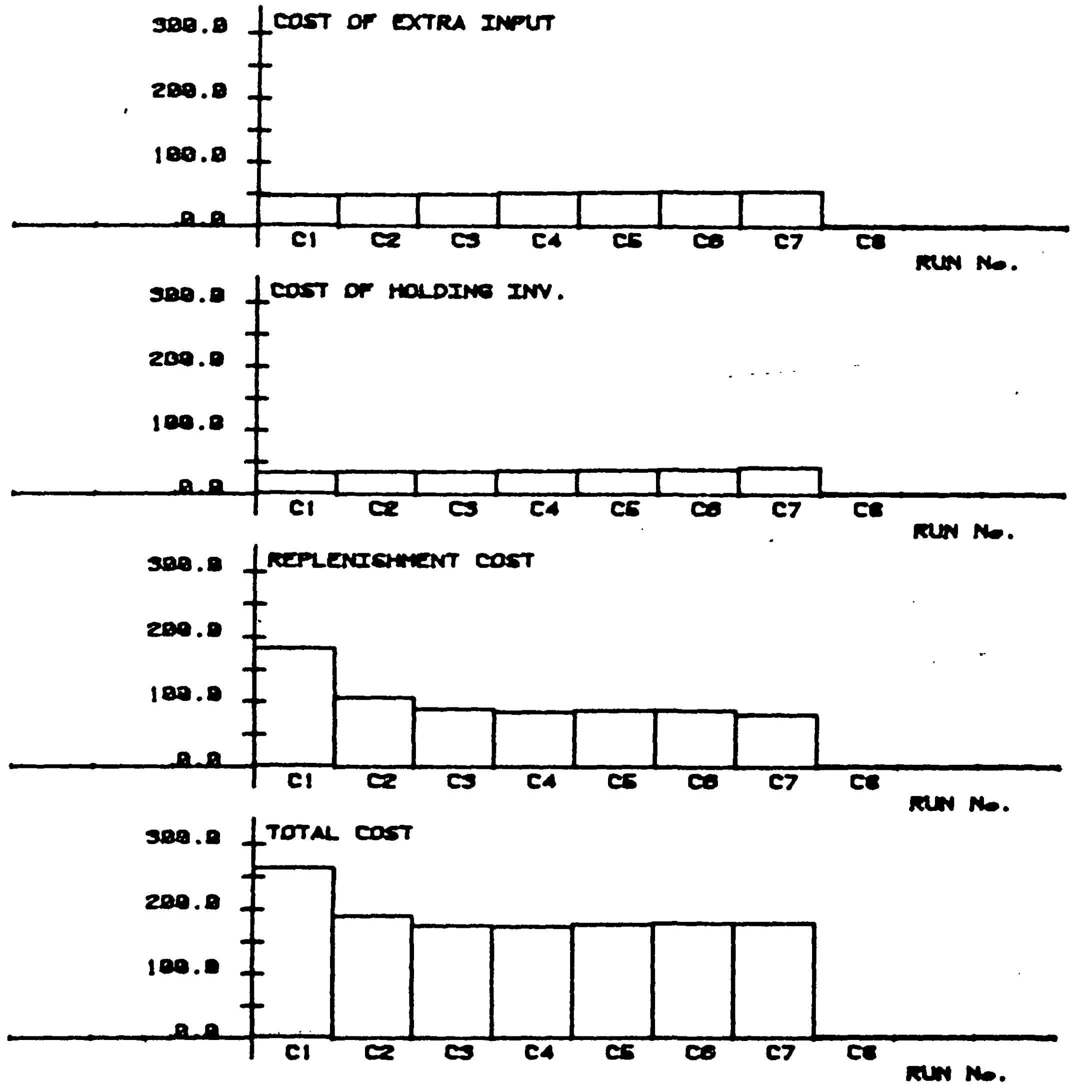


FIGURE 3.20a-d : RESULTS OF NORMALISED COST STRUCTURE RUNS C1 TO C7

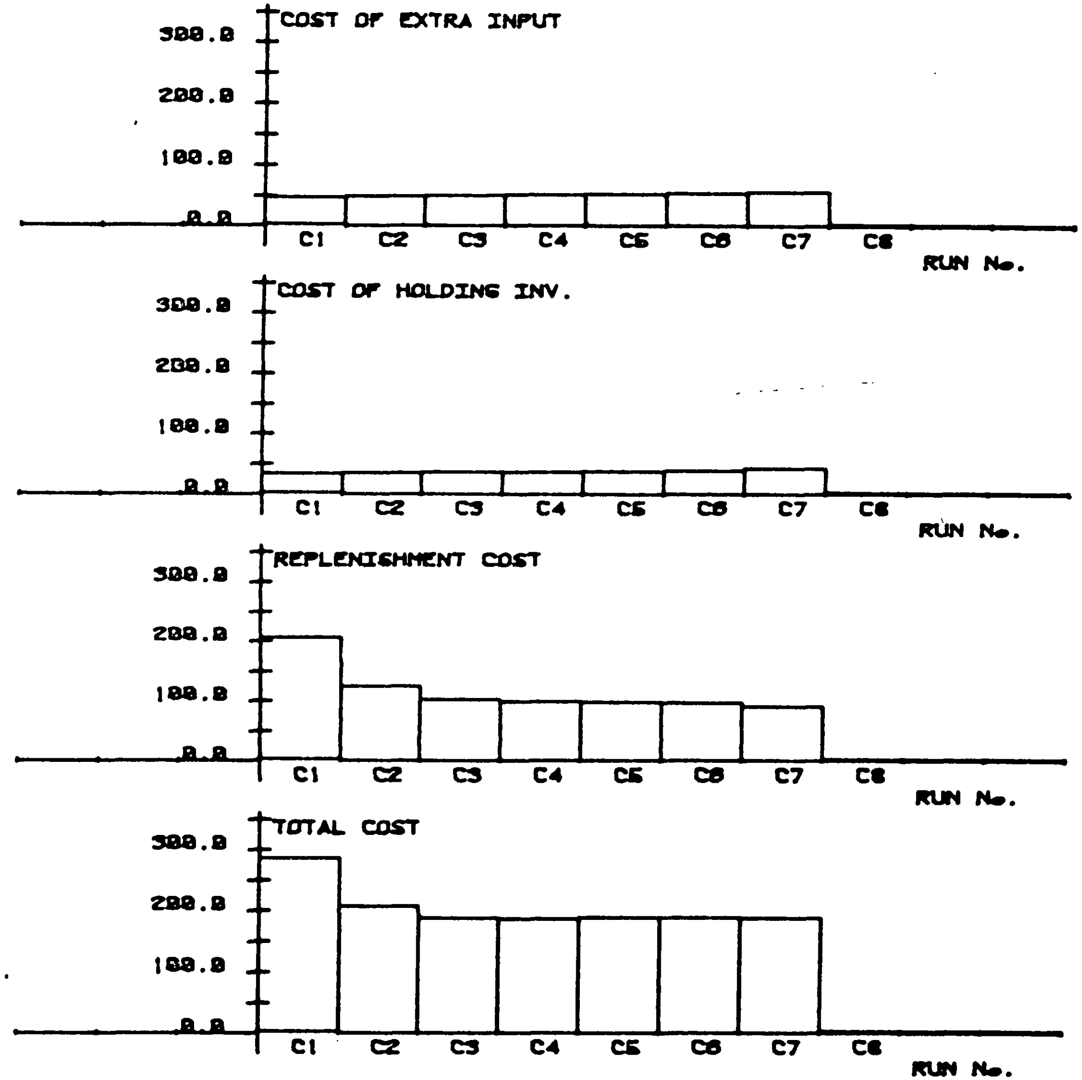


FIGURE 3.21a-d : RESULTS OF NORMALISED COST STRUCTURE
RUNS C1 TO C7 (a = 1.4)

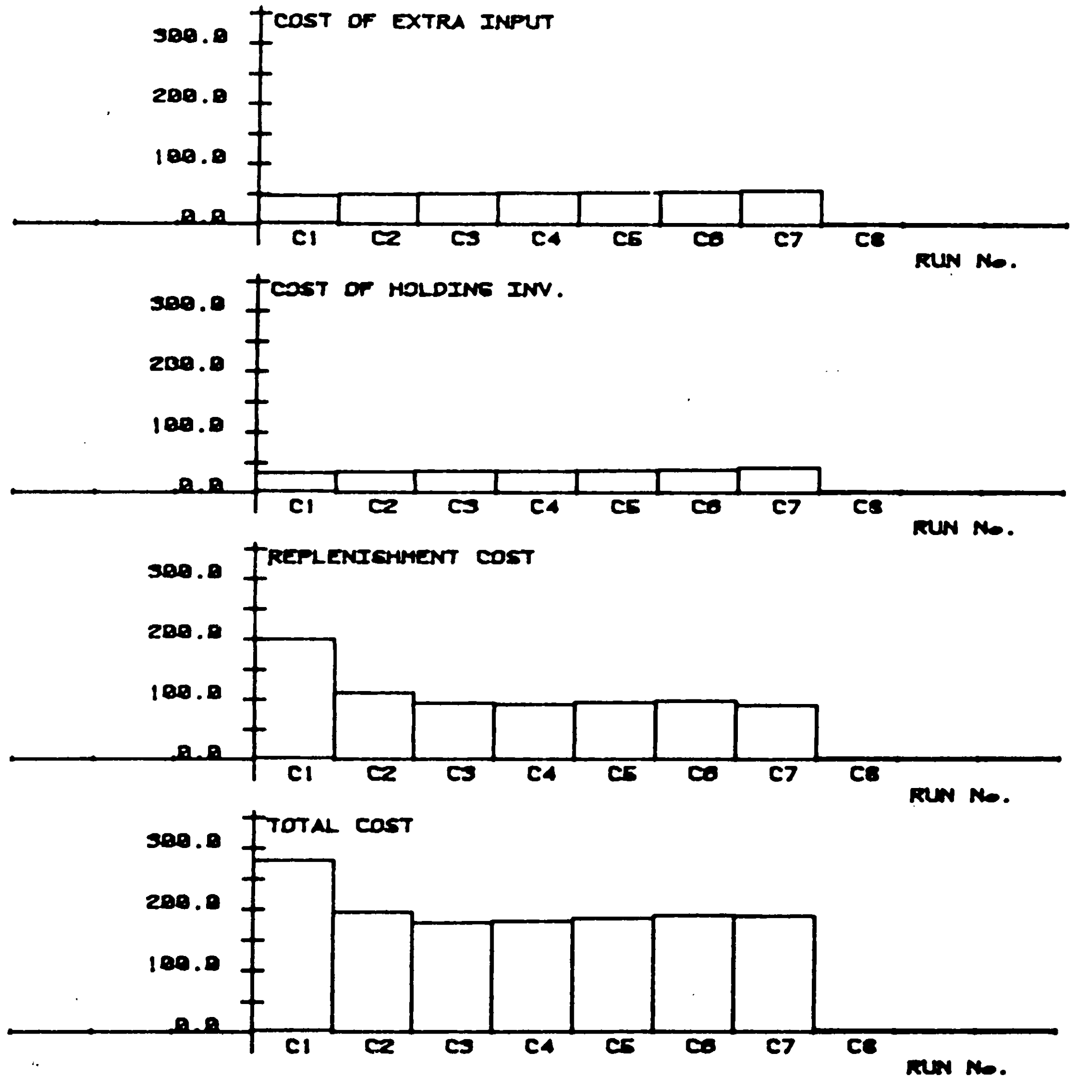


FIGURE 3.22a-d : RESULTS OF NORMALISED COST STRUCTURE RUNS C1 TO C7, NEW WEIGHTS IN MATRIX R

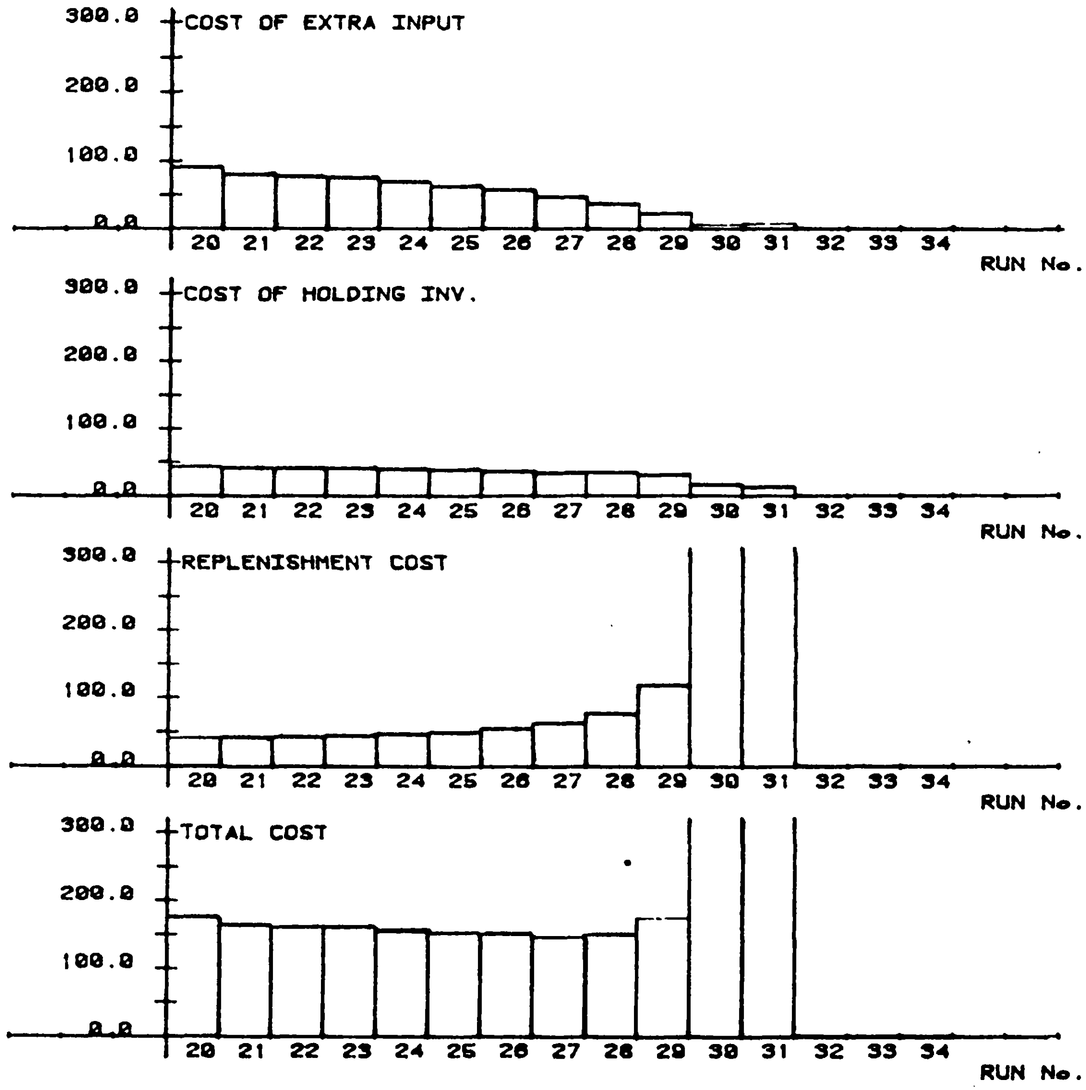


FIGURE 3.23a-d : RESULTS OF NORMALISED COST STRUCTURE FOR CPN 20-31

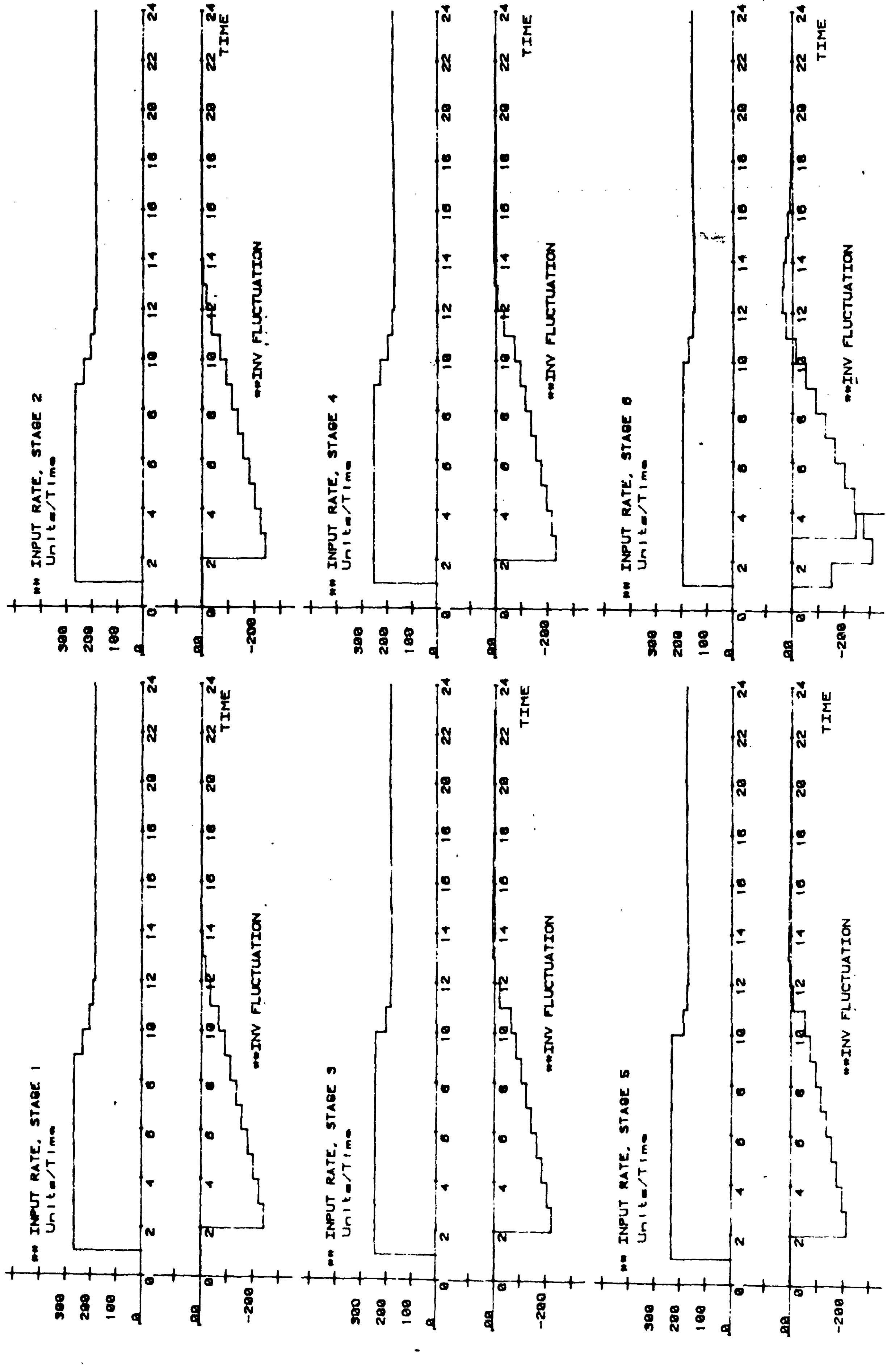


FIGURE 3.24 : DYNAMIC RESPONSE OF CPN 27

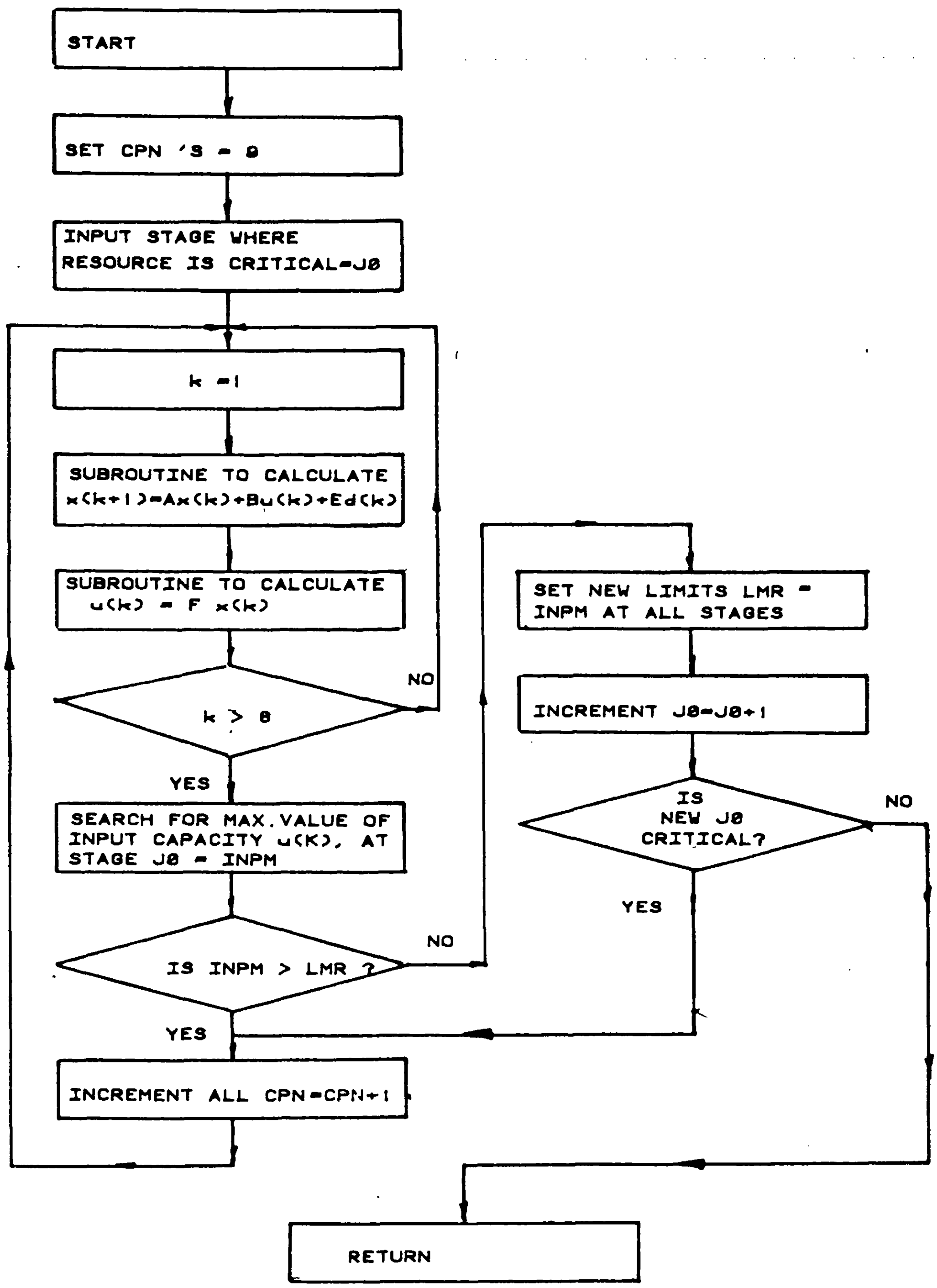


FIGURE 3.25 : SUBROUTINE 'ØDIS/ICCA' AUTO-SELECTION OF STRUCTURED POLICIES

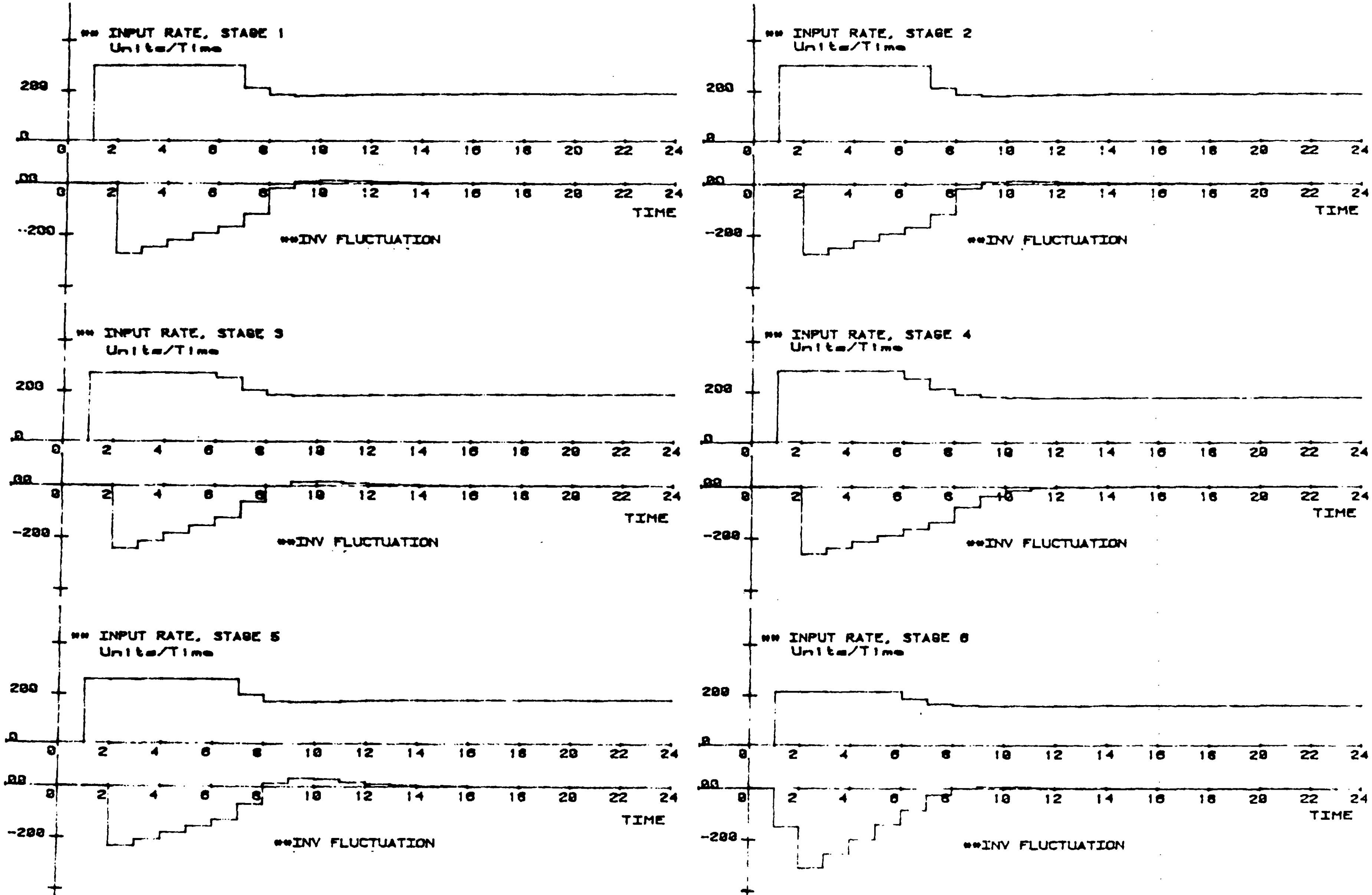


FIGURE 3.27 : ILLUSTRATION 2 FOR "@DIS/ICCA" ALGORIHM

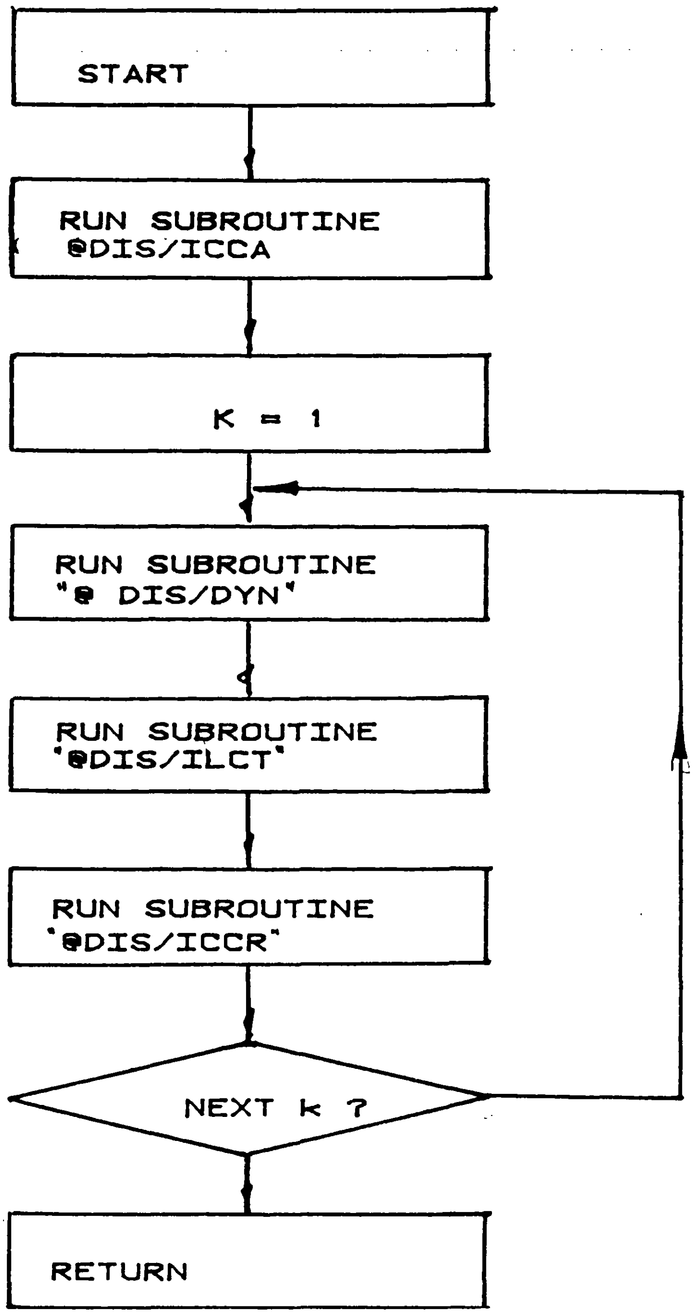


FIGURE 3.28 : OVERALL CONTROL SIMULATION.

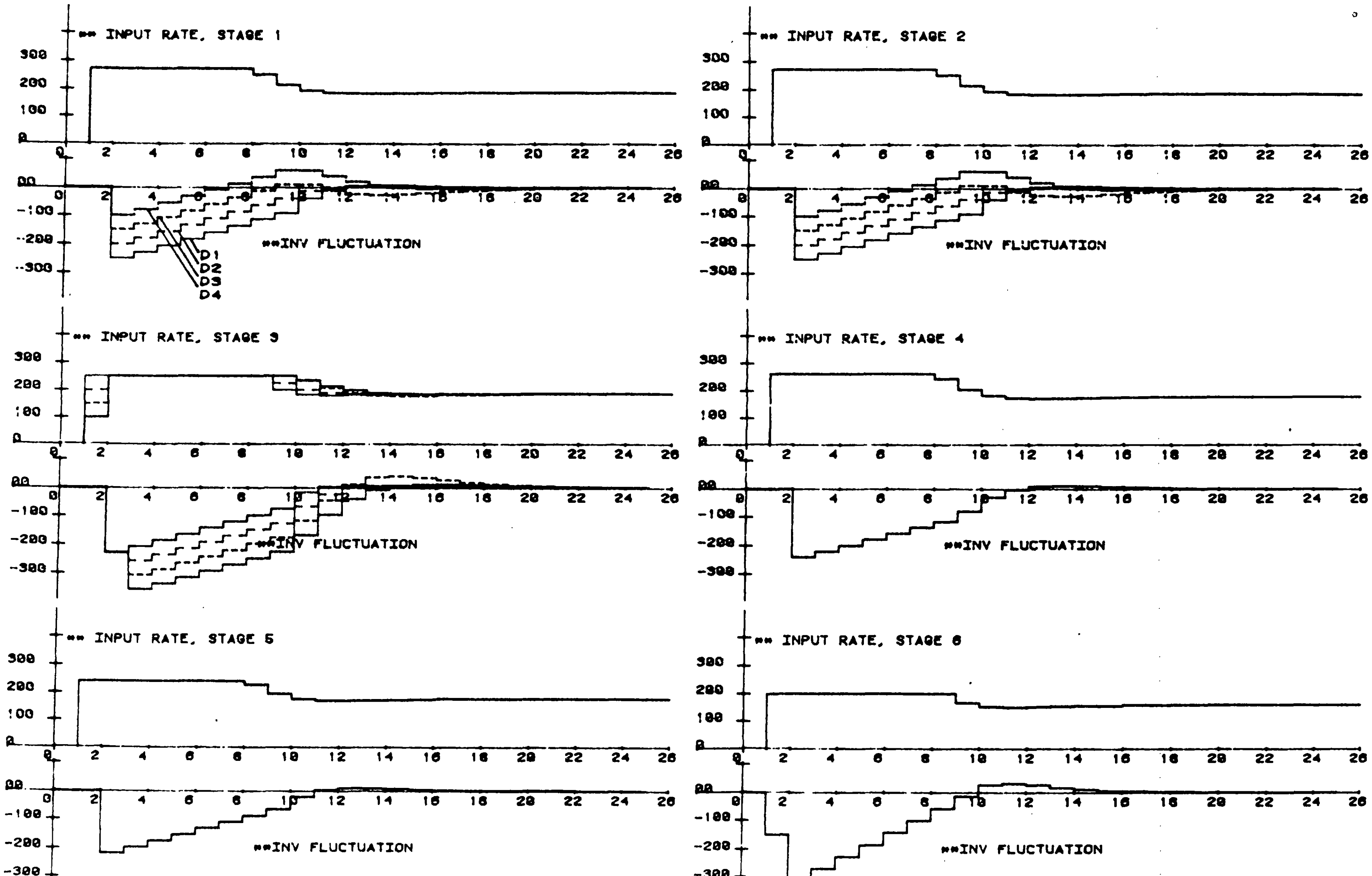


FIGURE 3.29 · ILLUSTRATIONS OF "ODIS/ILCT" ALGORITHM RUNS D1–D4

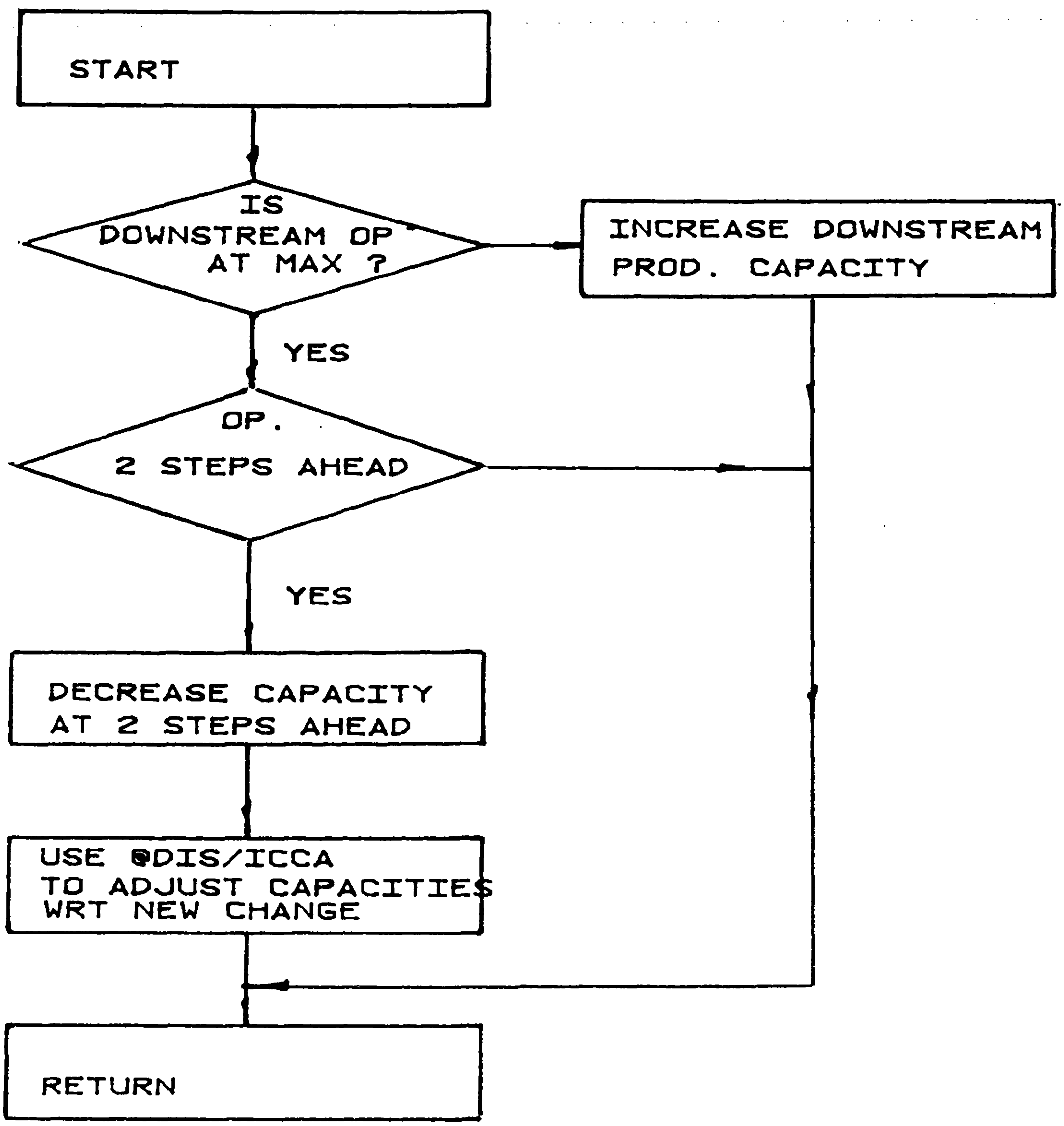


FIGURE 3.30 : SUBROUTINE "@DIS/INVC"

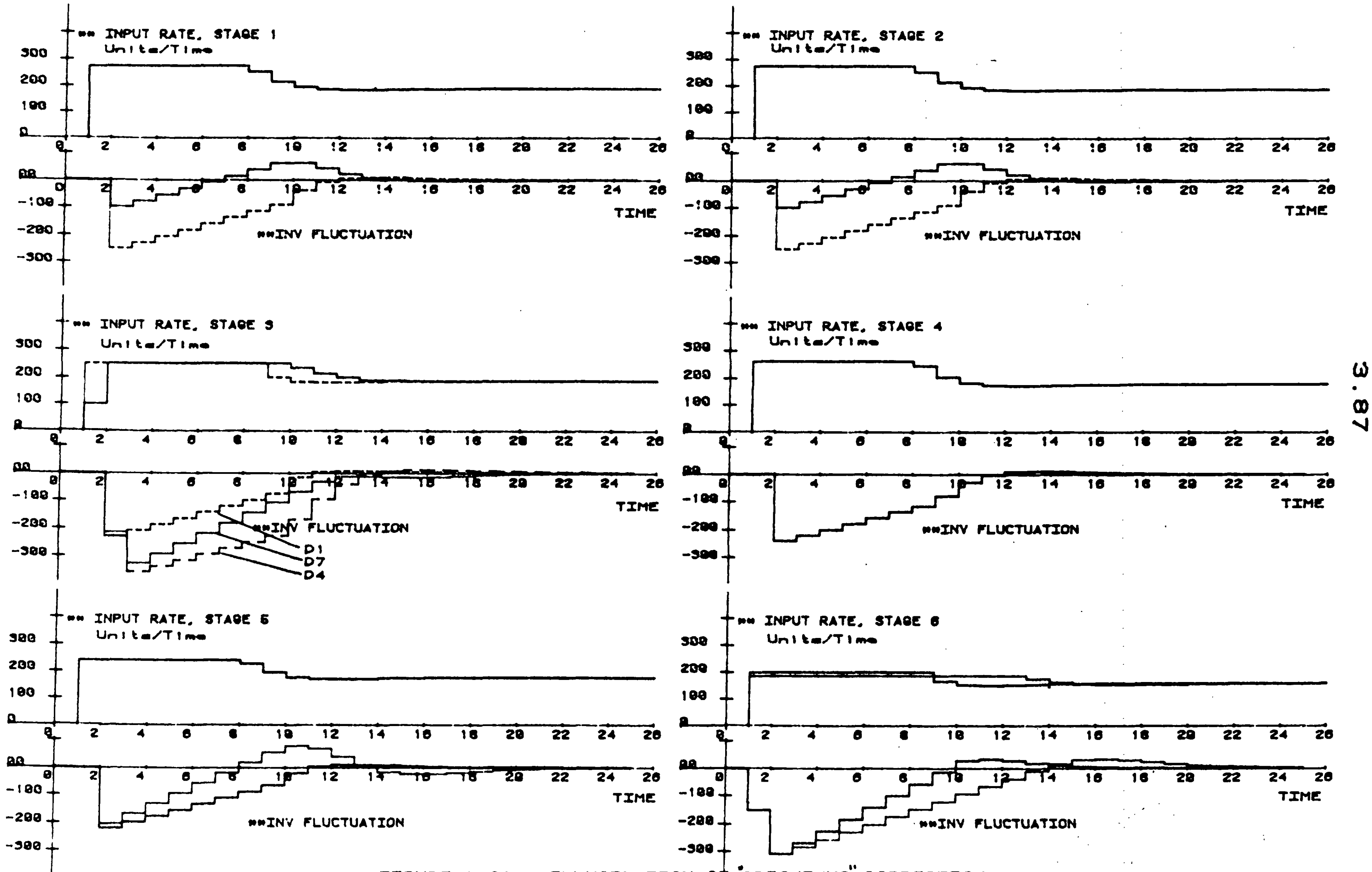


FIGURE 3.31 : ILLUSTRATION OF "DIS/INVC" CORRECTION
RUN D5 WITH D1 AND D4

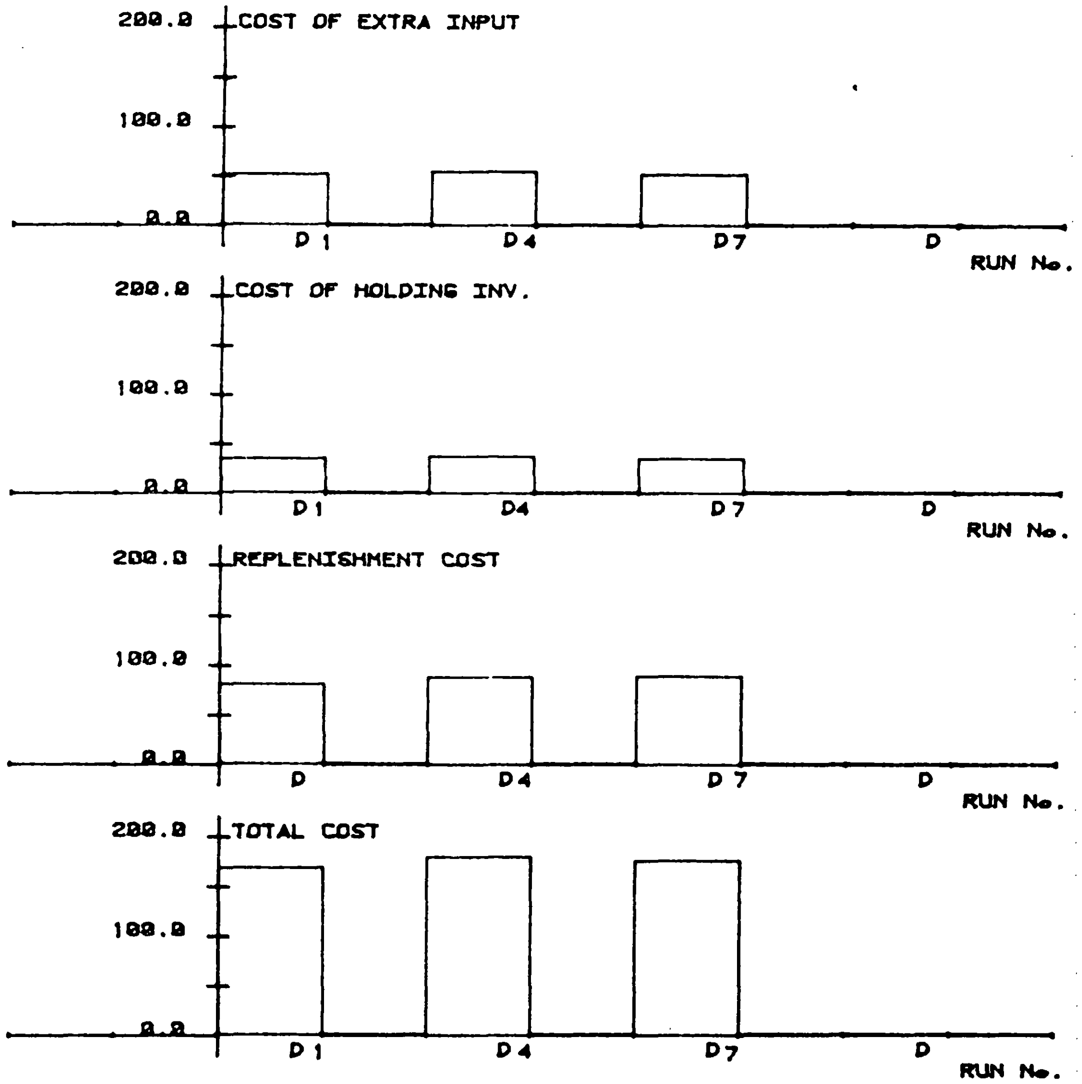


FIGURE 3.32a-d : RESULTS OF COST STRUCTURE FOR RUNS D1, D4 & D7.

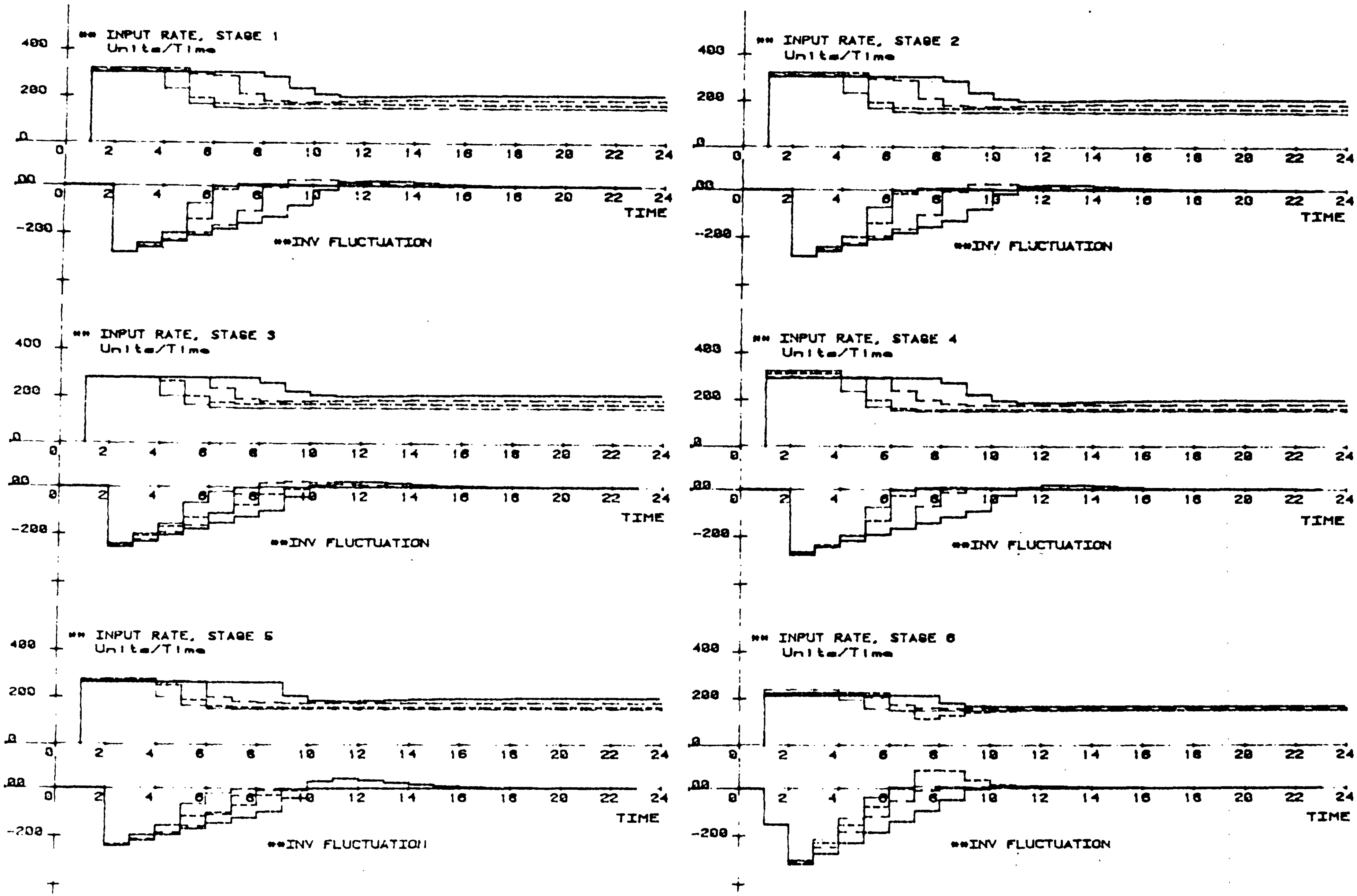


FIGURE 3.36 : DYNAMIC RESPONSES OF SYSTEM IN RUNS E1-E4

Gamma Distribution:

$$f(x) = \frac{\alpha}{\Gamma(r)} (\alpha x)^{r-1} e^{-\alpha x}$$

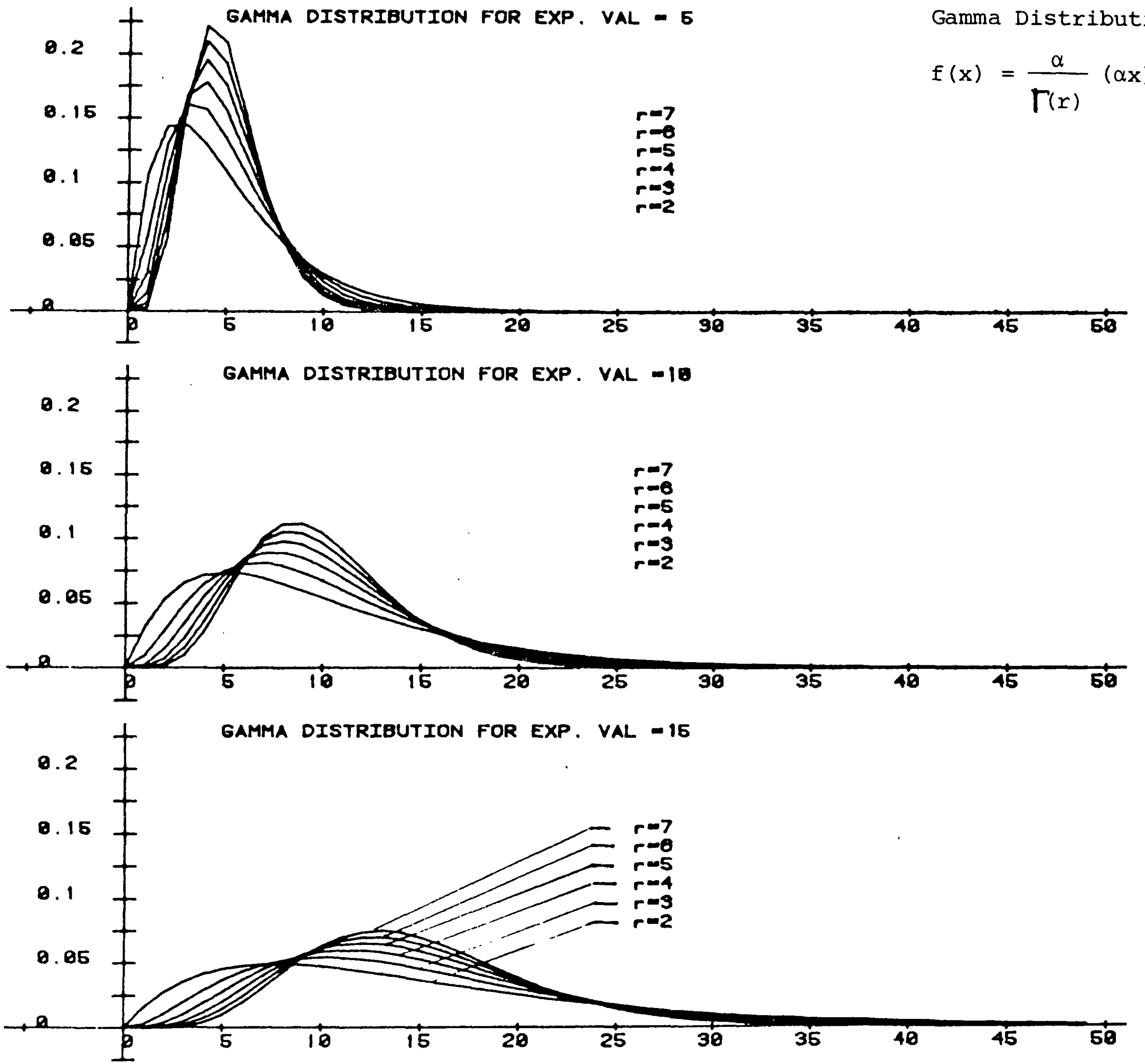


FIGURE 3.33 a–c : GAMMA DISTRIBUTIONS FOR 3 EXPECTED VALUES

Probability function:

$$P(X>x) = \sum_{k=0}^{r-1} e^{-\alpha x} (\alpha x)^k / k!$$

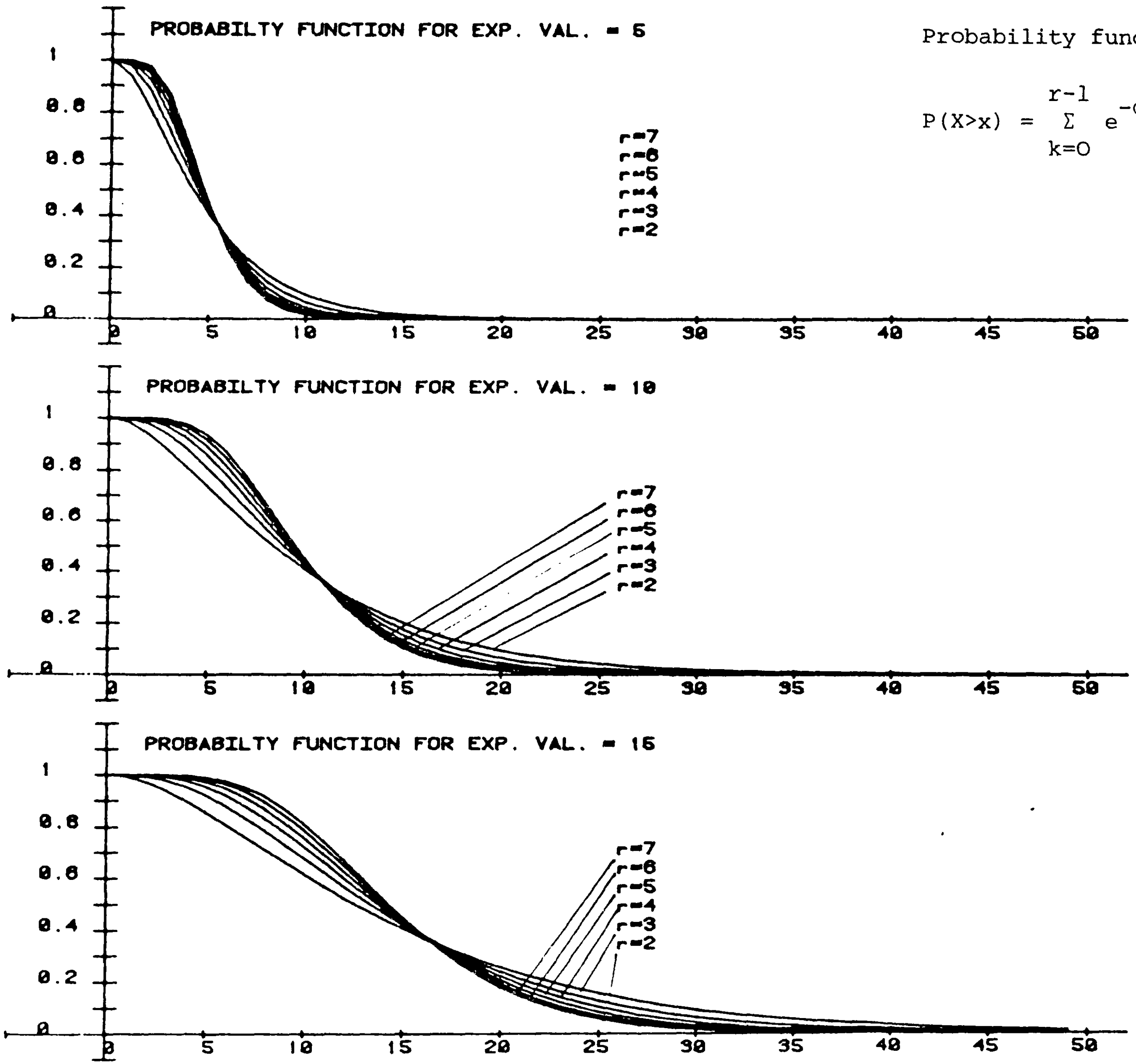


FIGURE 3.34b-c : GAMMA PROBABILTY FUNCTIONS FOR 3 EXPECTED VALUES

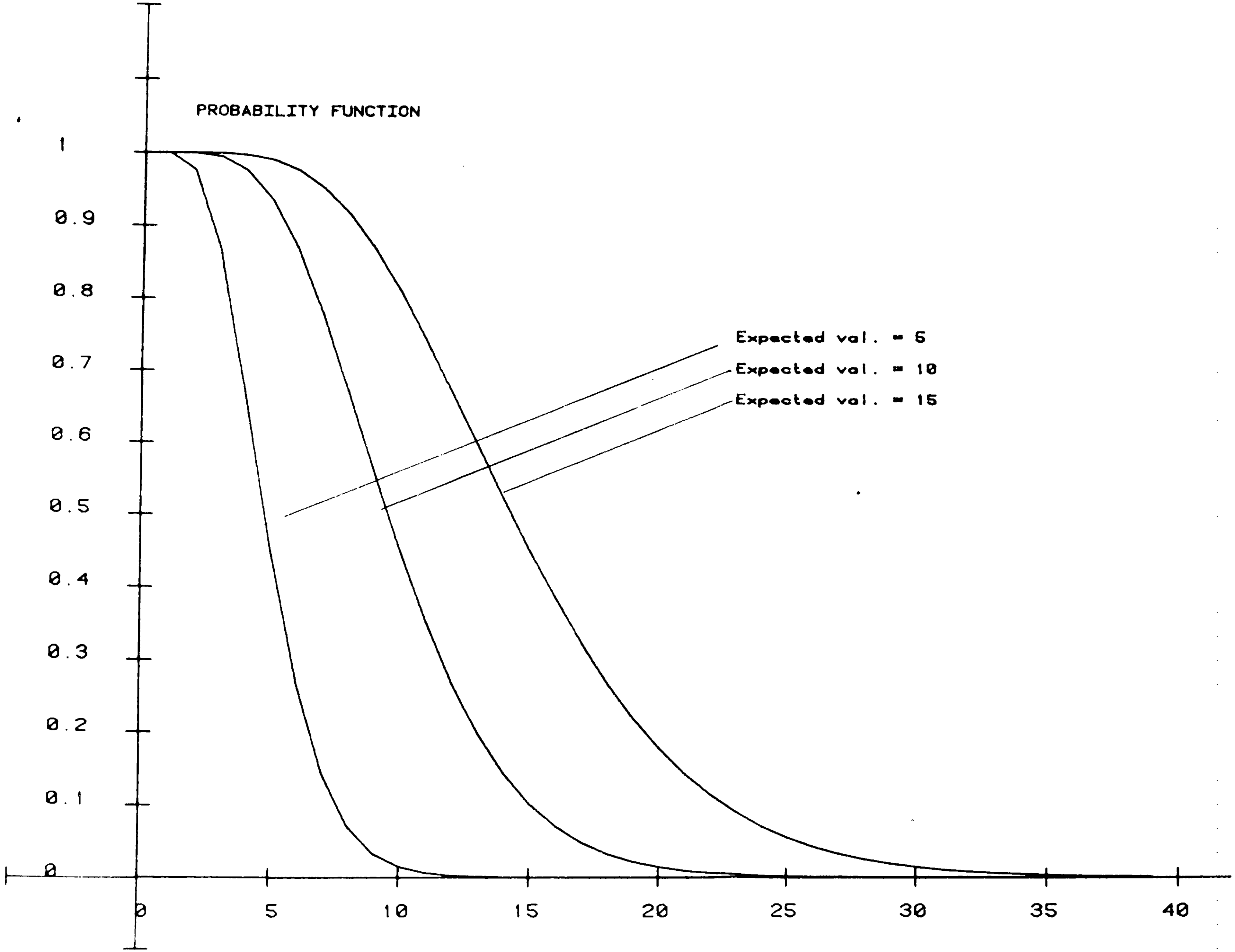


FIGURE 3.35 : PROBABILITY FUNCTION FOR r = 7

	STAGES	1	2	3	4	5	6
CPN 20	MAXIMUM INPUT RATES	336	336	299	321	286	231
	SAFE STOCKS	299	299	259	286	249	308
CPN 24	MAXIMUM INPUT RATES	306	306	278	291	265	219
	SAFE STOCKS	278	278	247	265	237	308

TABLE 3.1 : MAXIMUM INPUTS AND SAFE STOCKS FOR CPN 20 AND 24

STAGE \ RUN No	B1	B2	B3	B4	B5	B6	B7
1	273	273	273	273	273	273	273
2	273	273	273	273	273	273	273
3	220	230	240	250	260	270	280
4	261	261	261	261	261	261	261
5	240	240	240	240	240	240	240
6	200	200	200	200	200	200	200

TABLE 3.2 : MAXIMUM INPUTS FOR RUNS B1 TO B7

STAGES \ RUN No	C1	C2	C3	C4	C5	C6	C7
1	273	273	273	273	273	273	273
2	273	273	273	273	273	273	273
3	250	250	250	250	250	250	250
4	261	261	261	261	261	261	261
5	240	240	240	240	240	240	240
6	170	180	190	200	210	220	220

TABLE 3.3 : MAXIMUM INPUTS FOR RUNS C1 TO C7

	STAGES	1	2	3	4	5	6
1	MAXIMUM INPUT RATES	273	273	250	261	240	200
	SAFE STOCKS	250	250	228	240	218	300
2	MAXIMUM INPUT RATES	300	300	273	285	260	215
	SAFE STOCKS	273	273	243	260	233	308

TABLE 3.4 A-B : RESULTS OF @DIS/ICCA ALGORITHM

RUNS \ STAGES	1	2	3	4	5	6
D1	250	250	228	240	218	308
D2	200	200	256	240	218	308
D3	150	150	306	240	218	308
D4	100	100	356	240	218	308

TABLE 3.5 : MINIMUM SAFE STOCKS FOR RUNS D1-D4

EXP VAL	P(X>0)	P(X>5)	P(X>10)	P(X>15)	P(X>20)
5	1.00000	0.449711	0.014228	1.24E-4	-0
10	1.00000	0.934712	0.449711	0.101632	0.014228
15	1.00000	0.988932	0.808123	0.449711	0.178081

TABLE 3.6 : SELECTED PROBABILITY VALUES FROM FIGURE 3.33

RUN / REJECT % AT	E1	E2	E3	E4
1	0	0	0	0
2	0	0	0	0
3	15	10	20	0
4	0	0	0	0
5	10	5	15	0
6	5	0	10	0
FINAL DEMAND	150	150	150	150

TABLE 3.7 : 4 DIFFERENT REJECT SCENARIOS.

STAGES		1	2	3	4	5	6
E1	MAXIMUM INPUT RATES	309	309	280	294	268	222
	SAFE STOCKS	280	280	250	268	240	308
E2	MAXIMUM INPUT RATES	321	321	280	311	272	225
	SAFE STOCKS	280	280	242	272	233	300
E3	MAXIMUM INPUT RATES	305	305	280	290	265	212
	SAFE STOCKS	280	280	254	265	241	317
E4	MAXIMUM INPUT RATES	322	322	280	322	280	238
	SAFE STOCKS	280	280	238	280	238	300

TABLE 3.8 : MAXIMUM INPUTS AND SAFE STOCKS FOR RUNS e1 - e4

CHAPTER 4

CHAPTER 4.

Control Simulation For Multi-Product Manufacturing System.

4.1. Introduction.

In many manufacturing environments, production facilities are rarely geared to a single uniform product. While single item production may exist for certain fast consumer products, factors such as different models, sizes, and additional options create an environment of multi-product manufacture even in the field of mass production as found in the automobile industry. Such a state of affairs will lead to the necessity of sharing the available facilities according to the varying requirements in the production of the different products/models.

A survey of the literature has shown that the majority of the dynamic control studies carried out by various workers has concentrated on the single product systems. Examples of such analyses can be found in Christensen and Brogan (1971,/1/) and Porter et al (1976,/3/). This has contributed to an improved insight into production control problems, its practicality may be enhanced if it can also take into consideration the multi-product environment most often encountered in manufacturing industries. Control studies for multi-product control in a dynamic environment of multi-stage production-inventory have not been abundant. Examples of such class include Hitomi and Nakamura (1976,/59/), who descibed a synthetic case using functional space analysis; Drew (1975,/2/), who used some developments of the Lagrangian function to analyse a practical case. The approach described in this section makes use of the various extensions and observations of multi-variable control theory given in earlier chapters. The case presently considered is the production of two different car models which have to use some common manufacturing facilities due to the fact that the same power unit may be fitted to either model.

Various practical features of production control are considered in the analysis. The concept of structured control policies, which co-ordinate both capacity requirements and inventory fluctuations is again introduced in this multi-product environment. This is achieved by the development of a new algorithm that makes use of the properties described earlier on in this work. Practical constraints, such as bottleneck areas and float limits are again taken into consideration. Moreover, it is shown how the various lines can be subjected to a dynamic preferential weighting according to the assembly demand priority.

4.2 Description of the Control Problem.

In this particular case study, some of the current development of multivariable control theory is applied in a multi-product manufacturing environment such as the automotive industry. The analysis will consider the production of two different car models to be referred as models Y and Z. This is actually based on real cases taken from an automotive company, which has asked to remain unidentified. The simulation has to be carried out jointly for both models in view of the fact that they share some common resources. It is pointed out that while only two models are considered, the same approach can be adopted for manufacturing systems with more than two products. This is facilitated by the matrix analysis nature of the control simulation in this approach.

The schematic for the production of the two car models are given in Figure 4.1, where six major production-inventory stages are used to represent the manufacture of a car.

They are namely :

Stage 1 : Gearbox assembly.

Stage 2 : Engine assembly.

Stage 3 : Power unit assembly.

Stage 4 : Body in white welding.

Stage 5 : Painted body production.

Stage 6 : Trim and Final assembly.

The fact that both models use the same power unit is illustrated in Table 4.1. The body works follow different lines from start to final assembly. Table 4.1 also gives the average operation time at each operation stage, and the resources required which are coded as R1 to R9.

Here again, it is pointed out that these values are obtained as a result of the particular modelling approach that assumes a linear discrete-time structure of the system as explained in the previous chapter where a single product was considered. A linear discrete-time model has to be used so as to express the problem in more controllable terms following the hierarchical decomposition concept as outlined in chapter 2. Each "production-inventory" stage that is considered at this level of analysis is in fact a whole series of operations dedicated to the production of an assembly or some intermediate major groupings of operations along the assembly line. This linearisation approach has already been explained in the previous chapter.

The problem scenario considered in this section is the allocation of extra manufacturing facilities in the simultaneous start-up situations for the two models, and the associated control policies for the buffer banks. The problem is made more difficult when manufacturing processes for the two models require common facilities to be shared. There will thus exist an environment of dynamic competition for resources from the requirements of the two models.

The approach adopted in this analysis uses development of a new algorithm designed specifically to solve this class of problem. The various extensions of multivariable control theory as described in chapter 2 are used; namely the concept of individual and discrete CPN (Control Policy Number) for the control of individual modes. These CPN's are controls based on feedback (sub)matrices synthesised from discrete increments of eigenvalues. While these CPN control individual modes, they are still effected using the information of all the state variables of the system. The practical significance is that the allocation of manufacturing resources is being done at each production-inventory stages in accordance to the other individual responses of other stages. This approach offers the possibility of synthesising control measures at each stage and at each time-period so that the resource allocated produces the necessary sub-parts to pass on to the next production stage and to fill its own depleted inventory. It is to be realised that since so many stages are present in parallel and in series, an overall co-ordination would have been virtually impossible in the absence of the current multivariable control approach.

4.3 Mathematical Formulation of the Control Problem :

The two production-inventory systems are formulated jointly in a linear discrete time multivariable control problem as expressed in the following matrix equations:

$$x(k+1) = A\,x(k) + B\,u(k) + E\,d(k) \qquad \text{----(4.1)}$$

$$u(k) = F\,x(k) \qquad \text{----(4.2)}$$

where

$$A = \begin{bmatrix} A_y & \\ & A_z \end{bmatrix} \qquad x = \begin{bmatrix} x_y \\ x_z \end{bmatrix}$$

$$B = \begin{bmatrix} B_y & \\ & B_z \end{bmatrix} \qquad u = \begin{bmatrix} u_y \\ u_z \end{bmatrix}$$

$$E = \begin{bmatrix} E_y & \\ & E_z \end{bmatrix} \qquad d = \begin{bmatrix} d_y \\ d_z \end{bmatrix}$$

$$F = \begin{bmatrix} F_y & \\ & F_z \end{bmatrix}$$

Suffixes Y and Z relate the system matrices to their respective models. The details of the formulation has already been explained in a previous section where the concept of "control" and "state" variables was introduced.

A_Y , A_Z	=	Plant matrices.		(18 X 18)
B_Y , B_Z	=	Input Matrices.		(18 X 18)
E_Y , E_Z	=	Disturbance Matrices.		(18 X 18)
x_Y , x_Z	=	State vectors.		(18 X 1)
		(production rate, No of parts/unit time)		
		(inventory level, No of parts)		
u_Y , u_Z	=	Input vectors.		(18 X 1)
		(extra capacity required: No of parts/unit time).		
d_Y , d_Z	=	Disturbance vectors.		(18 X 7)
k	=	Argument denoting time.		

The detailed equations are similar to those in Chapter 3 except for the fact that there are now two sets of such equations corresponding to the two models. Prior to the actual control simulation, the matices are transformed into their canonical forms. Feedback matrices are also pre-synthesised with arbitrarily assigned eigenvalues to the closed loop plant matrix (A + BF) as before.

4.4 Method of Solution.

4.4.1 Dynamics Of Simulation Exercise.

The overall schematic for the control simulation is given in Figure 4.2, showing the various modules involved in the analysis. It consists of a main command program together with a suite of 3 modular programs, averaging 6 - 12K of memory developed on a TEKTRONIX 4052 desktop computer. A time horizon of 24 time-periods is used for the present study which is simulated on an iterative basis. At each iteration, two subroutines are run:

(i) "@SYS/DYN" which calculates the equivalence matrix equation:

$$x(k+1) = A\,x(k) + B\,u(k) + E\,d(k).$$

giving the resultant states of the systems. This is in actual fact an extended version of algorithm "@DIS/DYN" described in Chapters 2 and 3, but presently dealing with two systems.

(ii) "@SYS/DCAS" which calculates the new control input variables and adjusts them within the existing constraints. The CPN 's of the non - constraint production-inventory stages are then set equal to those with constraints in order to benefit the advantages of the structured limits as described in Chapter 2. This new algorithm is discussed in detail subsequently.

During the run of the program, it was found that a substantial amount of core memory is used up by the various different matrices involved. Some of these matrices which have a time suffix associated with them, have one additional dimension for every time period ,e.g.

x(k, k+1, k+2, k+3, k+4,.....) The matrix functions built into the firmware of the computer have been used extensively. The computation time of this control simulation exercise is 10-12 minutes.

The efficiency of the overall approach and the "@SYS/DCAS" algorithm is demonstrated for this practical problem of controlling the production of two models with the features and conditions considered under the following headings:

(i) Desired operating levels.

(ii) Bottleneck area - Power units.

(iii) Structured Capacity rates.

(iv) Structure of algorithm.

4.4.2 Desired Operating levels.

In this particular case study, a simultaneous start-up situation of step input in demand is considered for both models Y and Z. This condition may arise from either of the following scenarios:

(i) An actual step-up in production rate.

(ii) Start-up situation arising from a prior shut-down situation.

(iii) Introduction of new car models.

It is desired to have the operating levels (or as increments above the current values) :

MODEL Y 1200 units/week.

MODEL Z 4000 units/week.

at the steady state conditions.

The dynamic analysis for this problem assumes a 10-shift week and uses a decision time interval of one shift. Implicit in the use of a certain particular duration of the time period is the fact that transportation of the semi-finished assemblies is being effected intermittently at the beginning or end of the time period considered. The choice of this time interval as described in chapter 3, needs to be one that reflects as close as possible the actual practice, or one that is most recommendable for better practical results. At the steady state it will therefore be:

MODEL Y 120 units/shift.

MODEL Z 400 units/shift.

The above two demands are both the values at the 7th entry in the disturbance vectors d and d respectively. Demands and disturbances at the other production inventory stages are assumed to be zero at this initial stage of analysis.

<u>4.4.3 The bottleneck area : Power units assembly.</u>

For the two models considered, the current analysis assumes the usage of the same power unit. Variations as size, engine capacity within one model are not included in the analysis since the actual production processes are not significantly different in nature to justify any differentiation in the operation time. Nevertheless the engines, gearboxes and power units for the two models are assumed to still have variations such as size and tolerances that prevent direct interchangeability of units without any prior alteration. The labour involved is flexible to deal with either one just as efficiently. This will thus lead to the necessity of sharing out the manufacturing facility at these stages between the requirements of the two models.

In this practical case study the assembly of power units for this particular joint systems is assumed to be the bottleneck area. A maximum allocation of 340 minutes in one shift is assumed for the production of power units that will go into the two particular models, i.e. production of 680 units/shift for operation time of 1/2 minute per unit. It is noted that at the steady state it is required to produce a total of 520 units/shift, demanding an aggregate of 260 minutes in one shift.

4.4.4 Structured Capacity Rates.

The capacity rates at the various other production stages are assumed to be flexible as to be able to match in a "structured" way the individual fluctuating capacity rates of the two models that are competing for the facilities at the power unit assembly stage. The advantages of the structured limits have already been mentioned for the case of a single product in Chapter 3. The same advantages are sought for the present case of multi-product with common assemblies sharing common resources.

They are namely:

a) Generating control policies for the recovery of the system in capacity requirements and buffer banks. These policies are synthesised accordingly.

b) Smooth control on the individual responses with little or no overshoot and/or undershoot.

c) Cost economic control policies as characterised by a cost function developed in Chapter 2.

Ideally, the analysis could have treated each individual product separately and then work out the aggregate capacity requirements subsequently. However, in a practical situation where constraints on the resources exist, it will be of a continuous necessity to share out resources dynamically according to the requirements of the final demands of the products.

4.4.5 Structure of Algorithm. (Code name = "@SYS/DCAS")

Each system has 6 modes of response corresponding to the 6 production - inventory stages and each of which is controlled by a particular CPN (Control Policy Number) as described in a previous chapter. These individual CPN 's are in effect feedback matrices synthesised from discrete increments of eigenvalues. It has also been shown that small CPN's give rise to sharper recovery responses as compared to larger CPN which are associated to the relatively slow build-up responses.

The algorithm developed to cater with the present case searches iteratively at each time period for the appropriate respective CPN controlling each individual product response using the critical resource R3 (assembly of power units). The search is carried out so that the aggregate usage of resource R3 is restricted to a maximum of 340 minutes in one shift. The rest of the time has to be allocated to the assemblies of other models that are not included in the present analysis.

The steady state at R3 is 520 units/shift or the allocation 260 minutes/shift.

The extra amount of 80 minutes/shift is obtained through:

a) Working overtime for the tracks dedicated to the particular power unit specifications.

b) Releasing capacity from parallel tracks to the production of the assemblies for the particular models.

c) Combination of both.

The same situation applies for the other resources except for the fact that they are flexible enough to adjust themselves proportionally to the rate at R3.

At each time period, the CPN 's controlling the modes of response for each individual system are set equivalent to the one identified as satisfying the constraints at R3. This is the process that actually attempts to obtain controls with structured limits. The details of the algorithm code named "@SYS/DCAS" are given in Figure 4.3 .

Due to the limited resource at R3, it is required to decide on the relative importance of the recovery responses for the two systems at the initial stage of the run. In this particular example, where there is an aggresive marketing strategy for the MODEL Z for both home and foreign markets, the priority is given to the MODEL Z over the MODEL Y. This requirement is efficiently accomodated by the initial choice of the starting CPN values. From chapter 2, it has been shown how the magnitude of the individual CPN as associated with each production stage is indicative to the nature of the response. Small CPN values give sharper recovery responses compared with larger CPN values which are associated with the relatively slow responses. Thus the starting CPN's at each production stage in MODEL Z are chosen to be 4 scales lower than those of the MODEL Y to represent the higher priority given for a sharper control for system MODEL Z. The actual values are CPN's 12 for the modes of the MODEL Z and CPN 16 for the modes of the MODEL Y.

4.5 Results of Control Simulation.

The results of the simulation (RUN F1) are shown in Figure 4.4 and 4.5 for MODEL Y and MODEL Z systems respectively. Each figure consists of six sets of graphs showing the modes of responses at the six production-inventory stages. At each production-inventory stage, two major features are represented:

(i) The input capacity rate expressed in the number of units intended to be manufactured at each paticular time-period, i.e. one working shift.

(ii) The fluctuation of the inventory with respect to a certain arbitrary datum.

The analysis assumes, a priori, a limitless inventory. From the lowest value to which the inventory level dips as a result of the control simulation, a knowledge of the minimum safe inventory is obtained.

The use of the initial CPN value as a means of a priority weighting to the responses of the two systems is also successfully demonstrated. It is the current intention to favour the production of the MODEL Z at the expense of MODEL Y line at the original stage of the time horizon considered. This objective is indeed achieved: For the three resources R1, R2 and R3 which manufacturing facilities have to be shared out at production stages 1, 2 and 3 of both systems, it is seen how more capacity is given to the production of assemblies feeding the MODEL Z at the beginning of the run. For example, at the production stage 1 there is a high capacity input for the MODEL Z system compared to the MODEL Y. This state of affairs only ceases when the production rate and inventory level are reaching steady state, whereby the manufacturing facility is shifted favourably to the MODEL Y system. The same dynamic feature is observed at other production-inventory stages. In general, extra allocation of capacity is only made to the assemblies feeding the MODEL Y, when those of the MODEL Z are nearing stability. The use of the initial CPN value as a measure of priority weighting in the dynamic control of a multi-product environment is positively illustrated.

The usage of the resources are given in Figure 4.6. This is given in the number of minutes of the labour or machine time allocated in each shift for the particular manufacturing facility. It is seen that resource R3, i.e. assembly of power units is utilised at the maximum of 340 work hours / time period, while the other resources are dynamically adjusted to balance that of resource R3.
One of the parameter, R, in the algorithm "@SYS/DCAS" (Figure 4.3) , controlling the resetting capacity of the system is altered from the value of 1 to 2. This is an attempt to have a stronger resetting policy leading to a sharper response. (RUN F2) This is indeed reflected by a marginally faster response (in time periods) although at the expense of some slight overshoot as given in Figures 4.7, 4.8 and 4.9. The marginal improvement in the response is considered to be too minimal as to justify the need to alter the value of the parameter R to 2. Table 4.2 gives the maximum production rates incurred in number of units/time period and safe inventories for the above two cases. (Runs F1 and F2)

4.6 Modified Version Of Control Simulation Structure. ("@SYS/DSAS")

4.6.1 Structure of Modified Approach.

In the two above cases (RUN F1 and F2), it is seen that the usage of the resources other than R3, R4, R5, and R6 is relatively high at the initial stage. While this may be technical feasible, it is to the benefit of better working relationships and practices, if the extra load is smoothed. To alter the algorithm otherwise will mean sacrificing some benefits of the structured limits. Therefore a compromise approach is adopted with a further loss in mathematical optimality, but with a gain of practicality in the overall solutions. A two step approach is adopted. This is achieved by running the "@SYS/DCAS" algorithm for an initial 10 time periods. An average is then taken of the usage values of the resources, R1 to R9. These averages are then used as limits for a rerun of the control simulation exercise from time period k = 1 again. It is found from the actual simulation runs that the new average values as such, are too tight to allow acceptable control recovery resposes. Therefore the average values are incremented by 5% before the second stage of the simulation. The new structure of the overall control simulation code-named "@SYS/DSAS" is given in Figure 4.10 . A modified version of "@SYS/DCAS" algorithm code-named "@SYS/DICAS" is used for the synthesis of control policies where there are constraints at all the resources R1 to R9. This new algorithm is given in Figure 4.11.

4.6.2 Results of the Revised Control Structure.

The results of the new approach (RUN F3) are given in Figures 4.12

and 4.13 for the MODEL Y and MODEL Z respectively, using a resetting value of R = 1. (Figure 4.11).

The usage of the resources R1 to R9 is given in Figure 4.14. Compared with the results of the algorithm "@SYS/DCAS" (Figure 4.6), it is observed that the objective of smoothing the overall utilisation is achieved for each resource. The current overall maximum limits are worked out as :

565, 565, 340, 122, 113, 105, 444, 411, 378 .

Originally there was a higher requirement of manufacturing facilities for resource R1, R2, R7 and R8 at the initial transient period of the run. In addition to the above feature, some of the individual responses have also been improved as a result of the new approach. The input values for the MODEL Z are also individually smoothed (shown in Figure 4.13) compared to the responses prior to the modification(Figure 4.5). Some of the individual inventory responses reach the steady state faster too. Nevertheless, this feature does not apply to all responses, the inventory fluctuations at stages 3, 4 and 5 actually take a longer time before starting to recover. But the actual duration of the recovery period is still similar.

The input values for the MODEL Y system do not show the same smoothing effect as the MODEL Z. On the other hand, the responses of the inventory levels are improved in the ability to recover to the original level in a shorter length of time.

The above improvements have been obtained by using a 5% extra, in the aggregate capacity over the initial 10 time periods. The aggregate utilisation over the time horizon of 24 time periods for the 9 resources are compared for Runs F1 and F3 and are given in Figure 4.15 and Table 4.3. It is clearly seen that over the particular time horizon considered, the difference in the aggregate utilisation is insignificant. Therefore the original belief that the modified algorithm may lead to severe loss of optimality is shown to be only mildly justified. In fact, the revised version has led to both an improvement in (sub)optimality and practicality of the solutions as will be shown in the next section.

4.7 Mathematical Analysis of Modified Version.

While some of the features obtained from the modified version have been described in a subjective way in the previous section, they are now assessed quantitatively. The cost functional developed in Chapter 2 is used in this particular exercise. This cost functional J is given as :

$$J = J1 + J2 + J3. \qquad \text{---- } 4.1$$

J1, Cost of extra input.

$$= \sum_{k=1}^{T_s} P \{ (U(k) - U_{sn})^{[2]} \} \qquad \text{---- } 4.2$$

J2, Cost of inventory held.

$$= Q\{ (M)_n^{[2]} \} \qquad \text{---- } 4.3$$

J3, Replenishment delay penalty cost.

$$= R \{ I(k)_n^{[a + .1(k-t_r)]} \} \qquad \text{---- } 4.4$$

The details of the above cost structure have already been explained in Chapter 2and 3, where the same cost structure was applied to the same model with one single product.

It is pointed that the absolute magnitude of the values of weighting matrices P, Q and R are chosen such that each respective resulting cost, namely J1, J2 and J3 is on a similar scale, i.e. the costs have the same units. This is done so as to make inter-comparison possible. The choice of these values is effected from a combination of both their financial values where appropriate and their relative importance as viewed by the management.

These costs are applied to the two system responses for models MODEL Y and MODEL Z (Runs F1 and F3), and the different cost components are given in Figure 4.15.

a, Cost of Extra Inputs, (J1).

For this particular cost given in equation 4.2, it is found that the modified version gives rise to an increase of 5% and 10% in cost for the responses of the MODEL Y and MODEL Z respectively. It is pointed out that this actual increase does not contradict the analysis given in Table 4.4, where it was shown that the overall utilisation can be considered as exactly similar for each individual resource. This is so because the cost function J1 is based on the following features:

(i) The extra percentage required above steady state.

(ii) The dynamic timing of the allocation of the resources.

The analysis as shown in Table 4.4 is only the results of algebraic additions, without any priority weighting.

b, Cost of Safe Inventory Held, (J2).

This is the cost associated in holding the minimum amount of safe stocks to cater for the problem scenario (equation 4.3). The modified version carries a 6% increase for the response in the MODEL Y line, and non for the MODEL Z line. This is mainly due to the fact that the modified version, in reducing the handicap against the production of the MODEL Y assemblies, presently allocates more manufacturing facilities than previously, thus increasing the demands for inter-stage buffers.

c, Replenishment Delay Penalty Cost, (J3).

This cost as explained in chapter 2 is the one associated to the state of depletion of the inventories of assemblies and final product, and is structured in such a way as to penalise slow recovery to the original level, (equation 4.4). The results of this cost function are given in Figure 4.15c. In this particular case, the modified version shows a marked improvement of 23% for the MODEL Z system response, and only 2% extra cost for MODEL Y line.

d, Total Costs, (J).

The results of the total costs, J, in equation 4.1 are given in Figure 4.15d. The overall costs for the MODEL Y show an improvement of 12% while that for MODEL Z indicates a penalty of 5% in costs. Moreover, the aggregate cost of the two systems, MODEL Y and MODEL Z is shown to be improved marginally by 5% over the original approach.

In effect the new version has eased the previous preference given to the MODEL Z line. It provides a definite improvement to the responses of the MODEL Y line at the marginal expense of production of assemblies to the MODEL Z line.

4.8 Introduction Of Inventory Constraints.

4.8.1 Method Of Solution

Now that the modified control approach has been proved analytically to provide improved control responses, it is required to introduce the additional constraints on the initial availability of inter-stage buffers. The subroutine "@DIS/ILCB" as described in the previous chapters is again included in the simulation exercise.

In this example, it is assumed that there is a limit of 100 power units that would go in MODEL Y and 350 such units to MODEL Z. At other production stages, the buffers are sufficient to meet the required rates. A further assumption is that the bottle-neck area is still in the production of power-units and the maximum labour obtainable in one shift is 385/60 work hours. The demands for the finished cars are 120 and 400 for models Y and Z respectively. The production for MODEL Z is still to be favoured over MODEL Y, in view of the greater demand for the first model.

The control routine "@SYS/DSAS" is used again to work out the capacity requirements for the other production stages, so that they are structuredly balanced with respect to each other as explained in section 4.4. The results of this control simulation (RUN G1) are given in Figures 4.16, 4.17 and 4.18. The result when no constraint is present on the inventories (RUN G3) is also shown superimposed. The minimum buffers required for this situation should have been as given in RUN G3, and are shown in Table 4.4. From these figures, it is noted that the inventory constraints are indeed satisfied for both models at stage 3.The downstream stage, i.e. No 6 for the final car assembly is obviously affected since there has not been the initial required float. This is shown by the slower replenishment of the stock of finished cars. Moreover the depletion of the buffer of painted bodies is less severe than in RUN G3, because Stage 6 has not worked to its full maximum capacity at the initial period of the time horizon. However all the responses are still controlled appropriately to the steady states, illustrating the effectiveness of the simulation model.

4.8.2 Analysis of Control Results.

It is apparent that RUN G3 is more control-effective than Gl, since the latter had additional constraints on the buffer sizes, while the former did not have any. In this section, it is now examined how much the penalty actually is in the extra constraints in RUN Gl. This is carried out with the cost structure developed in Chapter 2, and is given in equations 4.1 - 4.4. The results of this exercise is shown in Figure 4.19 which consists of:

(a) Cost of Extra Inputs.

(b) Cost of Holding Inventory.

(c) Replenishment Cost.

(d) Total Costs.

For cost (a) and (b), it is seen that the results for the models under the two runs Gl and G3 are marginally similar (+ 3%). As for the replenishment cost (c), it is noted that for both models Y and Z, RUN G3 carried out without inventory constraints, provide more cost-effective responses. This has been largely due to the fact that RUN Gl has a slower replenishment for the inventory of finished cars. Cost (d), the total cost show a similar pattern, i.e. G3 is beeter than Gl by some 7 %. It is pointed out that this loss in effectiveness is the result of introducing 40% less than required inventory of power units for model Z. This represents inventory constraints only at 2 of the 10 inter-stage buffers.

In another series of runs, the constraints on the inventories of power unit for MODELS Y AND Z are assumed to be 100 each. In this particular case, it is a decrease of 40% of the required inventory of power units for MODEL Y and 80% decrease in power units for MODEL Z. The results of this run G2 is given in Figure 4.20 and 4.21. The minimum buffers required for this run is given in Table 4.5.

The cost functions in equations 4.1-4.4 are applied again and shown in Figure 4.22. From this figure, it is demonstrated that for MODEL Y, cost (a) and (b) are very similar for both runs G2 and G3, while for cost (c), G2 is 10% worse off.

In the case of MODEL Z, the penalty is indeed much higher with RUN G2. Cost (a) is 13% higher for G2 as compared to G3. As for cost (b), the results of RUN G2 is 25% over that of G3. The replenishment cost is seven times higher for G2 than G3, resulting from the fact that there has been a severe state of depletion of finished cars (MODEL Z) and a substantial overshoot in the production of power units destined for that particular model.

4.8.3 Discussion

The above exercise has shown that RUN G3 is the more cost-effective approach, which is to be expected since this has been derived from an assumption that the required inventory buffers are indeed available. This case shows the effectiveness of the control approach "@SYS/DSAS" in such a multi-product environment, in providing dynamic control policies on capacity requirements and inter-stage buffers. Nevertheless the approach still provides practical control policies even when additional constraints are included as witnessed in RUNS G1 and G2 testifying the effectiveness of the overall approach and the additional algorithm "@DIS/ILCB" in particular. The step in demands are still satisfied appropriately within the prevailing constraints, except that they are not as ideal as in RUN G3.

With such an approach, management can then have an increased insight in the control policies relating to the hoding of inter-stage buffers. These are:

(i) A knowledge of how particular float level will affect the response of the system.

(ii) The cost-benefit associated with different scenarios starting with different float levels.

4.9 Introduction Of Disturbances.

4.9.1 Analysis of Problem.

In this present series of simulations, the approach developed in the preceding section is applied again for a multi-product system with a certain amount of reject included. This illustrates the flexibilty of the model in its capacity to consider reject rates, a very common feature in a manufacturing environment. This is especially so in the automotive industry where the quality standards need to be very high for safety reasons and external competition. The same final demands are considered, i.e. 120 units of MODEL Y and 400 units of MODEL Z per shift and the scrap rates are as follows:

- Power assembly 15%
- Painting of car body 10%.
- Final Assembly 5%

for both systems.

These of course will mean extra demands in capacity above the steady state values, if the desired output rate is to be maintained. The calculation of the reject units is given in Appendix 6. The actual number of rejected units and the new steady states are:

	MODEL Y		MODEL Z	
	Reject units	Steady State	Reject units	Steady State
Stage 1 :	0	148	0	495
Stage 2 :	0	148	0	495
Stage 3 :	22	148	74	495
Stage 4 :	0	140	0	468
Stage 5 :	14	140	47	468
Stage 6 :	6	126	21	421

These will mean that values at the 3 rd , 5th, 6th and 7th entries are the disturbance vector d_Y are 22, 14, 6 and 120 respectively. Similarly, vector d_Z will have values of 74, 47, 21 and 400 at the 3rd, 5th, 6th and 7th entries. The rest being zero.

For this particular problem, it is assumed that it is possible to obtain a maximum of 385/60 work hours (i.e. 385 minutes in one shift) at the assembly of power units fitting the two models. The approach code-named "@BLT/DSAS" (Figure 4.10) is used. The results of the runs with and without disturbances (RUNS H1 and H2 respectively) are shown in Figure 4.23, 4.24 and 4.25 superimposed on the same graphs. The limits as worked out by the algorithm are given in Table 4.5. The safe inventories for the two models under the two conditions of with and without reject are given in Table 4.6.

At the critical stage of power unit assembly (stage 3), it is seen that the inventory takes far much longer to replenish for both models when reject is present. This severe depletion will be present, if the downstream production rates are to be maintained. It is again noted that resources are favourably allocated to the MODEL Z units in the initial part of the run, illustrating the arbitrary use of initial CPN as an effective means for deciding on such weighting. The invento ries of gearboxes and engines are depleted less severely with the disturbances in than without, although approximately the same length of time in restoring to the desired level is noted. This results from a combined effect of the following factors :

(i) The limits at resources 1 and 2 (R1 and R2) are higher.

(ii) The limit at R3 is still the same.

For both models, the inventories at stages 4, 5 and 6 take a longer time to replenish, as would be expected, nevertheless they are soon controlled to their original datum states.

4.9.2 Probabilistic Analysis.

It has already been mentioned that the approach used in the form of Brunovsky's canonical matrices treat the disturbances as acting on a constant value throughout the time horizon. Nevertheless, this fact does not prevent a study on the contingency measures for situations where the disturbances are known on a statistical basis.

In this section, the occurrence of disturbances are agian assumed to be represented by the Gamma distribution, as in Chapter 3. The three values of reject, i.e. 15%, 10% and 5% occurring at stages 3, 5 and 6 for both models are assumed to be the usual expected values. The same probability function as described in Chapter 3 is used, i.e.:

$$P(X > x) = \sum_{k=0}^{r-1} e^{-\alpha x} (x)^{k} / k!$$

where $r \geqslant 0, \alpha > 0$.

Parameter r is chosen to be 7 as representative for the actual case and is taken to be equal to 1. From Figure 3.33 and Table 3.6 the following probabilities are obtained.

P1 = 15 > x > 0 = 1 - .4497 = .5503

P2 = 10 > x > 0 = 1 - .4497 = .5503

P3 = 5 > x > 0 = 1 - .4497 = .5503

Consequently, the probability that the actual requirements of the stages 1 - 6 are between the responses of RUNS H1 and H2 for each respective model, the value is P1 X P2 X P3 = .16664
Obviously such an analysis may be repeated readily with other scenarios with different reject rates and different probability distributions. The model has been developed to allow such a flexibility in the investigation of different scenarios with recommended solutions together with their probabilistic occurrences.

4.10 Conclusion.

In this chapter, a two-product multi-stage production-inventory system has been considered with the use of the control approach developed during the course of the research. Such a multi-product manufacturing environment adds a further dimension to the problem, in the need to share out common manufacturing resources to dynamically competing demands from different models. This extra problem feature has been considered in conjunction with the other control parameters as:

- Dynamic capacity requirements and the limits imposed on them
- Availability of the necessary buffers.

Computational algorithms have been especially developed for such purposes. The introduction of individual CPN (Control Policy Number) that can control individual modes of the system, has rendered such a task readily implementable and effective. This is reflected in the results obtained from the control simulations.

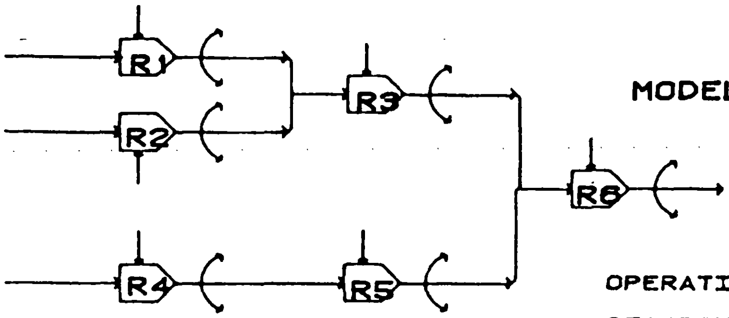
The concept of structured capacity has also been successfully transferred from a mono-product manufacture to one of multi-product. This is the situation where the production capacity at each stage is controlled in such a way that it is balanced with the other Production-Inventory stages. This objective has also been achieved in the multi-product environment where common resources have to be sharedout.

The use is made of the Gamma distribution in order to analyse the model on a probabilistic basis. This analysis provides a known probability value of how the system will repond given a particular production control scenario.

In conclusion, the research carried out has effectively extended the application of multi-variable control theory in a practical control environment of both single and multi-item manufacture in multi-stage production-inventory systems.

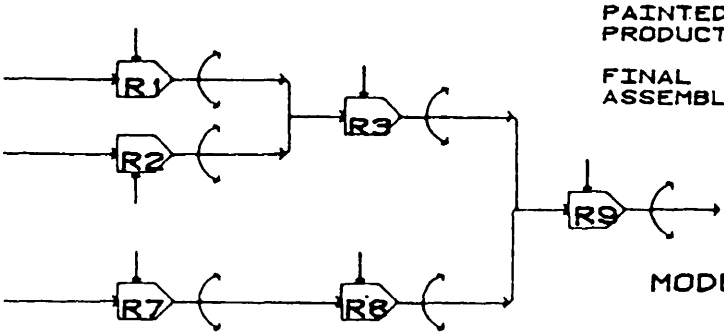
CHAPTER 4

FIGURES



MODEL Y

OPERATION	RESOURCE MODEL Y	RESOURCE MODEL Z
GEARBOX ASSEMBLY	R1	R1
ENGINE ASSEMBLY	R2	R2
POWER UNIT ASSEMBLY	R3	R3
BODY IN WHITE WELDING	R4	R7
PAINTED BODY PRODUCTION	R5	R8
FINAL ASSEMBLY	R6	R9



MODEL Z

KEY

PRODUCTION STAGE

RESOURCE INPUT

SEPARATE DEMAND OR TO INVENTORY

FEED INTO NEXT PRODUCTION STAGE

FIGURE 4.1 : SCHEMATIC REPRESENTATION MODELS Y AND Z

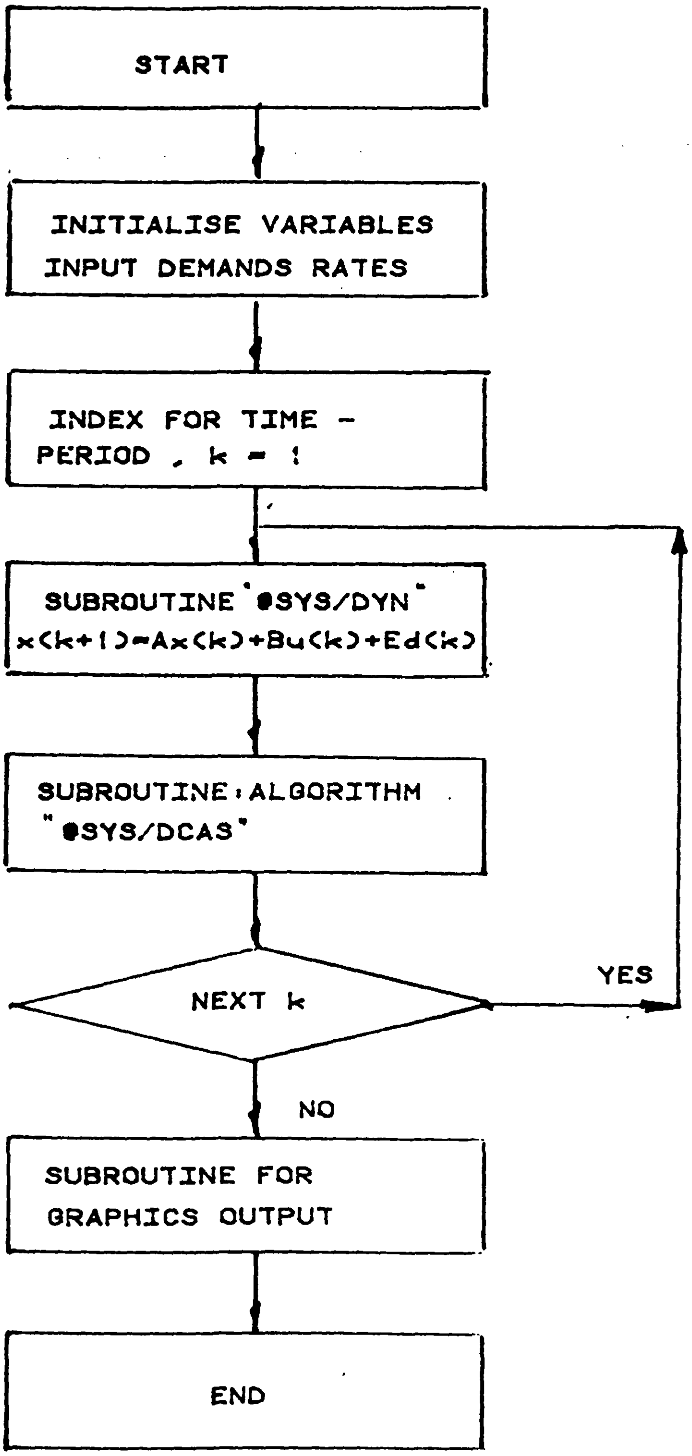


FIGURE 4.2 : STRUCTURE OF OVERALL CONTROL SIMULATION FOR DOUBLE SYSTEMS

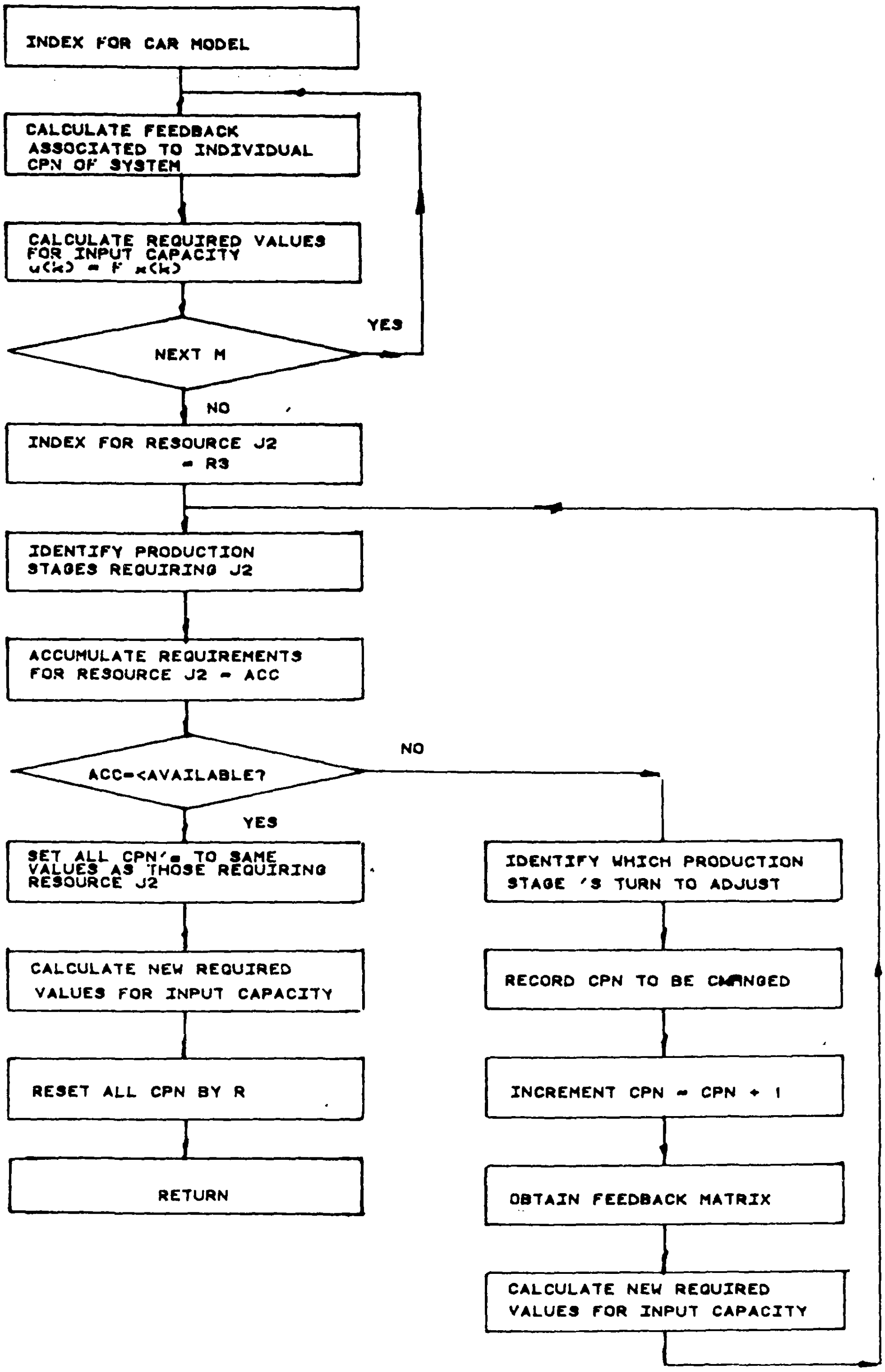


FIGURE 4.3 : SUBROUTINE '@SYS/DCAS'

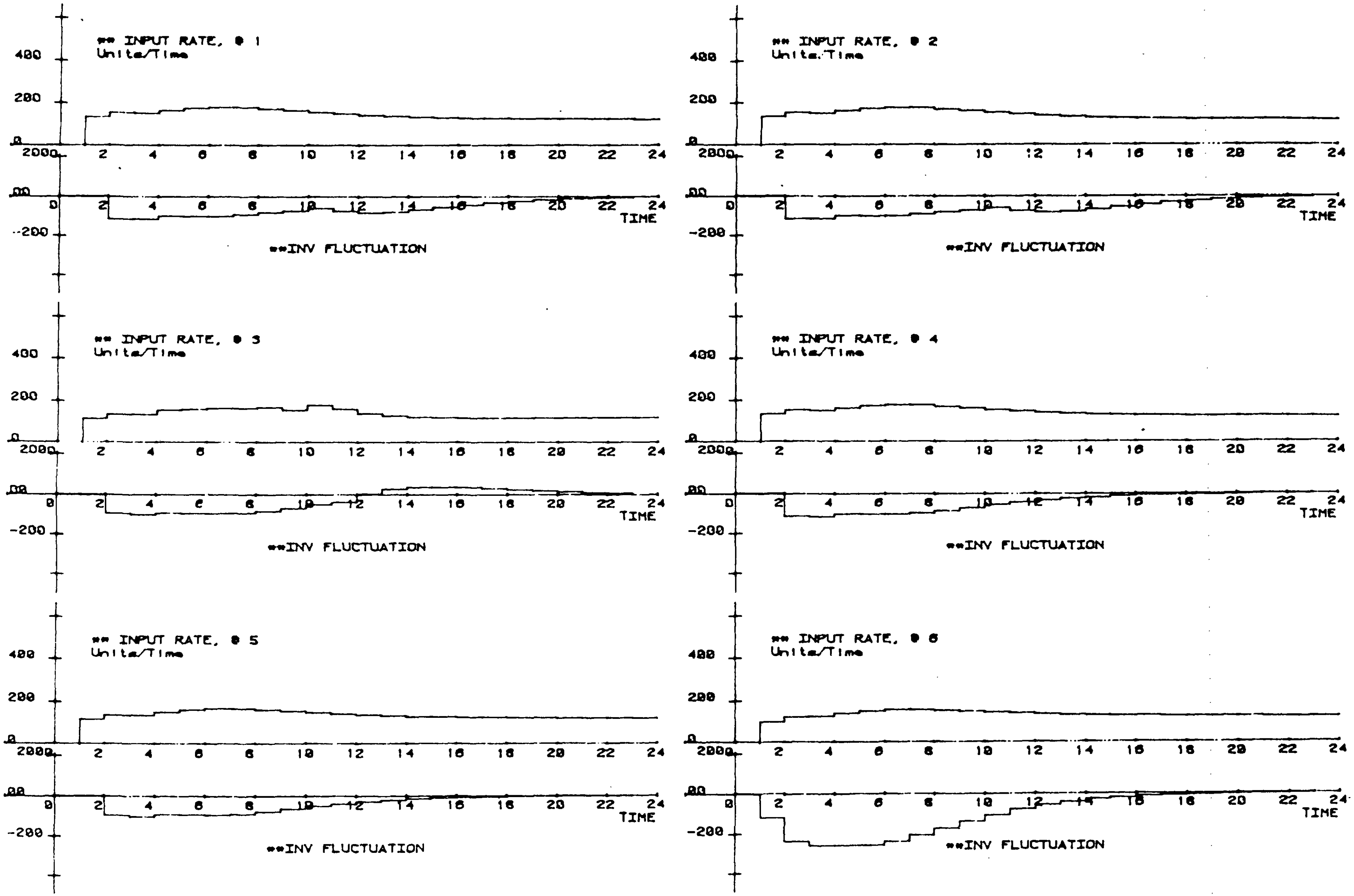


FIGURE 4.4 : RESULTS OF CONTROL SIMULATION IN RUN F1 FOR MODEL Y

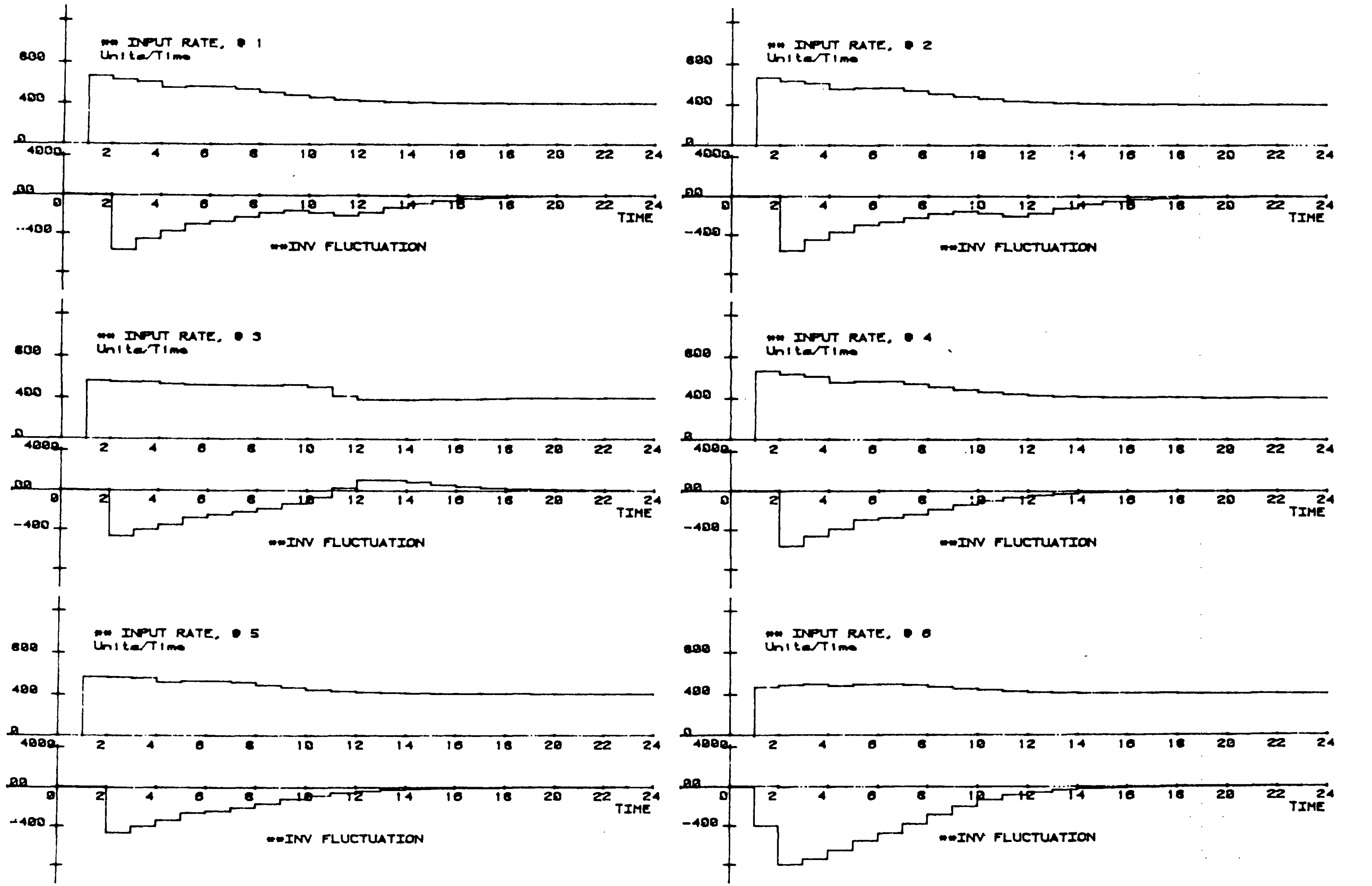


FIGURE 4.5 : RESULTS OF CONTROL SIMULATION IN RUN F1 FOR MODEL Z

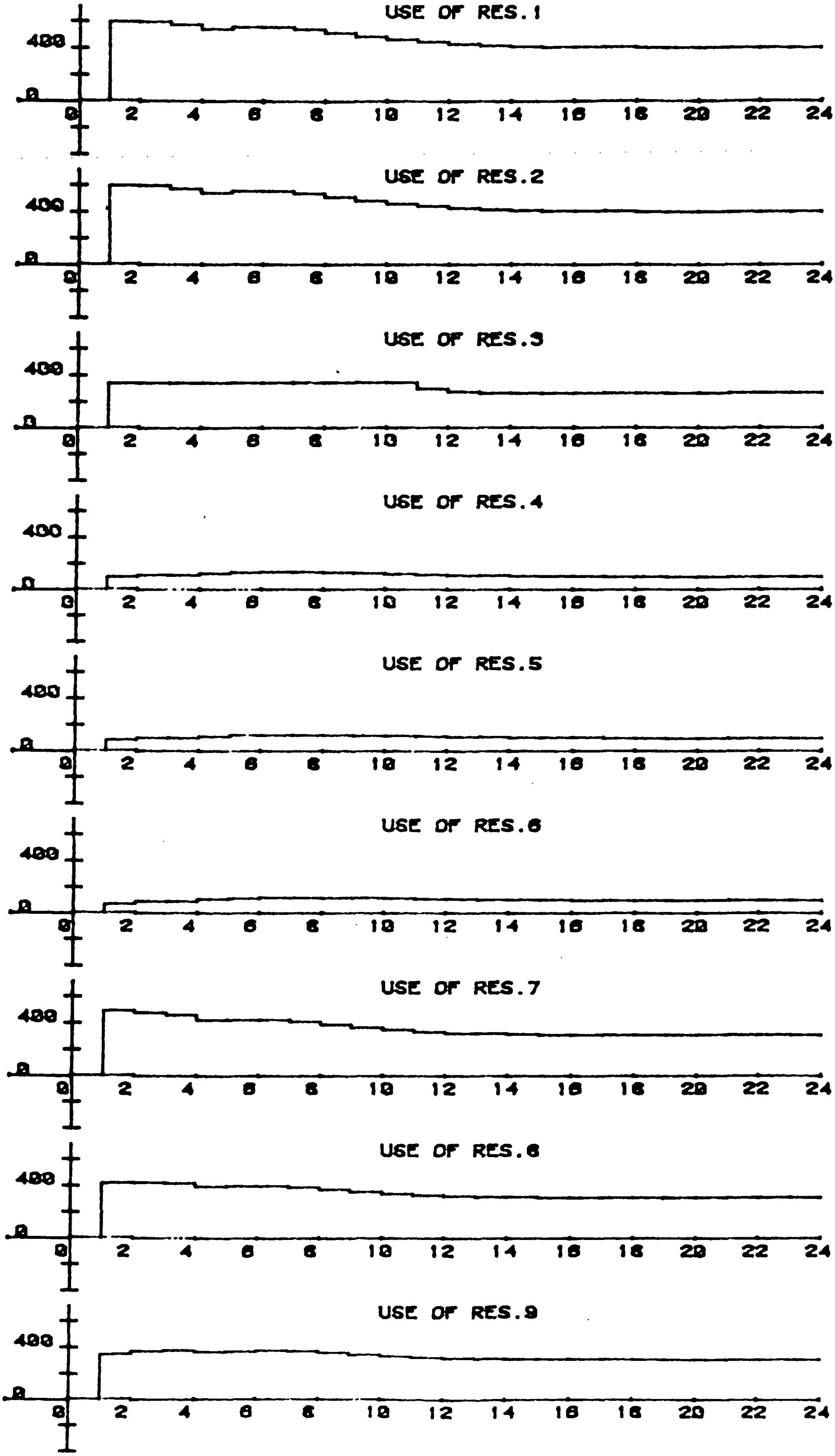


FIGURE 4.6: AGGREGATE UTILISATION OF RESOURCES IN RUN F1

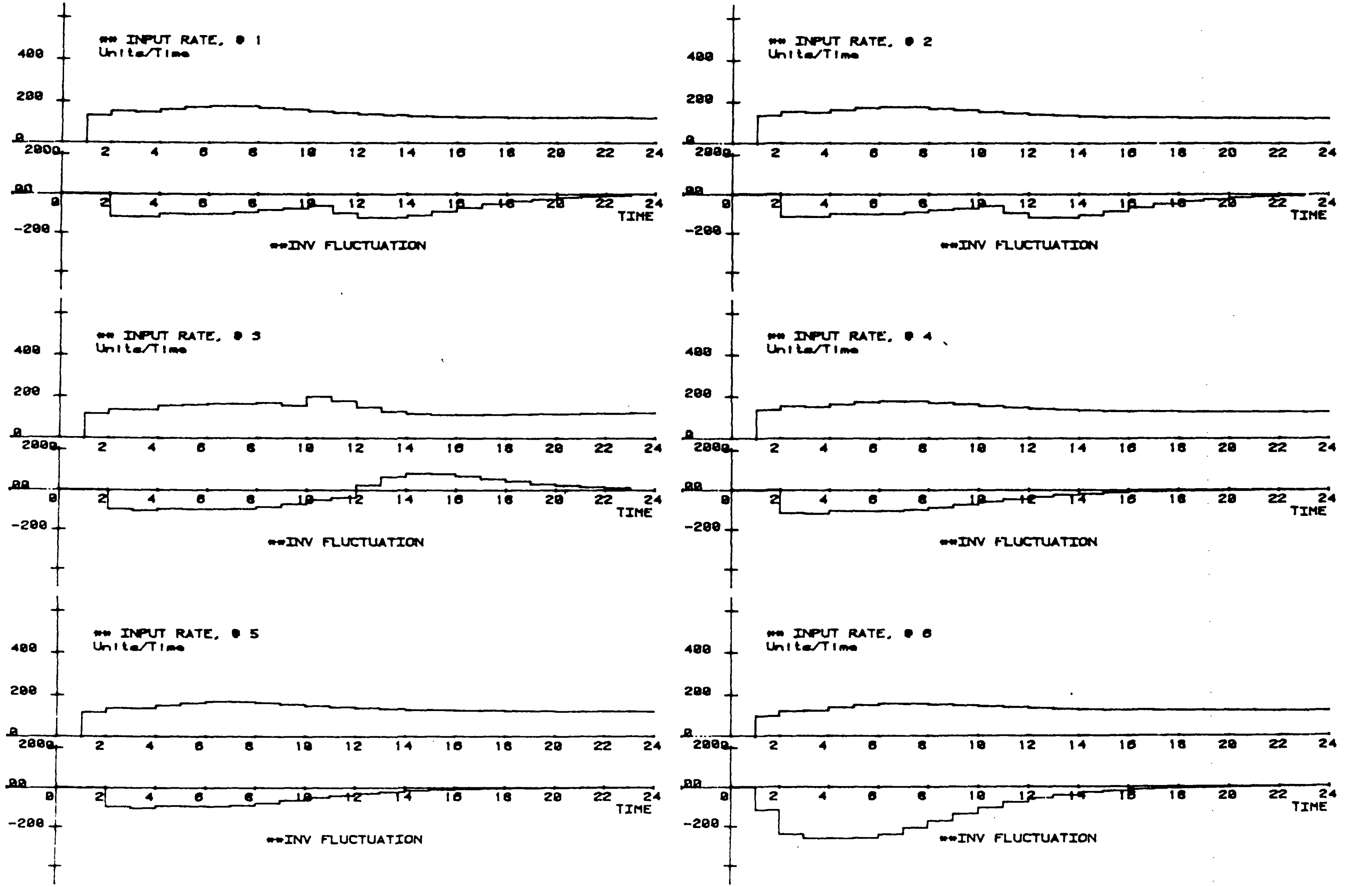


FIGURE 4.7 : RESULTS OF CONTROL SIMULATION IN RUN F2 FOR MODEL Y

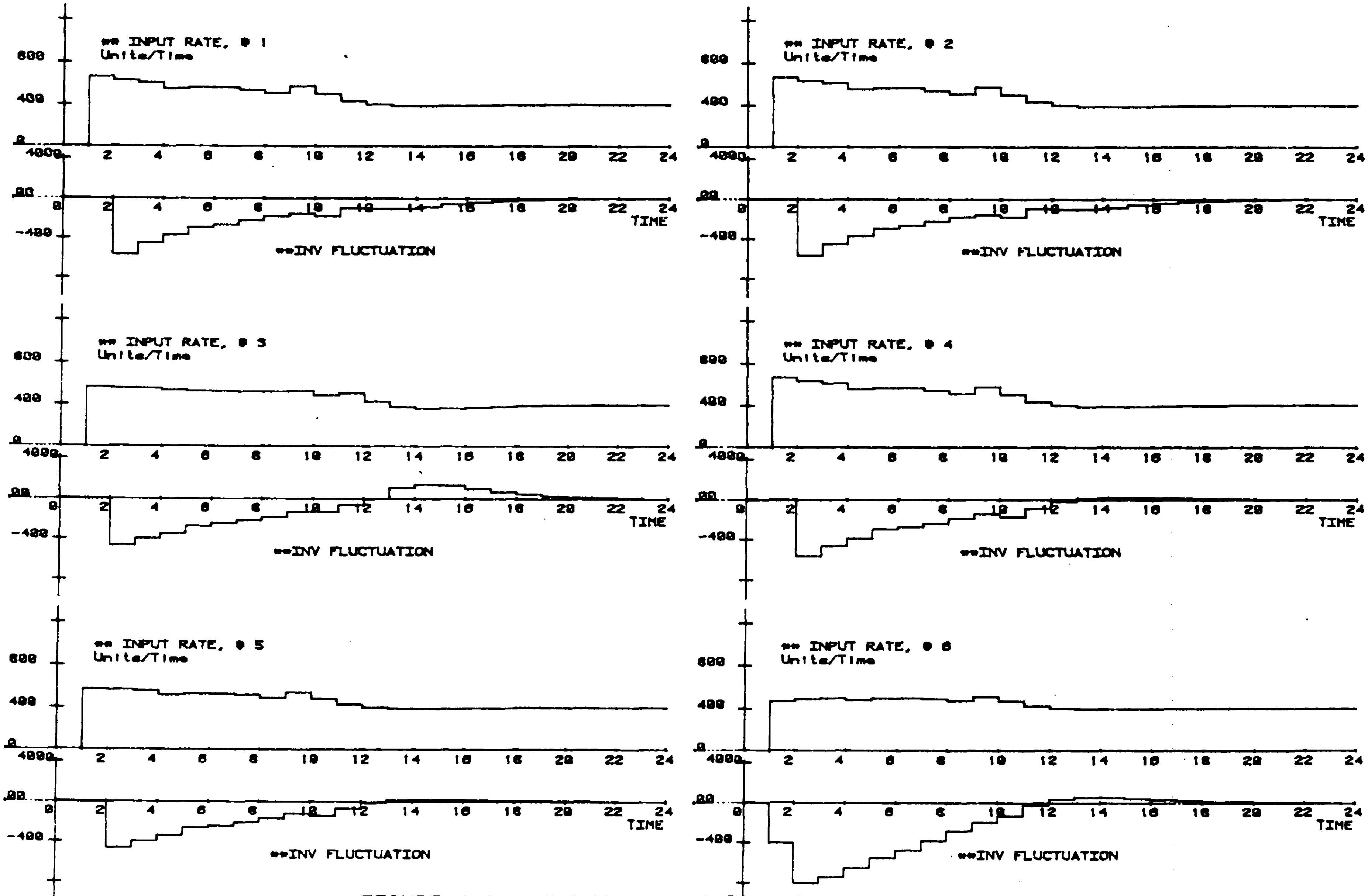
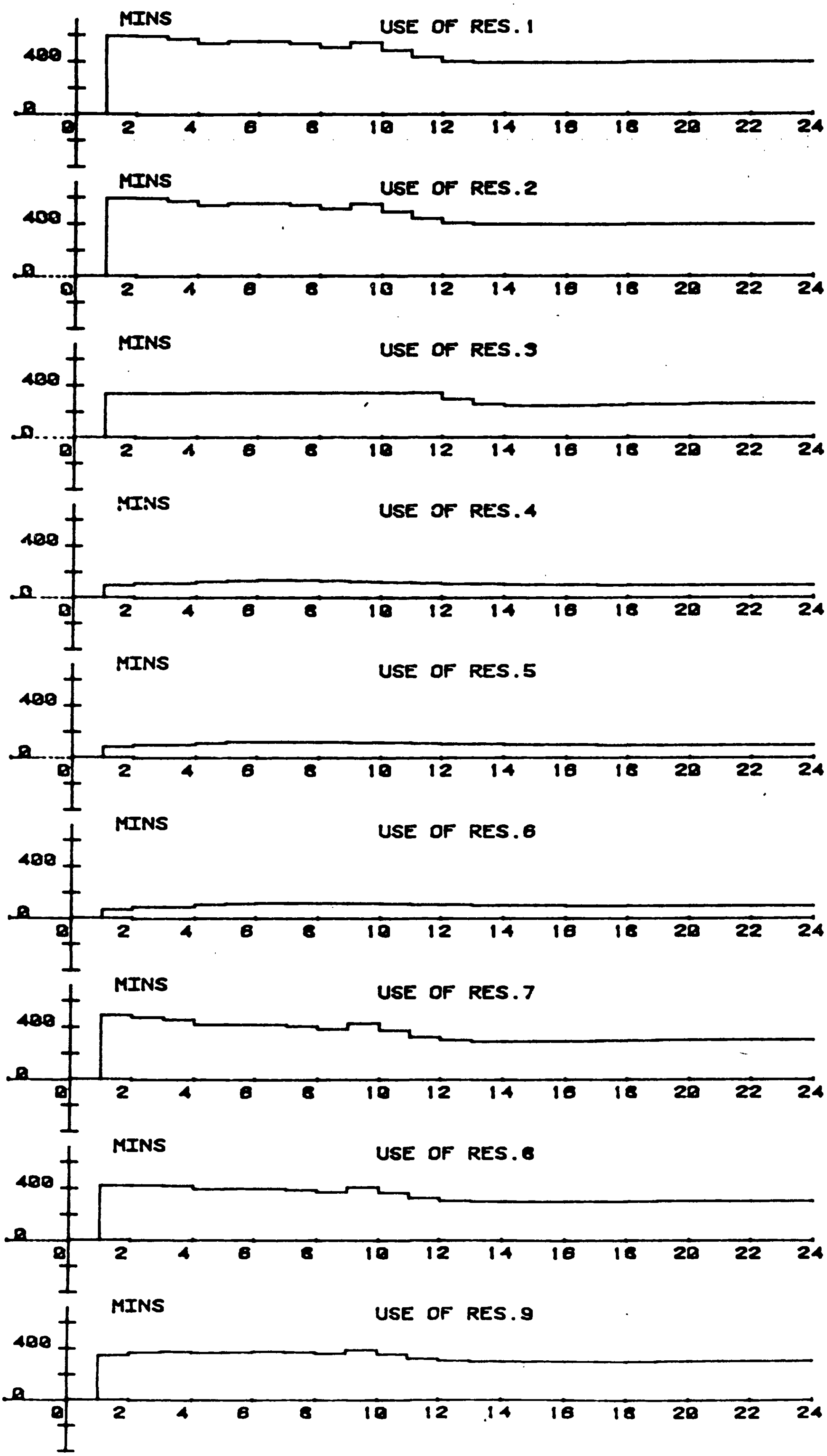


FIGURE 4.8 · RESULTS OF CONTROL SIMULATION IN RUN F2 FOR MODEL Z


MINS
USE OF RES.1
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0 2 4 6 8 10 12 14 16 18 20 22 24
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USE OF RES.2
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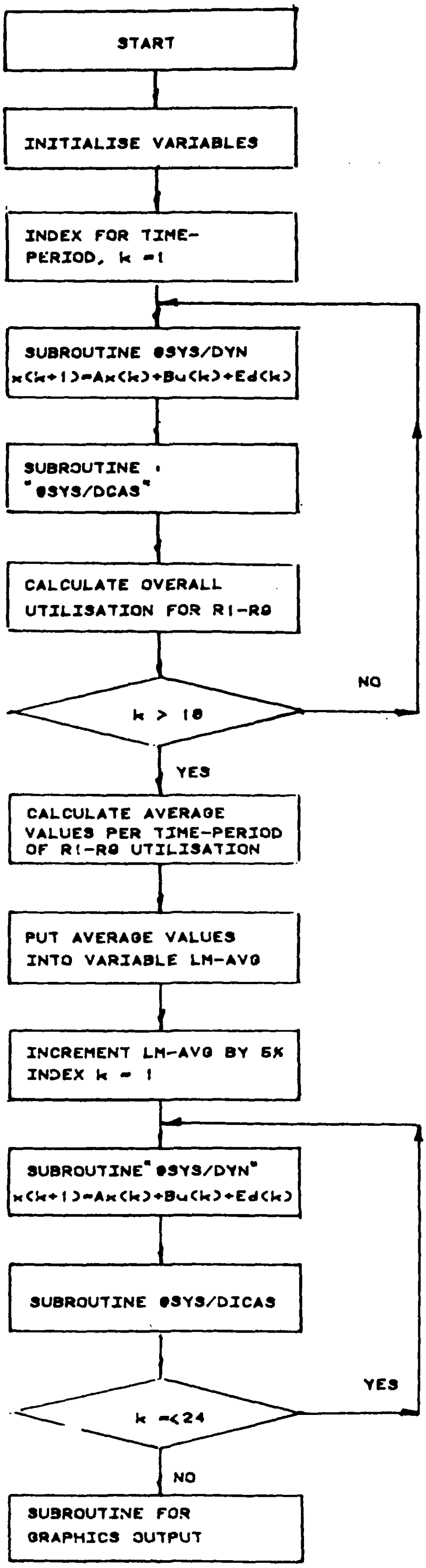


FIGURE 4.10 : NEW OVERALL CONTROL SIMULATION "@SYS/DSAS"

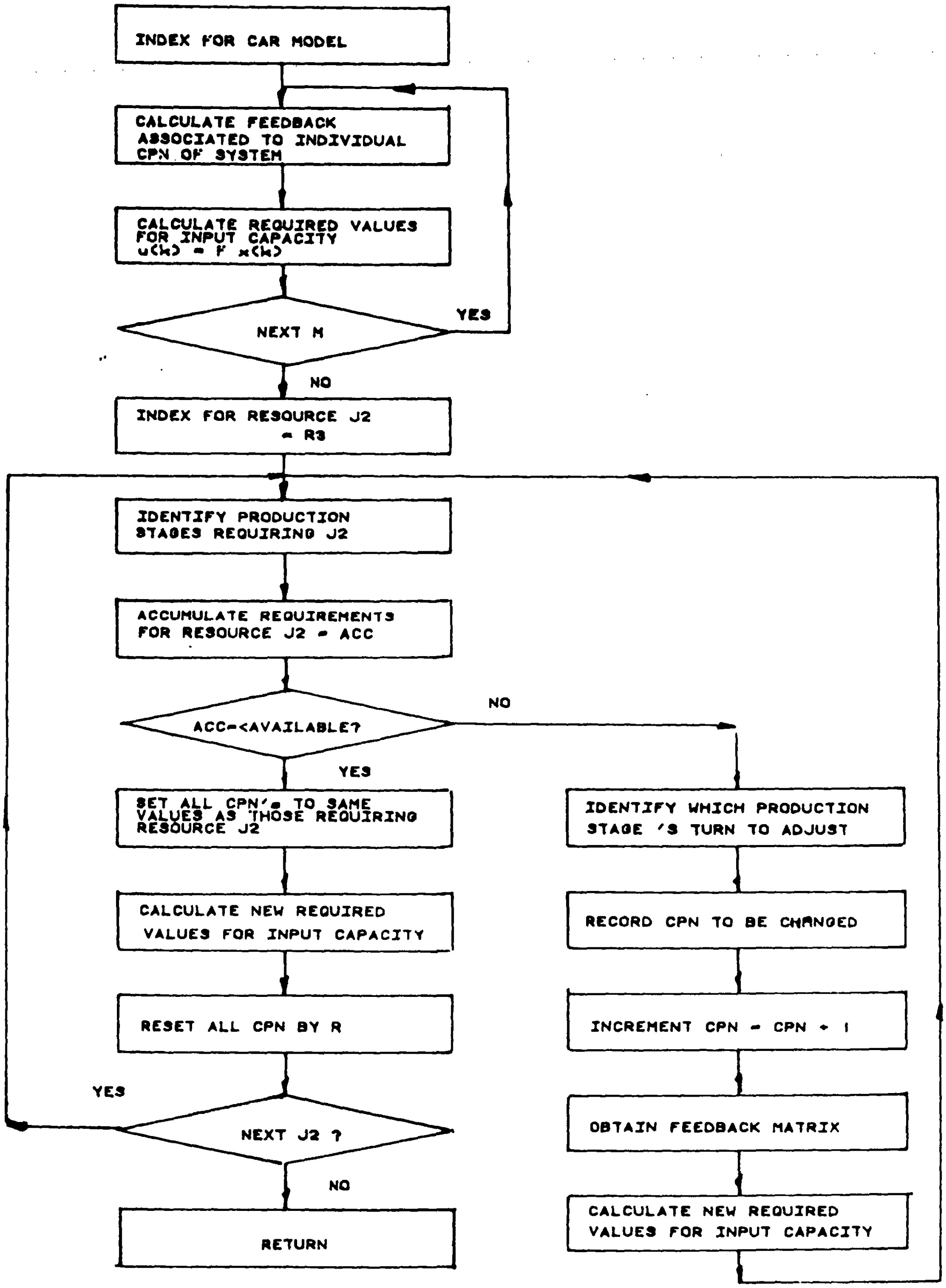


FIGURE 4.11 : SUBROUTINE "@SYS/DICAS"

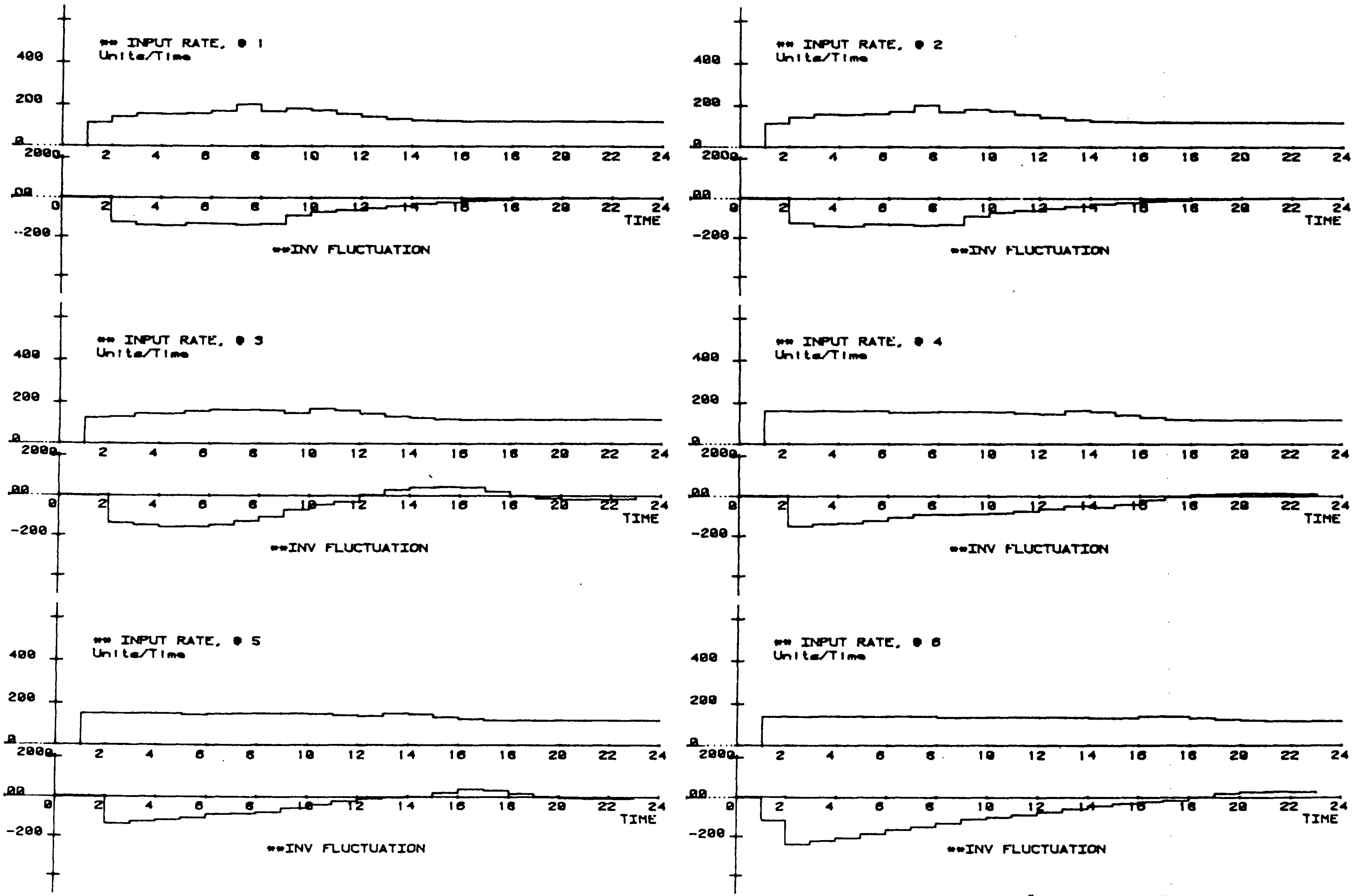


FIGURE 4.12: RESULTS OF MODIFIED CONTROL SIMULATION "@SYS/DSAS" FOR MODEL RUN F3

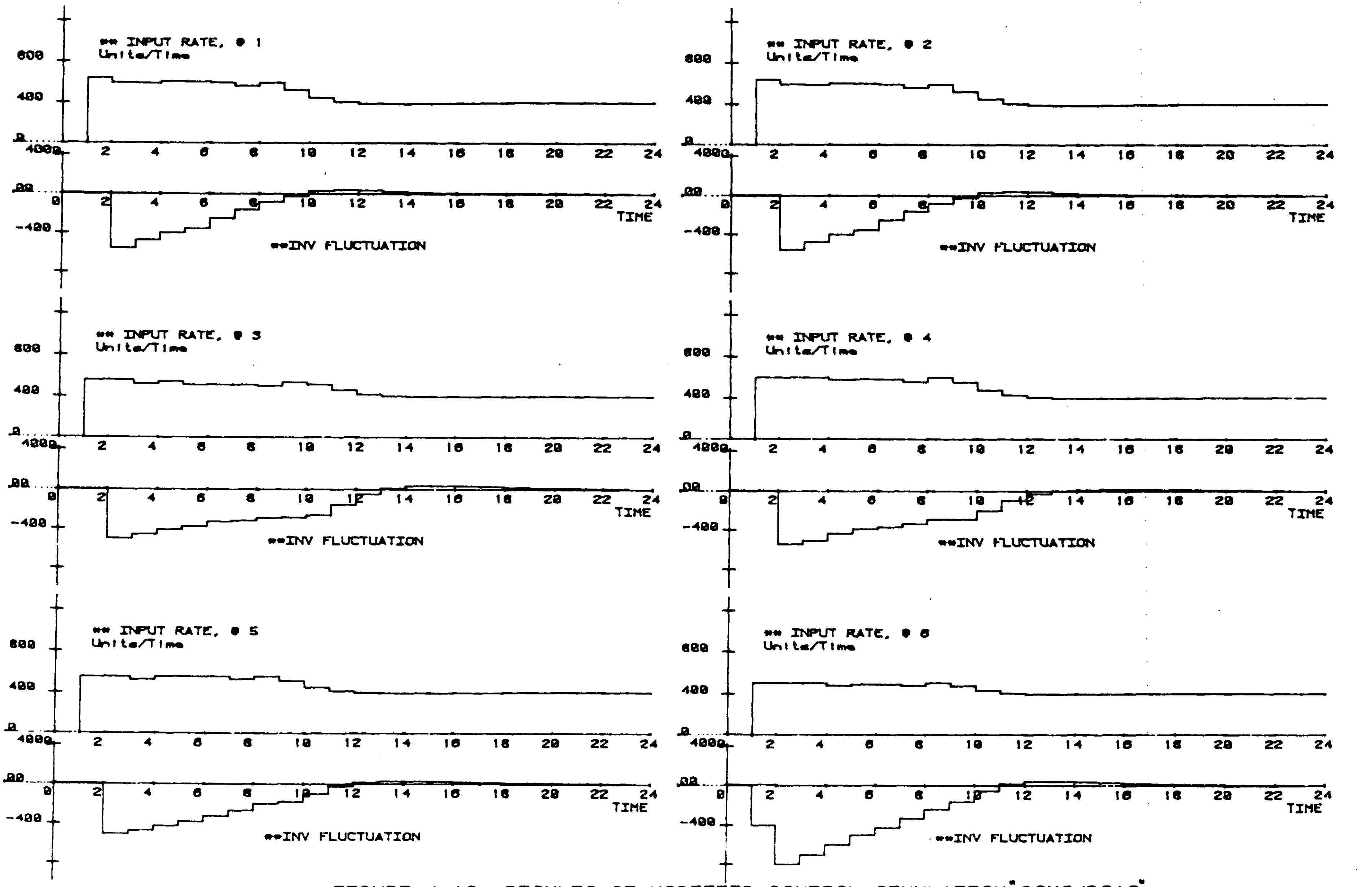


FIGURE 4.13: RESULTS OF MODIFIED CONTROL SIMULATION "@SYS/DSAS" FOR MODEL Z, RUN F3

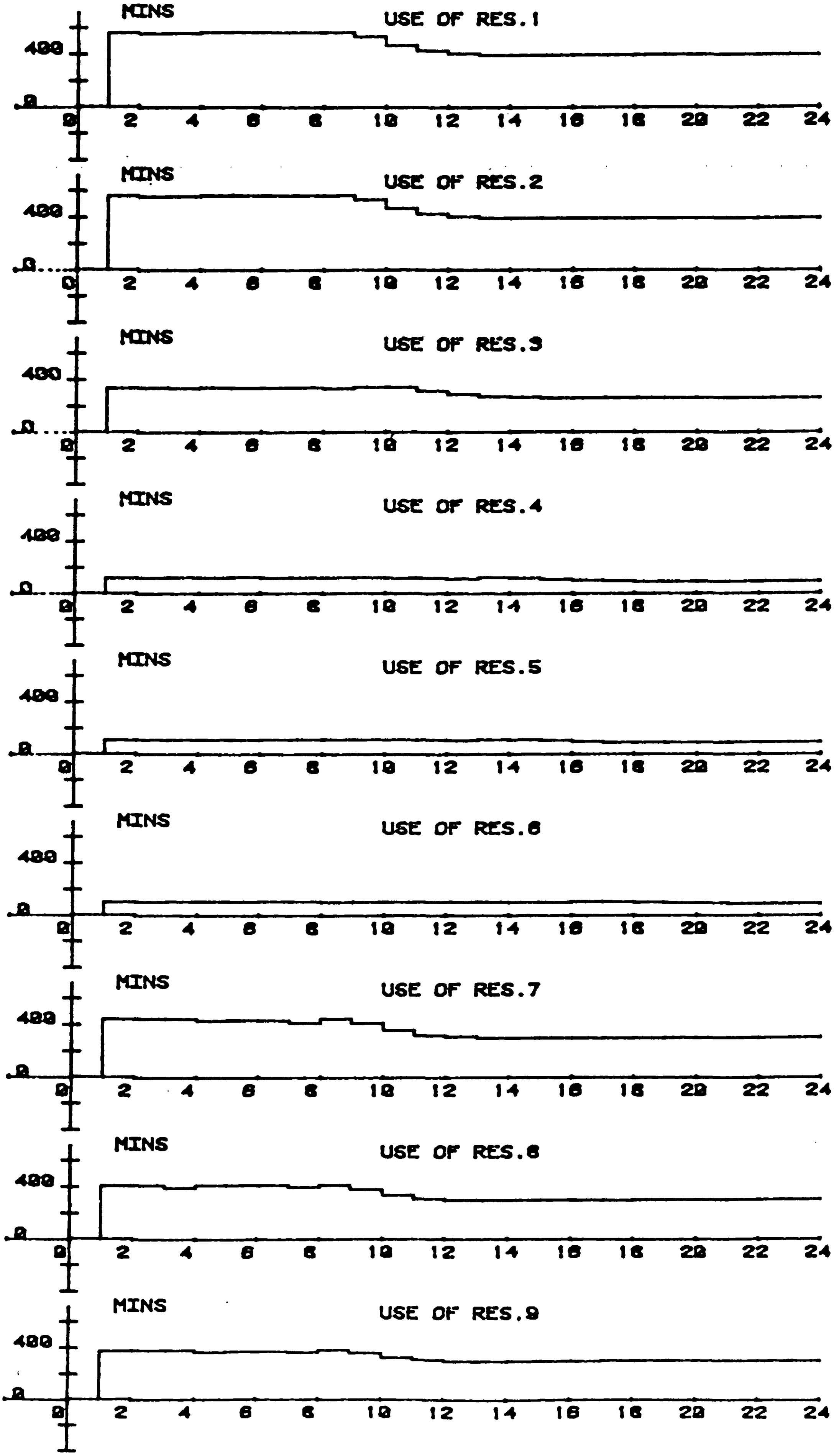
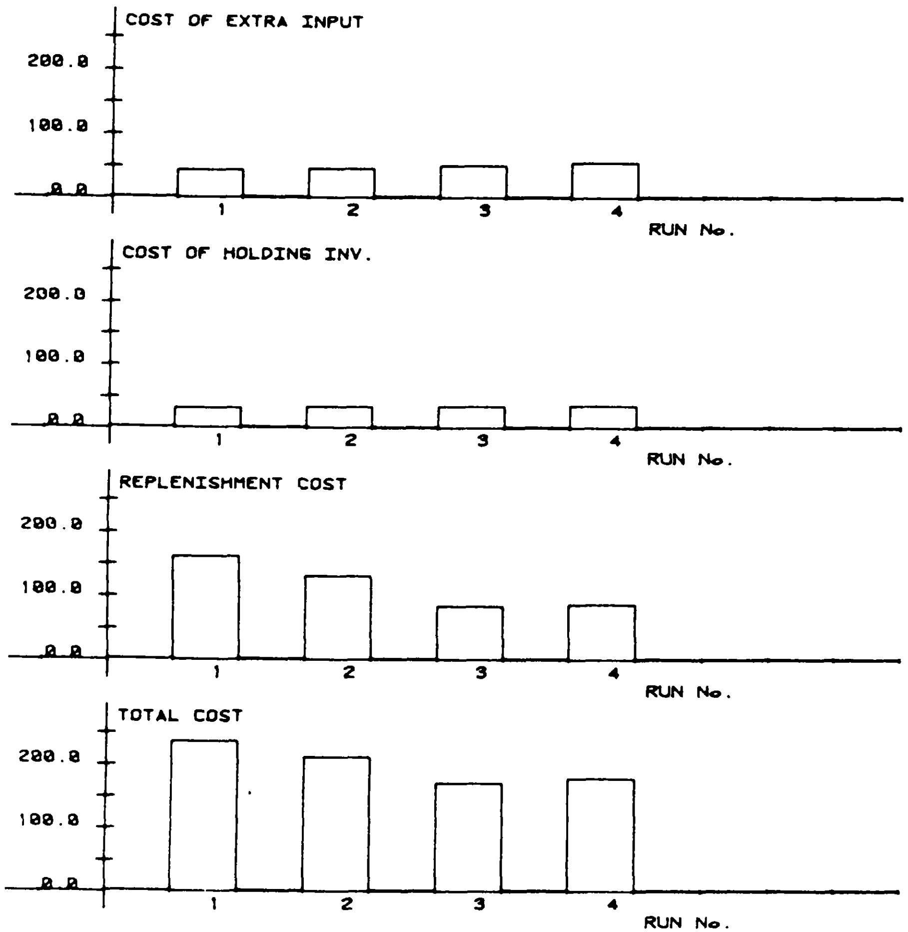


FIGURE 4.14 : AGGREGATE UTILISATION OF RESOURCES IN RUN F3



LABEL

1 . COST FOR MODEL Y BEFORE MODIFICATION, RUN F1
2 . COST FOR MODEL Z AFTER MODIFICATION, RUN F3
3 . COST FOR MODEL Z BEFORE MODIFICATION, RUN F1
4 . COST FOR MODEL Z AFTER MODIFICATION, RUN F3

FIGURE 4.15a-d : RESULTS OF COSTS FUNCTIONS COMPARING RUNS F1 AND F3

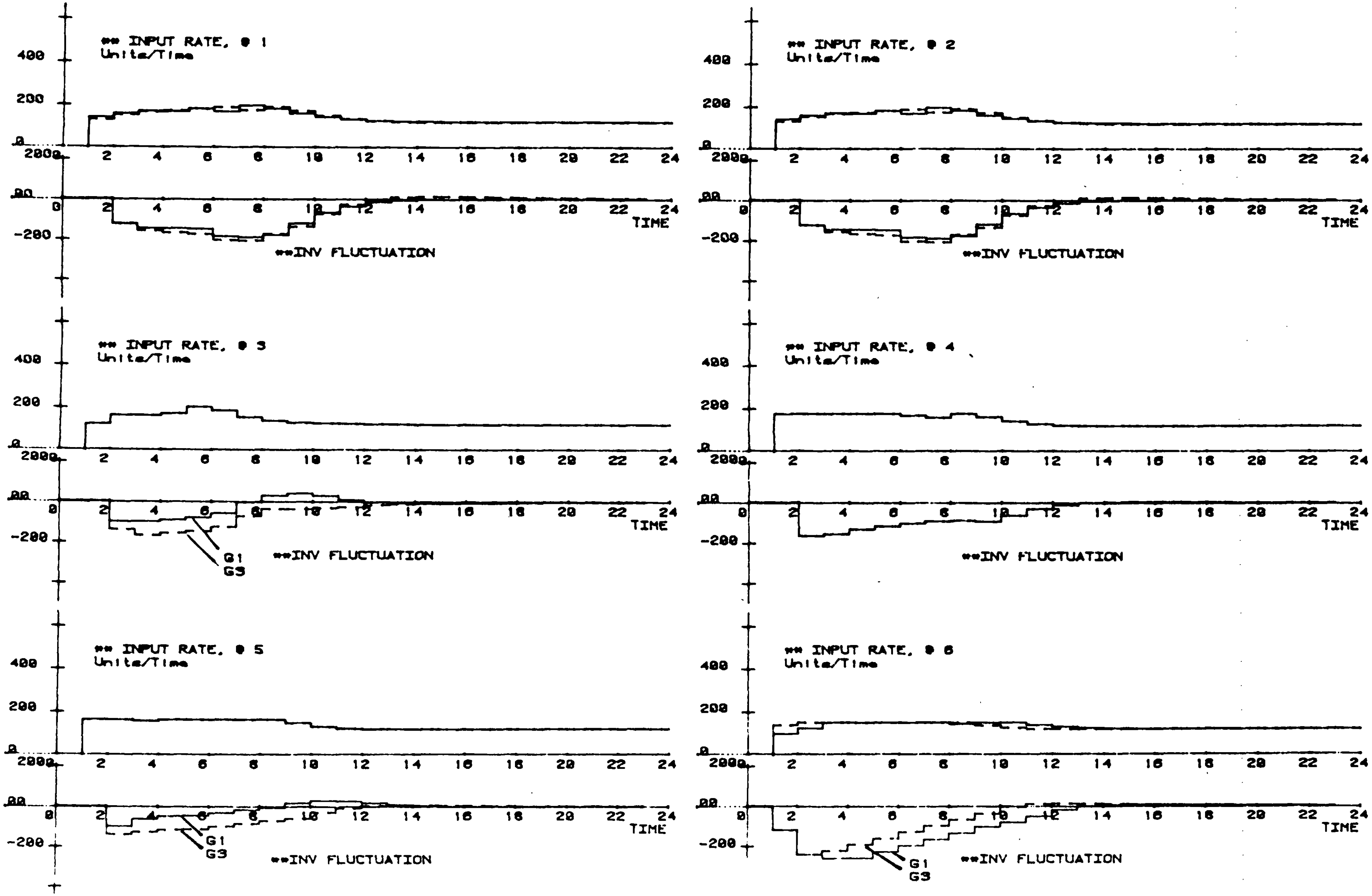


FIGURE 4.16 : RESULTS OF RUN G1 FOR MODEL Y
INTRODUCTION OF INV.CONSTRAINT @ STAGE 3

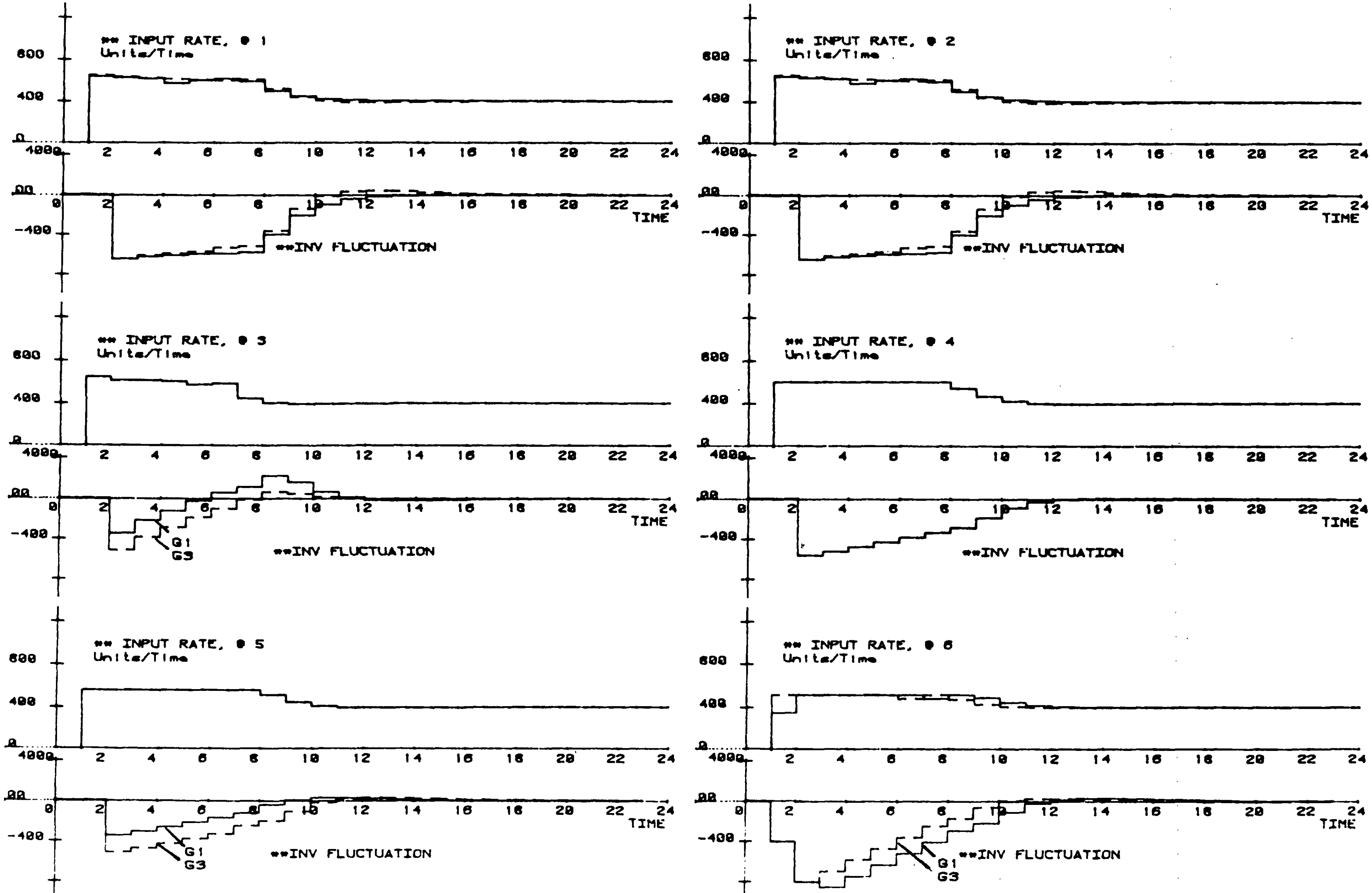


FIGURE 4.17 : RESULTS OF RUN G1 FOR MODEL Z
INTRODUCTION OF INV CONSTRAINT @ STAGE 3

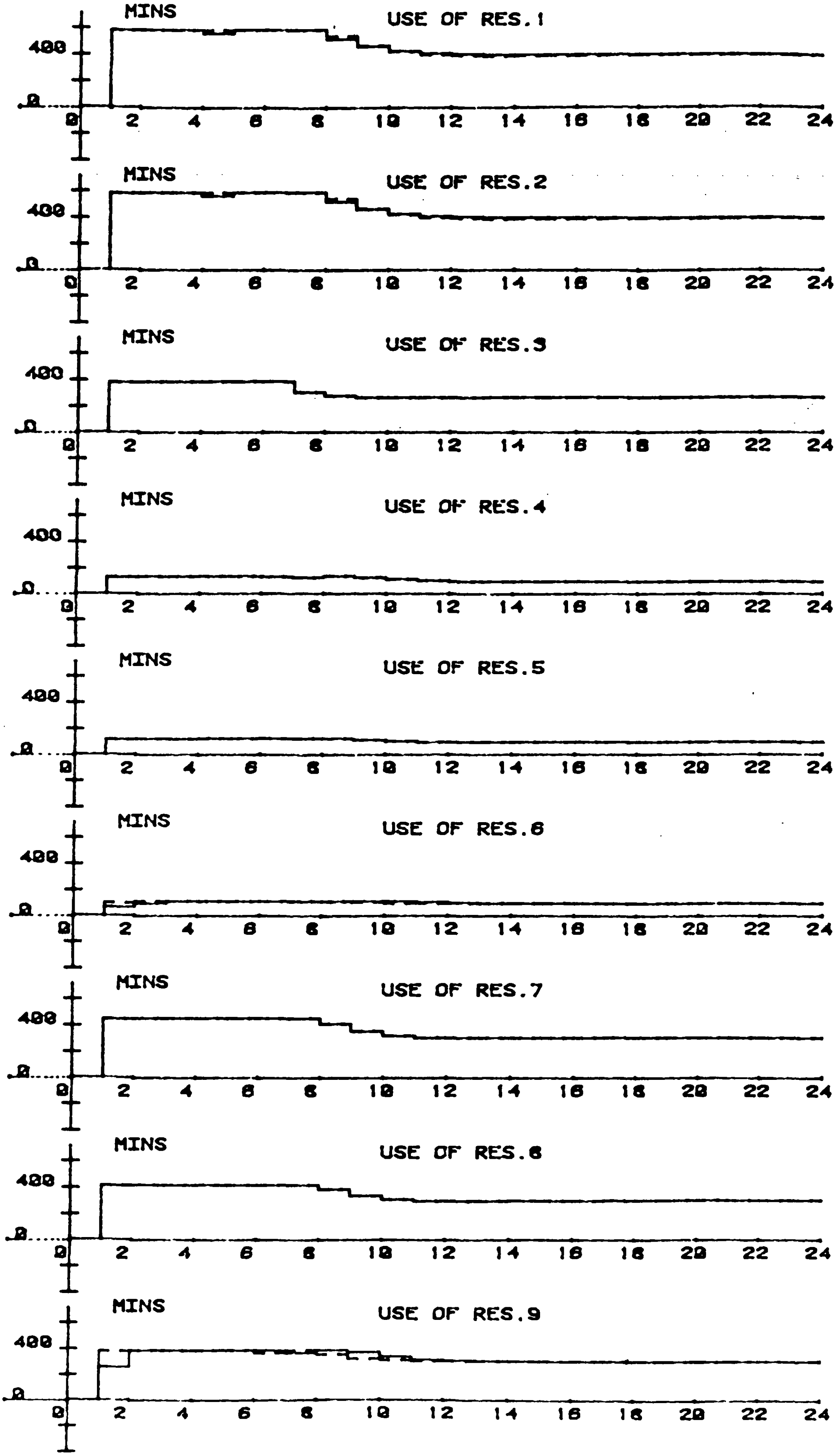
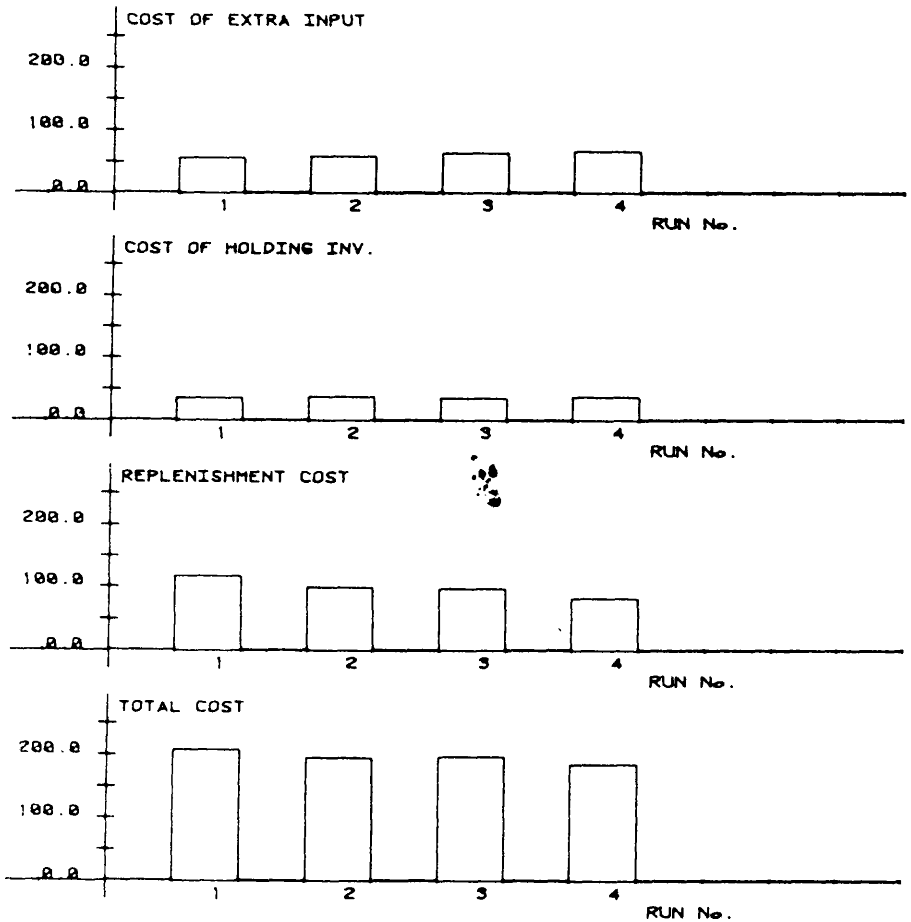


FIGURE 4.18 : AGGREGATE UTILISATION OF RESOURCES
RUNS G1 AND G3



LABEL

1 . COST FOR MODEL Y WITH INV. CONSTRAINT, RUN G1
2 . COST FOR MODEL Y WITHOUT INV. CONSTRAINT, RUN G3
3 . COST FOR MODEL Z WITH INV. CONSTRAINT, RUN G1
4 . COST FOR MODEL Z WITHOUT INV. CONSTRAINT, RUN G3

FIGURE 4.19a-d : RESULTS OF COSTS FUNCTIONS COMPARING RUNS G1 AND G3

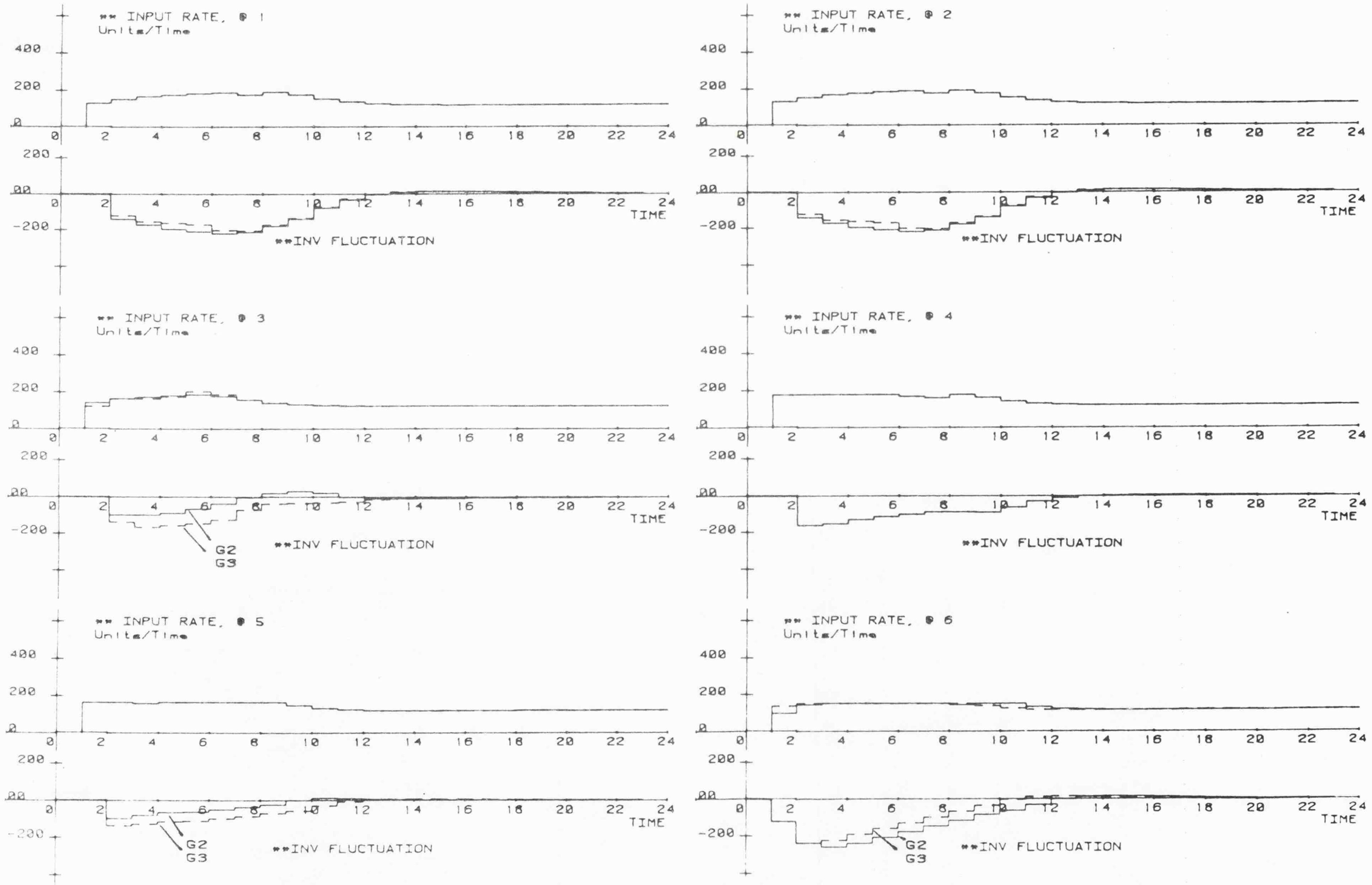


FIGURE 4.20 : RESULTS OF RUN G2 FOR MODEL
INTRODUCTION OF INV.CONSTRAINT @ STAGE 3

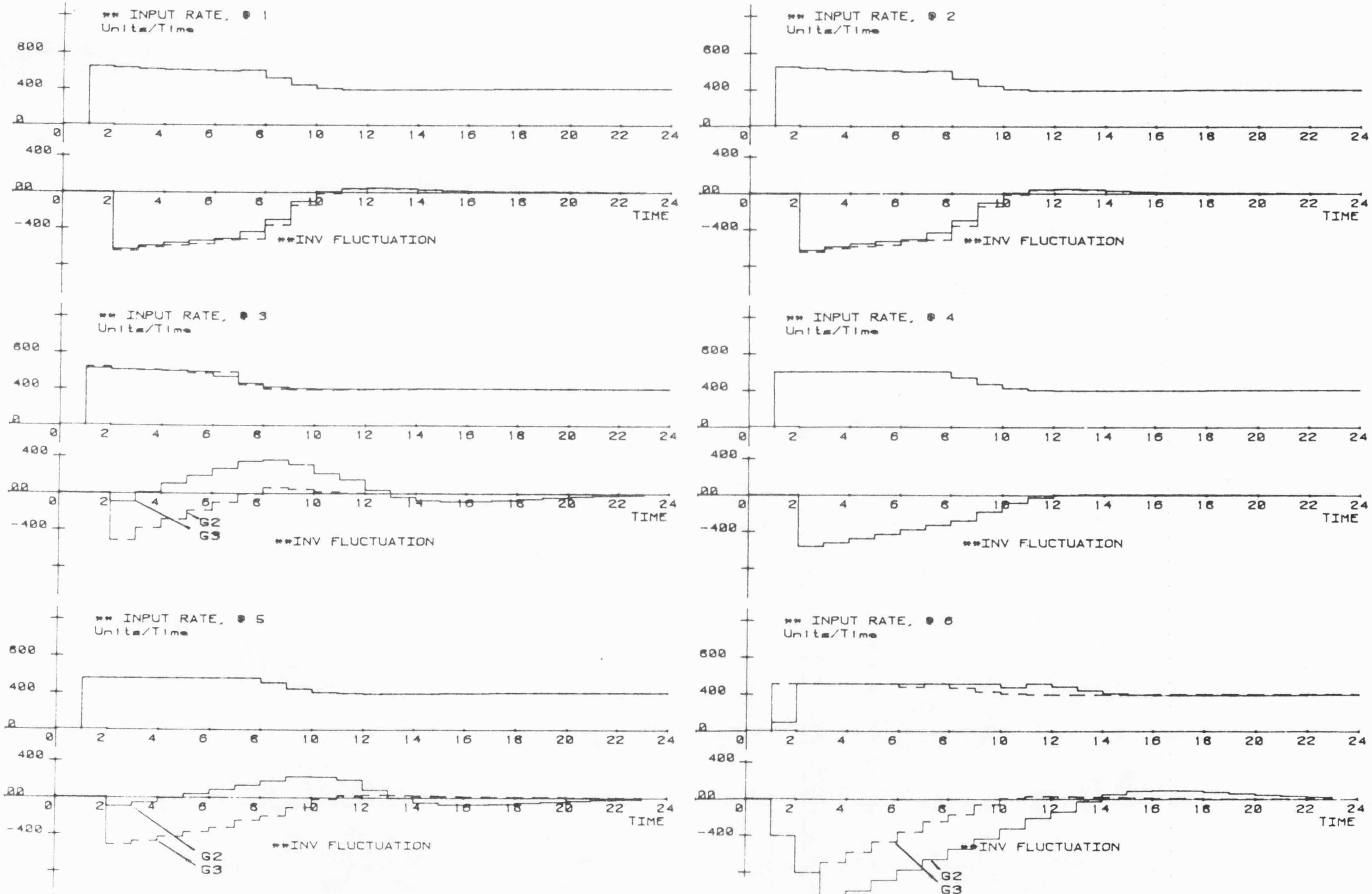
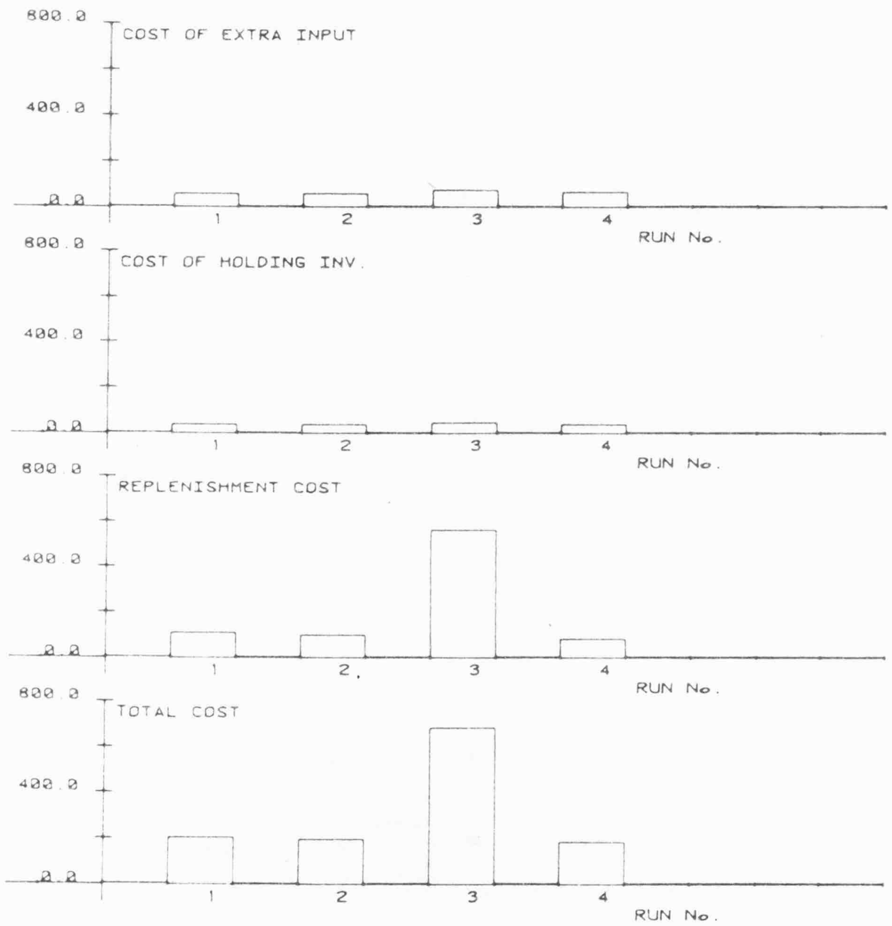


FIGURE 4.21 : RESULTS OF RUN G2 FOR MODEL Z
INTRODUCTION OF INV.CONSTRAINT @ STAGE 3



LABEL

1 . COST FOR MODEL Y WITH INV. CONSTRAINT, RUN G2
2 . COST FOR MODEL Y WITHOUT INV. CONSTRAINT, RUN G3
3 . COST FOR MODEL Z WITH INV. CONSTRAINT, RUN G2
4 . COST FOR MODEL Z WITHOUT INV. CONSTRAINT, RUN G3

FIGURE 4.22a-d : RESULTS OF COSTS FUNCTIONS COMPARING RUNS G2 AND G3

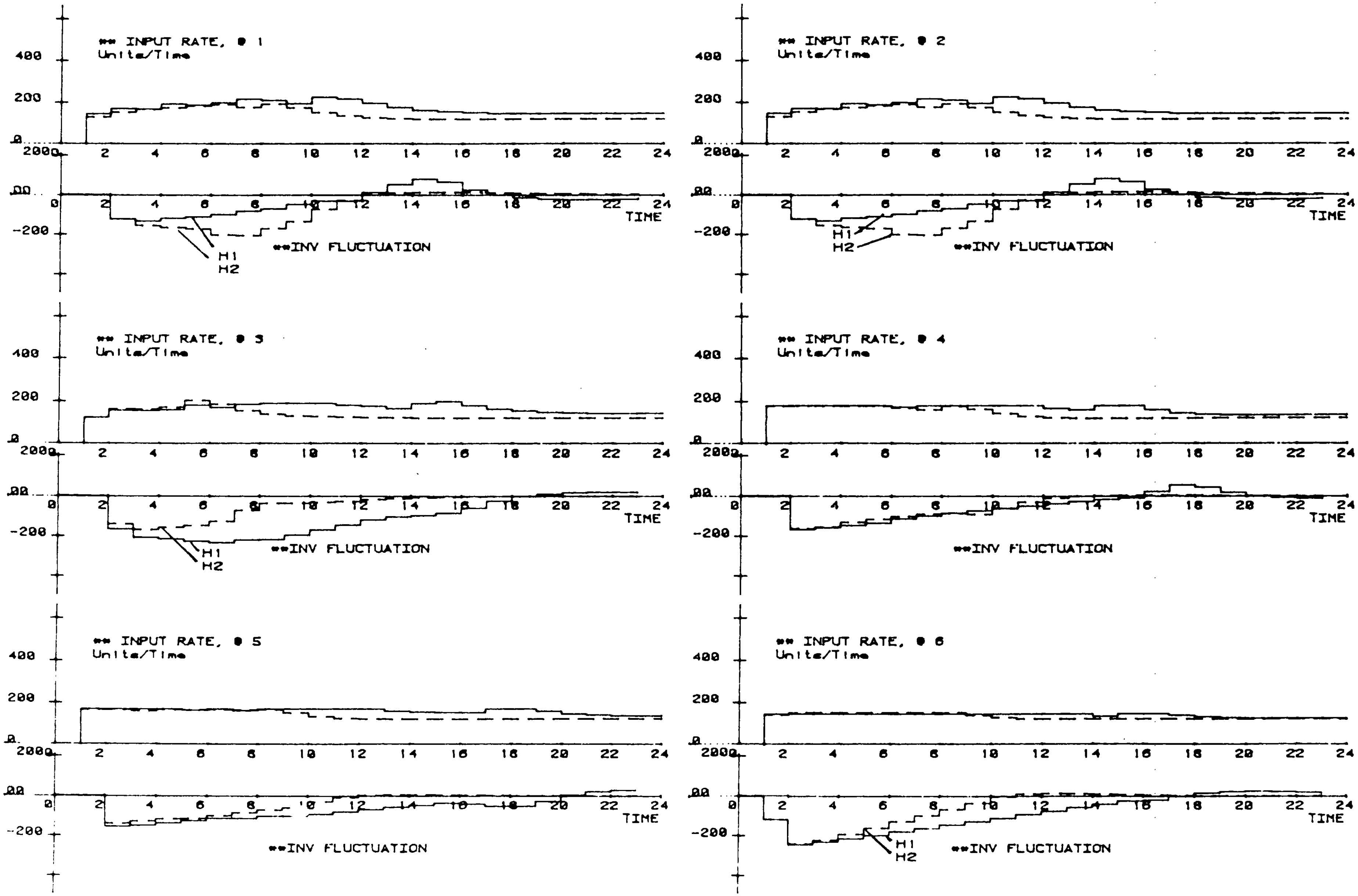


FIGURE 4.23 . RESULTS OF RUN H1 AND H2 FOR MODEL Y
INTRODUCTION OF REJECT IN RUN H1

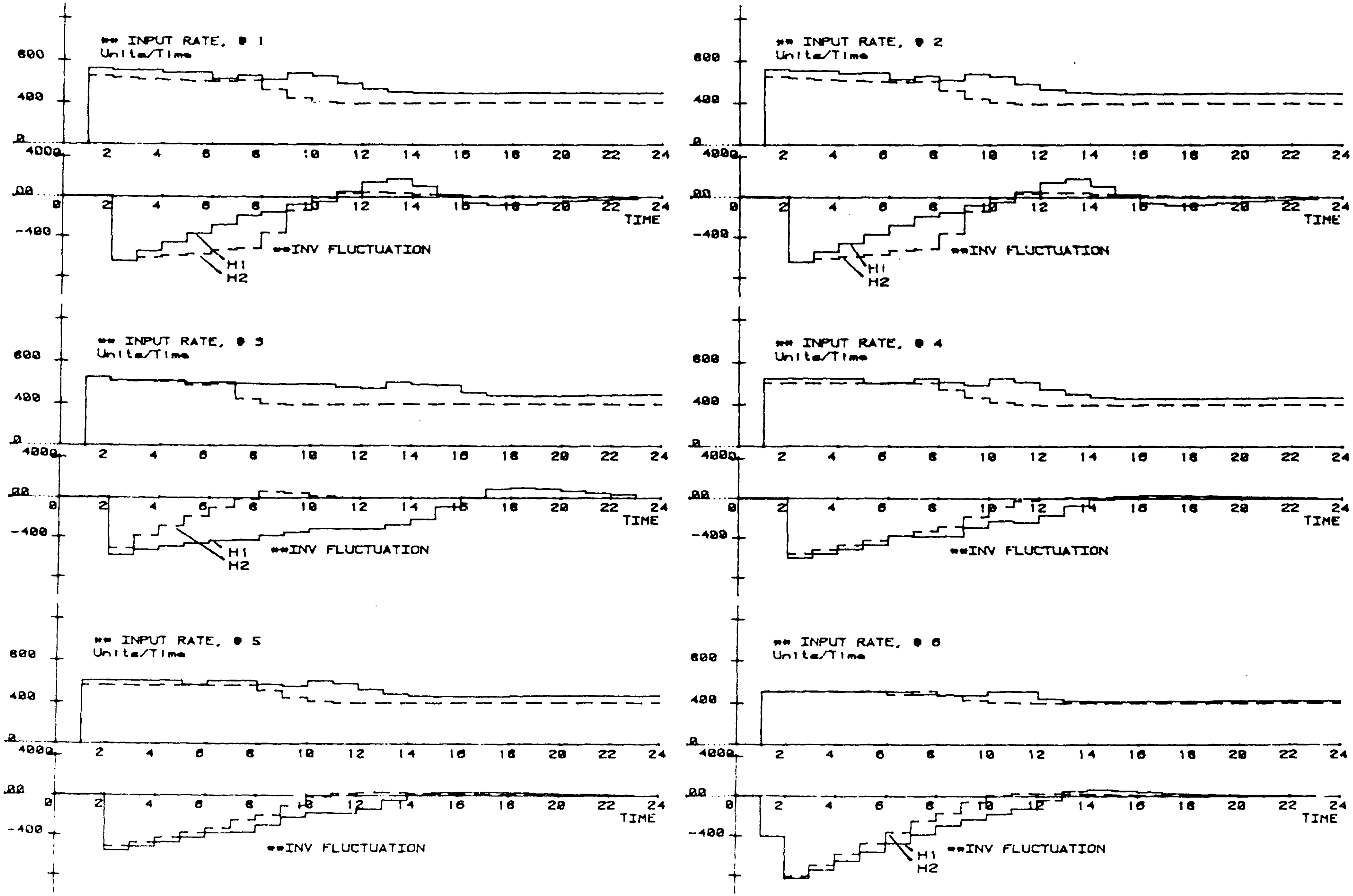


FIGURE 4.24 : RESULTS OF RUN H1 AND H2 FOR MODEL Z
INTRODUCTION OF REJECT IN RUN H1.

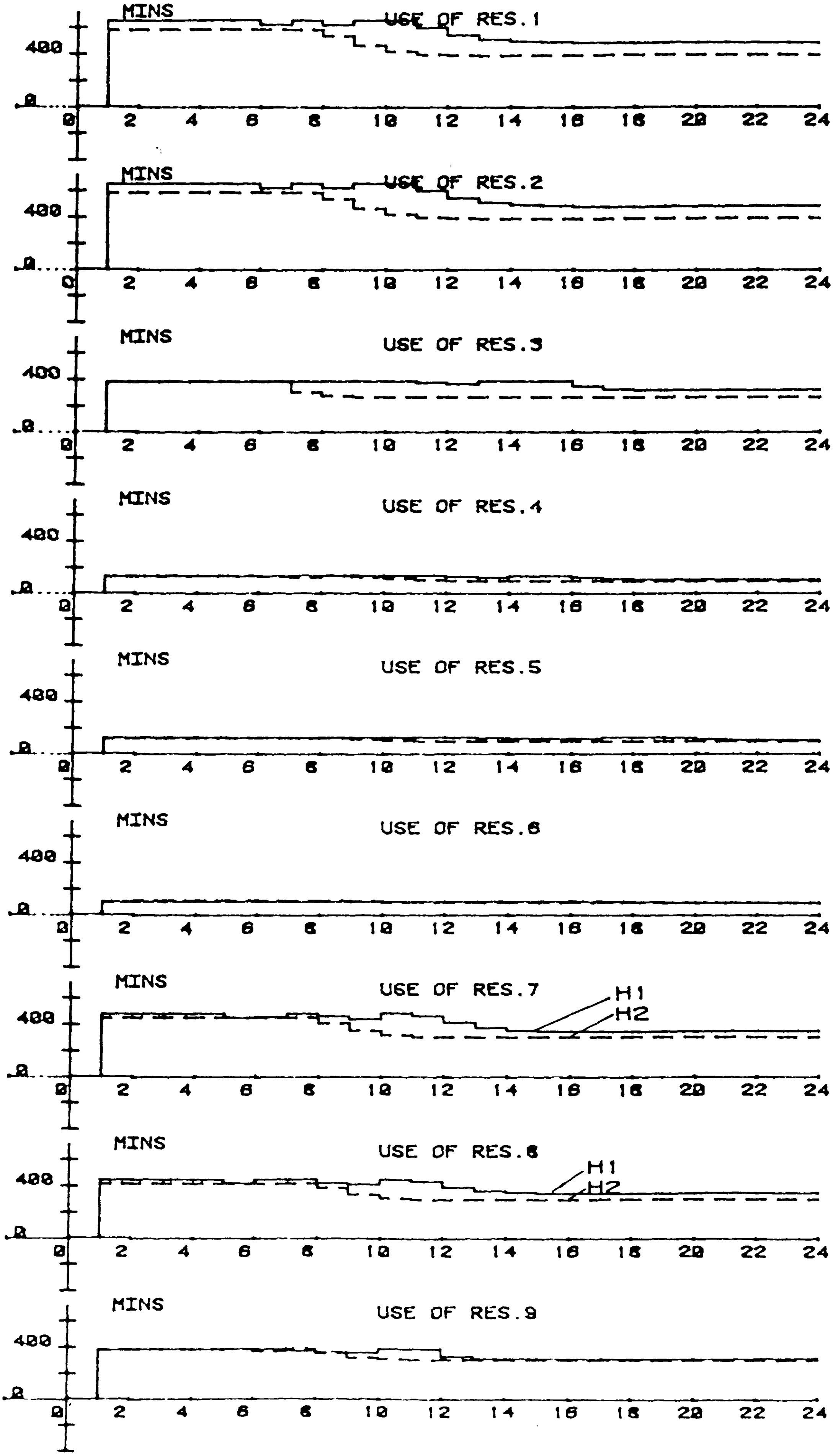


FIGURE 4.25 : AGGREGATE UTILISATION OF RESOURCES IN RUNS H1 AND H2

RESOURCE	MODEL Y STAGE	MODEL Z STAGE	OPERATION TIME (MIN)
R1	1	1	0.75
R2	2	2	0.75
R3	3	3	0.5
R4	4	0	0.75
R5	5	0	0.75
R6	6	0	0.75
R7	0	4	0.75
R8	0	5	0.75
R9	0	6	0.75

TABLE 4.1 : RESOURCE REQUIREMENTS FOR MODELS Y AND Z

		STAGES	1	2	3	4	5	6
MAX. PROD. RATES	RUN F1	MODEL Y	177	177	179	177	167	156
	RUN F1	MODEL Z	664	664	564	664	566	498
	RUN F2	MODEL Y	177	177	200	177	167	156
	RUN F2	MODEL Z	664	664	564	664	566	498
	RUN F3	MODEL Y	201	201	163	162	151	140
	RUN F3	MODEL Z	639	639	555	592	548	504
MIN. SAFE INV	RUN F1	MODEL Y	116	116	105	120	105	261
	RUN F1	MODEL Z	564	564	468	566	468	800
	RUN F2	MODEL Y	116	116	105	120	105	261
	RUN F2	MODEL Z	564	564	468	566	468	800
	RUN F3	MODEL Y	142	142	159	151	140	240
	RUN F3	MODEL Z	554	554	503	547	503	800

TABLE 4.2 : MAX. PRODUCTION RATES AND MINIMUM SAFE INVENTORIES FOR RUNS F1, F2 AND F3

RESOURCE	RUN F1	RUN F3
R1	10530	10531
R2	10530	10531
R3	6761	6761
R4	2430	2435
R5	2339	2343
R6	2249	2258
R7	8100	8101
R8	7800	7800
R9	7500	7500

MINS

TABLE 4.3 : UTILISATION OF RESOURCES ACCUMULATED OVER 24 TIME-PERIODS, RUNS F1 AND F3

	STAGES	1	2	3	4	5	6
RUN G1	MODEL Y	189	189	100	164	100	260
RUN G1	MODEL Z	646	646	350	558	350	850
RUN G2	MODEL Y	220	220	100	164	100	260
RUN G2	MODEL Z	626	626	100	558	100	1100
RUN G3	MODEL Y	205	205	167	164	140	240
RUN G3	MODEL Z	646	646	511	558	511	800

TABLE 4.4: MINIMUM SAFE INVENTORIES FOR RUNS G1, G2 AND G3

RESOURCE	RUN H1 REJECT	RUN H2 NO REJECT
R1	652	586
R2	652	586
R3	385	385
R4	135	133
R5	126	123
R6	107	113
R7	486	453
R8	452	418
R9	382	384

TABLE 4.5 : MAXIMUM CAPACITY REQUIREMENTS FOR RUNS H1 AND H2

	STAGES	1	2	3	4	5	6
RUN H1	MODEL Y	133	133	233	168	167	246
RUN H1	MODEL Z	646	646	583	602	556	821
RUN H2	MODEL Y	205	205	167	164	140	240
RUN H2	MODEL Z	646	646	511	558	511	800

<u>TABLE 4.6</u>: <u>MINIMUM SAFE INVENTORIES FOR RUNS H1 AND H2</u>

CHAPTER 5

Chapter 5 : Conclusions and Suggestions For Future Work.

5.1 Research Objectives.

The objectives of the research have been the investigation and development of mathematical control theory and their applications in the dynamic control of multi-stage production inventory systems. Such an exercise has been carried out from an initial close understanding of the production control problem leading to the initial design considerations of the necessary information infrastructure. This chapter discusses the main milestones attained during the course of the research.

5.2 Method of Approach.

The approach adopted has been to model the multi-stage production-inventory system with multivariable control theory. This technique has previously shown the following potential:

- The synthesis of production control policies from feedback information. This synthesis of decision rules is carried out in such a way that they are co-ordinated to each other, in the multivariable environment.

This control tool has been applied to one particular level in the hierarchical production control problem, where it is required to co-ordinate both the production of the various assemblies and their float levels. The model described in this thesis adopted the Brunovsky 's (1966,/67/) canonical forms following the work of Porter et al (1976,/3/), and implemented the design of controllers through the arbitrary pole assignment technique of closed-loop eigenvalues. The research identified certain particular properties of the control forms that were exploited beneficially into more practical control models that led to more practical solutions. This was in the possibility of identifying individual relevant controls for particular models of the systems. Such an identification, rendered computationally easy by the ordered structure of the formulation, led to the developments of numerous algorithms. These algorithms were specifically designed to obtain practical solutions for individual controls within their practical constraints. The practical nature of such constraints are:

- Limits in the capacity requirements.
- Limits in inter-stage buffers.

This is believed to be the major contributions of the research carried out.

5.3 Applications.

For the research to be of practical value, the model was then developed with an actual car manufacturing company as case study. The multi-stage production-inventory nature of the company was modelled into a linear discrete - time control problem. This was to achieve the advantages of hierarchical decomposition techniques in sub-dividing problems into different levels. The control problem was focused on the co-ordination of the various major assemblies as opposed to the more detailed level of actual scheduling of operations.

In order for the manufacturing system to achieve a certain controllabilty objective in response to a step disturbance, innumerable policies of resource allocation and inventory holding exist. In the case of a multi-stage production-inventory system as in car manufacture such disturbance may be triggered by a change in the demand rate of the final product (e.g. units of cars) and/or any of the assemblies (e.g. power units). An appropriate policy is therefore needed for the safe levels of stocks of both final product and sub-assemblies. Similarly, a correct dynamic allocation policy for the various resources as man-power and machine time is also necessary.

It has been demonstrated with the help of a cost function that local sub-optimum solutions do exist with the use of structured limits. These local sub-optimum solutions have then been considered as short-listed options for another selection exercise. Such an approach proved to be an efficient way to cut down the number of alternative policies that have to be considered in such control problem.

It is also noted that all the control policies are synthesised while taking into account the various prevailing constraints. The same model has been extended to deal with a multi-product environment. In such a case, limited resources have to be shared out to the production requirements of the various products. Therefore the problem of controlling the responses of the system is augmented with the need to share out the limited capacities as effectively as possible. Here again the flexibility of the control model developed is demonstrated by its ability to cope with this additional dimension of the control problem.

The control problem has been considered at the first level of hierarchical problem, i.e. how to match the major assemblies into the final product. Therefore the results obtained from the model are then used as the dynamic constraints for the problem lower in the hierarchy. This is because the production of one assembly itself involves a whole series of operations that may be arranged in a linear mode or batch mode. At the lower level, a new control situation exists that is separated from the major one, where local rules are possible. This approach therefore the approach provides decentralisation in the decision making process.

5.4 Suggestions for Future Work.

The research described in this dissertation has focused on the characteristics of car manufacturing. Numerous other manufacturing systems may be modelled into such multi-stage production-inventory structure. These are mainly in the high volume, low mix production as in electric appliance goods (white consumer goods such as washing machines, refrigerators, etc.), semiconductor devices, television sets, etc. It would be therefore very worthwhile to consider how far the control model developed during the course of the research, is still applicable, and how much of local adaptation is necessary so as to deal with any particular characteristics of the manufacturing concern.

Orientation of further study would include:

* The investigation of the actual duration of the unit time-period in the control simulation.
* The investigation of the extent to which the model can be applied to the different levels of production control.
* The investigation as whether the Gamma distribution is still applicable for the studies for contingency measures in other

manufacturing cases.

While it is believed that the flexible nature of the control model developed will allow a great ease of "local" adaptiveness, only definite research can acertain this.

Research may also be extended to models with an additional dimension, that of distribution, i.e. to consider multi-stage production - inventory - distribution systems.

5.5 Conclusion.

This thesis fills a gap in the modelling and control of multi-stage production - inventory systems, whereby the theoretical analyses of previous workers have been extended to provide a dynamic control simulation model for certain practical manufacturing systems. Its actual value, of course, rests on its actual implementation by industrial management in the search for a better understanding of the manufacturing system. Therefore, the next major exercise would be the more widespread communication of such control tools as developed in this research and by other previous workers to industrial management. During the course of the research, the author has experienced a favourable welcome by an increasingly enlightened and receptive manufacturing management, illustrating the fact that the initial communication barrier between academic and industrial practitioners is being slowly overcome.

The assistance of numerous manufacturing companies in the U.K. during the course of the research is gratefully acknowledged. The author takes this opportunity to wish subsequent workers in practical research equal success in obtaining a welcome from manufacturing industries.

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APPENDICES

APPENDIX 1

In this appendix the Prepelita algorithm (1971,/67/) is described in detail. This is the algorithm that transforms the system matrices into their canonical structures, $\bar{A}$ and $\bar{B}$. The derivation of the transformation matrix C is also given.

Wherever appropriate, the steps of the algorithm are also explained with the computational aspects of the exercise in mind. The algorithm has been programmed in Basic language for the TEKTRONIX desktop computer and consists of a suite of three programs, each of about 12K size of memory. The listings of the three programs are also included.

1.1 Prepelita Algorithm

1. Form a basis β for R^n from the first n linearly independent columns of the matrix

   $$(B, AB, \ldots\ldots, A^{n-1}B)$$

   in such a way that

   (i) the columns $b_i, \ldots\ldots b_m$ of the matrix B make up part of β

   and

   (ii) $A^j b_s \in \beta$ $(s = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n-1)$, if the vector is not a linear combination of $A^j b_k$ $(i = 0, 1, \ldots, j-1;\ k = 1, 2, \ldots, m)$, $(i = j;\ k = 1, 2, \ldots, s-1)$.

A.2

The subroutine used for the check for linear independence of a given set of vectors is based on the Gram Schmidt Orthogonalisation technique (Reference : Noble, 1969,/107/).

This gives the following basis for β:

$$
\begin{array}{llll}
b_1, & Ab_1, & \ldots & \ldots, A^{n_1-1} b_1 \\
b_2, & Ab_2, & & , A^{n_2-1} b_2 \\
\vdots & & & \\
b_m, & Ab_m, & \ldots & , A^{n_m-1} b_m,
\end{array}
$$

where $n_s \geqslant 1$ $(s = 1, 2, \ldots, m)$

Also, let the maximum n_s $(s = 1, 2, \ldots, m)$ equals s_o

Note that multiplication has to be done with the vectors with the same power of matrix A first, before moving to the next power of A.

The main reason is that if one proceeds along the basis β along its rows, no test can be performed because some of the previous vectors have not been calculated yet.

The linearly independent vectors of basis β are put in Matrix V1 in the program and is nxn in size.

Another matrix F2 is used as a flag to indicate which vector $A^i b_k$ is in the basis. This is in order to minimise the amount of computer memory needed, which would have been very big indeed considering the fact that the matrix V1 is 3-dimensional and very sparse.

N1(s) gives the n_s index for s = 1, ..., m.

2. Write the vectors $A^{n_s} b_s$ in terms of the basis β in the form

$$A^{n_1} b_1 = \sum_{j=0}^{n_1-1} \sum_{\ell=1}^{m} \gamma^1_{\ell,j} A^j b_\ell$$

and

$$A^{n_s} b_s = \sum_{j=0}^{n_s-1} \sum_{\ell=1}^{m} \gamma^s_{\ell,j} A^j b_\ell + \sum_{\ell=1}^{s-1} \gamma^s_{\ell,n_s} A^{n_s} b_\ell$$

$$(s = 2, 3, \ldots, m)$$

where

$$\gamma^s_{\ell,j} = 0$$

if

$$A^i b_\ell \not\in \beta$$

The following instructions determine the matrices C and $\bar{A}$.

Vectors $A^{n_s} b_s$ are in matrix B9 in the program.

The inverse function for matrices which is inbuilt in the computer system is used to solve for the values of γ s.

γ's are in Y matrix, and T and Y1 are temporary matrices. Instead of selecting which vector may be scaled to represent $A^{n_s} b_s$ (eg. those n where $n < n_s$), the whole basis is used so as to reduce the programming load. Moreover it avoids the problem of ending up with non-square matrices with redundant equations.

3. Calculate the columns c_{k_s} of the matrix C where

$$k_s = \sum_{i=1}^{s} n_j \qquad (s = 1, 2, \ldots., m)$$

and

$$k_o = 0$$

using the formula

$$c_{k_1} = b_1$$

and

$$c_{k_s} = b_s - \sum_{\ell=1}^{s-1} \gamma^s_{\ell,n_s} b_\ell \qquad (s = 2, 3, \ldots, m)$$

Vectors c_{k_s} are in matrix C7 in program, while index k_s is K4(s).

4. Form the following basis β' for R^n

$$c_{k_1},\ Ac_{k_1},\ \ldots.,\ A^{n_1-1}\ c_{k_1}$$

$$c_{k_2},\ Ac_{k_2},\ \ldots.,\ A^{n_2-1}\ c_{k_2}$$

....

$$c_{k_m},\ Ac_{k_m},\ \ldots.,\ A^{n_m-1}\ c_{k_m}$$

Indices n_i, $(i = 1, \ldots, m)$ are the same as those for basis β new basis β' is V2 in program.

5. Write the vectors

$$A^{n_s} c_{k_s} \quad (s = 1, 2, \ldots, m)$$

in terms of the basis β' in the form

$$A^{n_s} c_{k_s} = \sum_{j=1}^{n_s} A^{n_s - j} \sum_{\ell=1}^{m} \alpha_{\ell, k_s - j+1} c_{k_\ell}$$

$$(s = 1, 2, \ldots, m)$$

where

$$\alpha_{\ell, k_s - j+1} = 0$$

if

$$A^{n_s - j} c_{k_\ell} \notin \beta'$$

Solution of α's, follows the same method as that of γ in the program.

Values of α's are put in X5.

6. Calculate remaining columns of

$c_{k_{(s-1)}+i}$ of the matrix C using

the formula

$$c_{k_{(s-1)}+i} = A^{n_{s-i}} c_{k_s} - \sum_{j=1}^{n_s-i} A^{n_s-i-j} \sum_{\ell=1}^{m} \alpha_{\ell,k_s-j+1} c_{k_\ell}$$

$$s = (1, 2, \ldots, m)$$

$$i = (1, 2, \ldots, n_s-1)$$

Since index j is being subtracted from n_s and it is required to start with the least power of A, then the do loop starts with index J from n_s-1 to 1 step -1. U1 and U2 are used to identify which vectors of the basis are being used for the calculation of the columns.

7. Construction of matrix $\bar{A}$. (referred to A∅ in program).

Matrix $\bar{A}$, is itself built up of its companion sub-matrices containing the elements $\alpha_{i,j}$.

$$\bar{A} = \begin{matrix} \bar{A}_{1,1}, & \bar{A}_{1,2}, & \ldots, & \bar{A}_{1,m} \\ \bar{A}_{2,1}, & \bar{A}_{2,2}, & \ldots, & \bar{A}_{2,m} \\ \ldots & \ldots & \ldots & \ldots \\ \bar{A}_{m,1}, & \bar{A}_{m,2}, & \ldots, & \bar{A}_{m,m} \end{matrix}$$

and

$$\bar{A}_{ii} \begin{bmatrix} 0 & , & 1 & , & 0 & , \ldots & 0 & 0 \\ 0 & , & 0 & , & 1 & & 0 & 0 \\ \vdots & & & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ 0 & , & 0 & , & 0 & & 0 & 1 \\ \alpha_{i,k_{(i-1)}+1}, & & \alpha_{i,k_{(i-1)}+2}, & & \alpha_{i,k_{(i-1)}+3}, & \ldots; & \alpha_{i,k_i-1}, & \alpha_{i}k_{i} \end{bmatrix}$$

$(i \neq j;\ i,j=1,2, \ldots m)$

and

$$\bar{A}_{i,j} = \begin{bmatrix} 0 & 0 & & & \\ 0 & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & 0 & 0 & 0 & 0 \\ \alpha_{i,k_{(j-1)}+1}, & \alpha_{i,k_{(j-1)}+2}, & \alpha_{i,k_{(j-1)}+3}, & \ldots, \alpha_{i,k_j} & , \alpha_{i}k_j \end{bmatrix}$$

$(i \neq j,\ i,j=1,2, \ldots m)$

8. a. Determination of matrices G_o and G_1

$$
G_o \quad \begin{bmatrix}
1 & -\gamma^{2}_{1,n_2} & -\gamma^{3}_{1,n_3}, & \ldots, & -\gamma^{m}_{1,n_m} \\
0 & 1 & -\gamma^{3}_{2,n_3}, & \ldots, & -\gamma^{m}_{2,n_m} \\
0 & 0 & 1\ , & \ldots, & -\gamma^{m}_{3,n_m} \\
. & . & . & \ldots, & . \\
0\ , & 0\ , & 0\ , & , & -\gamma^{m}_{m-1,n_m} \\
0\ , & 0\ , & 0\ , & , & 1
\end{bmatrix}
$$

b. Determine matrix G_1

G_1 = 1NV(GØ).

9. Determination of matrix B and subsequently of $\bar{B}$.

Matrices B and $\bar{B}$ are referred to as B7 and BØ respectively in the program.

$$\hat{B} = \text{diag}\ (\hat{B}_1, \hat{B}_2, \ldots \hat{B}_m)$$

where $(n_i \times 1)$ matrix,

$$\hat{B}_i = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \qquad i = 1, \ldots, m$$

and $\bar{B}$ is given as

$$\bar{B} = \hat{B}G_1$$

APPENDIX 1.2 : PROGRAM FOR PREPELITA ALGORITHM

```
1000 INIT
1005 REM
1010 REM         "@PREP/PGM1"    FILE 1
1015 REM         : PREPELITA ALGORITHM PROGRAM 1.1
1020 REM         : DATE 26-OCT-81
1025 REM
1030 C$="00"
1035 O1=32
1040 P=6
1045 N=P*3
1050 REM SIZE OF MATRIX A(N,N)::::: MATRIX B(N,P):::::::***
1055 REM V9(N)     TEMPORARY VECTOR  *********************************
1060 REM V1(N,P1) BASIS CREATED P1 GIVEN BY LIN IND VECTORS**********
1065 DIM A(N,N),B(N,P),V9(N),V1(N,N+P),Z1(N,N+P),I1(N),Y1(N)
1070 DIM T(N,N+1)
1075 DIM Z9(N,N+P)
1080 A=0
1085 V1=0
1090 T=0
1095 B=0
1100 REM         ESTABLISHING SYSTEM MATRICES
1105 REM         FOR CAR MANUFACTURING SYSTEM USED
1110 REM         IN CHAPTERS 3 AND 4 OF THESIS.
1115 FOR I=P+1 TO P*3
1120 A(I,I)=1
1125 A(I,I-P)=1
1130 NEXT I
1135 FOR I=1 TO P
1140 B(I,I)=1
1145 NEXT I
1150 B(7,3)=-1
1155 B(8,3)=-1
1160 B(9,4)=-1
1165 B(10,5)=-1
1170 B(11,6)=-1
1175 S9=0
1180 FOR I=1 TO N
1185 V1(I,1)=B(I,1)
1190 S9=S9+B(I,1)*B(I,1)
1195 T(I,N+1)=0
1200 NEXT I
1205 Z1(1,1)=S9
1210 REM S=COUNT OF V1*******************************************
1215 S1=1
1220 REM PUT NEXT COLUMN OF MATRIX B INTO TEMP. VECTOR V9***********
1225 IF P=1 THEN 1305
1230 FOR J9=2 TO P
1235 FOR I=1 TO N
1240 V9(I)=B(I,J9)
1245 NEXT I
1250 REM GO FOR LIC SUBROUTINE************************************
1255 GOSUB 7000
1260 PRINT @O1:"F1=";F1
1265 REM IF LIN. IND. THEN PUT INTO BASIS*************************
1270 IF F1=0 THEN 1300
1275 S1=S1+1
1280 FOR I=1 TO N
1285 V1(I,S1)=V9(I)
```

```
1290 NEXT I
1295 REM
1300 NEXT J9
1305 M=S1
1310 PRINT @01:"M=";M
1315 DELETE 1000,1310
1320 REM PUT FLAG F2 FOR "IN BASIS"**********************************
1325 DIM F2(M,N+1),B9(N,M),N1(M),K1(M),K4(M)
1330 FOR I=1 TO M
1335 F2(I,1)=1
1340 N1(I)=0
1345 NEXT I
1350 PRINT @01:"CALC A (1) b"
1355 REM PUTTING         A b(1)****************************************
1360 FOR J=2 TO N
1365 FOR I=1 TO M
1370 F2(I,J)=0
1375 NEXT I
1380 NEXT J
1385 W=0
1390 P1=1
1395 FOR S=1 TO M
1400 FOR J=1 TO N
1405 S8=0
1410 FOR I=1 TO N
1415 S8=S8+A(J,I)*V1(I,S)
1420 NEXT I
1425 V9(J)=S8
1430 NEXT J
1435 REM CALL LIC SUBROUTINE******************************************
1440 GOSUB 7000
1445 IF F1=0 THEN 1495
1450 S1=S1+1
1455 FOR I=1 TO N
1460 V1(I,S1)=V9(I)
1465 NEXT I
1470 F2(S,P1+1)=1
1475 F2(S,P1+2)=1
1480 IF S1=N THEN 1885
1485 N1(S)=P1+1
1490 GO TO 1540
1495 F2(S,P1+1)=0
1500 REM PUTTING FOR B9=MATRIX CONTAINING A^(ns).bs*****************
1505 FOR I=1 TO N
1510 B9(I,S)=V9(I)
1515 NEXT I
1520 N1(S)=P1
1525 W=W+1
1530 IF W=)M THEN 1870
1535 GO TO 1575
1540 FOR J=1 TO N
1545 S9=0
1550 FOR I=1 TO N
1555 S9=S9+A(J,I)*V9(I)
1560 NEXT I
1565 B9(J,S)=S9
1570 NEXT J
1575 NEXT S
```

```
1580 Q=M
1585 S5=0
1590 PRINT @01:"CALC A(p) b"
1595 REM CALCULATING FOR A(p)B*****************************************
1600 P1=P1+1
1605 REM PRIOR CHECK
1610 PRINT @01:"PRC"
1615 FOR S=1 TO M
1620 REM IF F2(S,P1+1)()0 THEN 2185
1625 IF F2(S,P1+1)=0 THEN 1860
1630 REM Q=Q+1
1635 REM FOR J=1 TO N
1640 REM S9=0
1645 REM FOR I=1 TO N
1650 REM S9=S9+A(J,I)*V1(I,Q)
1655 REM NEXT I
1660 REM B9(J,S)=S9
1665 REM NEXT J
1670 REM GO TO 2520
1675 Q=Q+1
1680 FOR J=1 TO N
1685 S8=0
1690 FOR I=1 TO N
1695 S8=S8+A(J,I)*V1(I,Q)
1700 NEXT I
1705 V9(J)=S8
1710 NEXT J
1715 REM CALL LIC SUBROUTINE ***************************************
1720 GOSUB 7000
1725 IF F1=0 THEN 1780
1730 S1=1+S1
1735 FOR I=1 TO N
1740 V1(I,S1)=V9(I)
1745 NEXT I
1750 F2(S,P1+1)=1
1755 F2(S,P1+2)=1
1760 REM CHECK FOR FULL RANK**************************************
1765 REM IF S1=N THEN 2542
1770 N1(S)=P1+1
1775 GO TO 1860
1780 F2(S,P1+1)=0
1785 S5=S5+1
1790 REM INPUT INTO LAST VECTOR  A(n s)b s
1795 FOR I=1 TO N
1800 B9(I,S)=V9(I)
1805 NEXT I
1810 N1(S)=P1
1815 IF S5=M THEN 1885
1820 GO TO 1860
1825 FOR J=1 TO N
1830 S9=0
1835 FOR J=1 TO N
1840 S9=S9+A(J,I)*V9(I)
1845 NEXT I
1850 B9(J,S)=S9
1855 NEXT J
1860 NEXT S
1865 GO TO 1595
```

```
1870 REM
1875 C$="11"
1880 REM********************************
1885 DELETE 1000,1880
1890 DELETE 7000,8000
1895 FIND 2
1900 CALL "BAPPEN",2500
2500 REM
4000 REM
7000 REM      USING GRAM SCHMIDT METHOD
7005 REM      FOR LINEAR INDEPENDENCE CHECK FOR SET OF VECTORS.
7010 REM
7015 REM
7020 REM PUT PREVIOUS LIN IND VECTORS INTO Z1
7025 Z=S1+1
7030 PRINT @01:Z
7035 FOR J=1 TO S1
7040 FOR I=1 TO N
7045 Z1(I,J)=V1(I,J)
7050 NEXT I
7055 NEXT J
7060 REM FOR NEW TEMPORARY VECTOR
7065 FOR I=1 TO N
7070 V1(I,Z)=V9(I)
7075 NEXT I
7080 FOR J=1 TO Z
7085 FOR I=1 TO N
7090 Z9(I,J)=0
7095 NEXT I
7100 NEXT J
7105 FOR K=1 TO Z
7110 IF K=1 THEN 7190
7115 FOR I8=1 TO K-1
7120 S9=0
7125 FOR I=1 TO N
7130 S9=S9+Z1(I,I8)*V1(I,K)
7135 NEXT I
7140 Z9(I8,K)=S9
7145 NEXT I8
7150 FOR I=1 TO N
7155 S9=0
7160 FOR I8=1 TO K-1
7165 S9=S9+Z9(I8,K)*Z1(I,I8)
7170 NEXT I8
7175 Z1(I,K)=V1(I,K)-S9
7180 NEXT I
7185 IF K=Z THEN 7235
7190 S9=0
7195 FOR I=1 TO N
7200 S9=S9+Z1(I,K)*Z1(I,K)
7205 NEXT I
7210 Z9(K,K)=SQR(S9)
7215 FOR I=1 TO N
7220 Z1(I,K)=Z1(I,K)/Z9(K,K)
7225 NEXT I
7230 NEXT K
7235 REM CHECK FOR LIN IND OF LAST VECTOR
7240 D2=1.0E-8
```

```
7245 FOR I=1 TO N
7250 IF ABS(Z1(I,Z))>D2 THEN 7270
7255 NEXT I
7260 F1=0
7265 GO TO 7275
7270 F1=1
7275 PRINT @01:"F1=";F1
7280 RETURN
```

```
2500 REM
2505 REM
2510 REM      PROGRAM "@PREP/PGM2" FILE 2
2515 REM      PREPELITA ALGORITHM
2520 REM      DATE : 26-OCT-81
2525 REM
2530 REM
2535 PRINT @01:"N1",N1
2540 PRINT @01:"CALC OF K1,& OF K4"
2545 K1(1)=N1(1)+1
2550 K4(1)=N1(1)
2555 IF M<2 THEN 2580
2560 FOR X=2 TO M
2565 K1(X)=N1(X)+K1(X-1)+1
2570 K4(X)=N1(X)+K4(X-1)
2575 NEXT X
2580 PRINT @01:"K1";K1
2585 PRINT @01:"K4";K4
2590 DELETE Z1,Z9
2595 DIM Y(M,K1(M)),X5(M,K4(M))
2600 F2=0
2605 FOR S=1 TO M
2610 FOR J2=1 TO N1(S)
2615 F2(S,J2)=1
2620 NEXT J2
2625 NEXT S
2630 FOR I=1 TO M
2635 FOR J=1 TO K1(M)
2640 Y(I,J)=0
2645 NEXT J
2650 FOR J=1 TO K4(M)
2655 X5(I,J)=0
2660 NEXT J
2665 NEXT I
2670 REM SOLVING FOR A n1  b1****************************************
2675 REM PUTTING B1 VECTORS OF V1 INTO NEW MATRIX T(SUB BASIS OF BETA)
2680 FOR J=1 TO N
2685 FOR I=1 TO N
2690 T(I,J)=V1(I,J)
2695 NEXT I
2700 NEXT J
2705 FOR S=1 TO M
2710 FOR J2=1 TO N1(S)
2715 FOR L=1 TO M
2720 IF F2(L,J2)=0 THEN 2755
2725 IF S<>1 THEN 2745
2730 X5(L,J2)=1
2735 Y(L,J2)=1
2740 GO TO 2755
2745 Y(L,J2+K1(S-1))=1
2750 X5(L,J2+K4(S-1))=1
2755 NEXT L
2760 NEXT J2
2765 NEXT S
2770 IF M=1 THEN 2805
2775 FOR S=2 TO M
2780 FOR L=1 TO S-1
2785 IF F2(S-L,N1(S)+1)=0 THEN 2795
```

```
2790 Y(S-L,K1(S))=1
2795 NEXT L
2800 NEXT S
2805 REM SOLVING FOR Y PROPER****************************************
2810 REM WE HAVE MATRIX T****************************************
2815 REM RESULT B9**********************************************
2820 REM SOLUTION Y1      **************************************
2825 REM AUG MENT MATRIX T BY B9  ******************************
2830 REM THEN CALL INVERSE FUNCTION  ****************************
2835 PRINT @01:"SOL LIN EQ"
2840 REM PRINT @01:"B9",B9
2845 FOR I=1 TO N
2850 T(I,N+1)=B9(I,1)
2855 NEXT I
2860 PRINT @01:"T"
2865 REM PRINT @01:T
2870 T=INV(T)
2875 FOR I=1 TO N
2880 Y1(I)=T(I,N+1)
2885 NEXT I
2890 PRINT @01:"Y1";Y1
2895 PRINT @01:"DONE"
2900 REM GOING THRU THE POWER BLOCK
2905 I=0
2910 FOR J2=1 TO K1(1)
2915 FOR L=1 TO M
2920 IF Y(L,J2)=0 THEN 2940
2925 I=I+1
2930 Y(L,J2)=Y1(I)
2935 IF I=N THEN 2950
2940 NEXT L
2945 NEXT J2
2950 PRINT @01:"Y";Y
2955 IF M<2 THEN 3160
2960 PRINT @01:"SOL FOR Ans bs"
2965 REM SOLVING FOR Ans bs*****************************************
2970 FOR S=2 TO M
2975 FOR L=1 TO N
2980 FOR I=1 TO N
2985 T(I,L)=V1(I,L)
2990 NEXT I
2995 NEXT L
3000 PRINT @01:"SOL FOR Y"
3005 PRINT @01:"T"
3010 REM PRINT @01:T
3015 REM PRINT @01:"B9",B9
3020 REM SOLVE FOR Y 's
3025 Z=N
3030 PRINT @01:"Z=";Z
3035 FOR I=1 TO N
3040 T(I,Z+1)=B9(I.S)
3045 NEXT I
3050 FOR J=1 TO Z+1
3055 T=INV(T)
3060 FOR I=1 TO N
3065 Y1(I)=T(I,Z+1)
3070 IF ABS(Y1(I))>1.0E-5 THEN 3080
3075 Y1(I)=0
```

```
3080 NEXT I
3085 PRINT @01:"Y1";Y1
3090 PRINT @01:"DONE"
3095 I=0
3100 FOR J2=1 TO N1(S)+1
3105 FOR L=1 TO M
3110 IF Y(L,J2+K1(S-1))=0 THEN 3130
3115 I=I+1
3120 Y(L,J2+K1(S-1))=Y1(I)
3125 IF I=N THEN 3145
3130 NEXT L
3135 NEXT J2
3140 GO TO 3150
3145 REM
3150 PRINT @01:"Y";Y
3155 NEXT S
3160 DELETE 1000,3160
3165 FIND 3
3170 CALL "BAPPEN",4000
```

```
4000 REM
4005 REM
4010 REM         PROGRAM "@PREP/PGM3"   FILE 3
4015 REM         DATE 26-OCT-81
4020 REM
4025 DIM C7(N,K4(M))
4030 REM Ck1 =b1
4035 FOR I=1 TO N
4040 C7(I,K4(1))=V1(I,1)
4045 NEXT I
4050 REM FOR Cks*********************** TAPE 3-2    19-MAY-80   **
4055 DIM V7(N)
4060 IF M=1 THEN 4140
4065 FOR S=2 TO M
4070 FOR I=1 TO N
4075 FOR I9=1 TO N
4080 V9(I9)=0
4085 NEXT I9
4090 FOR I=1 TO N
4095 FOR L=1 TO S-1
4100 V7(I)=V1(I,L)*Y(L,K1(S))
4105 V9(I)=V9(I)+V7(I)
4110 NEXT L
4115 NEXT I
4120 FOR I=1 TO N
4125 C7(I,K4(S))=V1(I,S)-V9(I)
4130 NEXT I
4135 NEXT S
4140 REM REM
4145 REM
4150 REM***********************
4155 REM TO FORM BASIS B'********************************************
4160 REM PUT Ck i INTO BASIS ASSUMING LIN IND***********************
4165 REM CALL NEW BASIS V2(   )    ***********************************
4170 DIM V2(N(K),N(K))
4175 FOR J=1 TO M
4180 FOR I=1 TO N
4185 V2(I,J)=C7(I,K4(J))
4190 NEXT I
4195 NEXT J
4200 REM************************************************************
4205 K=2
4210 Q=M
4215 R=0
4220 S5=0
4225 FOR L=1 TO M
4230 IF F2(L,K)<>0 THEN 4295
4235 IF F2(L,K-1)=0 THEN 4345
4240 R=R+1
4245 FOR J=1 TO N
4250 S9=0
4255 FOR I=1 TO N
4260 S9=S9+A(J,I)*V2(I,R)
4265 NEXT I
4270 B9(J,L)=S9
4275 NEXT J
4280 S5=S5+1
4285 IF S5=M THEN 4395
```

```
4290 GO TO 4345
4295 Q=Q+1
4300 R=R+1
4305 FOR J=1 TO N
4310 S9=0
4315 FOR I=1 TO N
4320 S9=S9+A(J,I)*V2(I,R)
4325 NEXT I
4330 V2(J,Q)=S9
4335 NEXT J
4340 IF Q=N THEN 4360
4345 NEXT L
4350 K=K+1
4355 GO TO 4225
4360 FOR J=1 TO N
4365 S9=0
4370 FOR I=1 TO N
4375 S9=S9+A(J,I)*V2(I,Q)
4380 NEXT I
4385 B9(J,L)=S9
4390 NEXT J
4395 REM FOR A^(ns)    c (Ks)***********************************
4400 FOR S=1 TO M
4405 FOR J=1 TO N
4410 FOR I=1 TO N
4415 T(I,J)=V2(I,J)
4420 NEXT I
4425 NEXT J
4430 PRINT @01:"SOL FOR X5"
4435 REM PRINT @01:"B9",B9
4440 REM SOLVING USING INV FUNCTION***********************************
4445 Z=N
4450 PRINT @01:"Z=",Z
4455 FOR I=1 TO N
4460 T(I,Z+1)=B9(I,S)
4465 NEXT I
4470 PRINT @01:"T"
4475 PRINT @01:T
4480 FOR I=1 TO N
4485 T(I,Z+1)=B9(I,S)
4490 NEXT I
4495 T=INV(T)
4500 FOR I=1 TO N
4505 Y1(I)=T(I,Z+1)
4510 IF ABS(Y1(I)))1.0E-6 THEN 4520
4515 Y1(I)=0
4520 NEXT I
4525 PRINT "Y1";Y1
4530 REM PUTTING SOLUTIONS BACK INTO X5/**********************
4535 I=0
4540 FOR J=1 TO N1(S)
4545 FOR L=1 TO M
4550 IF S()1 THEN 4575
4555 IF X5(L,J)=0 THEN 4595
4560 I=I+1
4565 X5(L,J)=Y1(I)
4570 GO TO 4590
4575 IF X5(L,K4(S-1)+J)=0 THEN 4595
```

```
4290 GO TO 4345
4295 Q=Q+1
4300 R=R+1
4305 FOR J=1 TO N
4310 S9=0
4315 FOR I=1 TO N
4320 S9=S9+A(J,I)*V2(I,R)
4325 NEXT I
4330 V2(J,Q)=S9
4335 NEXT J
4340 IF Q=N THEN 4360
4345 NEXT L
4350 K=K+1
4355 GO TO 4225
4360 FOR J=1 TO N
4365 S9=0
4370 FOR I=1 TO N
4375 S9=S9+A(J,I)*V2(I,Q)
4380 NEXT I
4385 B9(J,L)=S9
4390 NEXT J
4395 REM FOR A^(ns)   c (ks)************************************
4400 FOR S=1 TO M
4405 FOR J=1 TO N
4410 FOR I=1 TO N
4415 T(I,J)=V2(I,J)
4420 NEXT I
4425 NEXT J
4430 PRINT @01:"SOL FOR X5"
4435 REM PRINT @01:"B9",B9
4440 REM SOLVING USING INV FUNCTION*************************************
4445 Z=N
4450 PRINT @01:"Z=",Z
4455 FOR I=1 TO N
4460 T(I,Z+1)=B9(I,S)
4465 NEXT I
4470 PRINT @01:"T"
4475 PRINT @01:T
4480 FOR I=1 TO N
4485 T(I,Z+1)=B9(I,S)
4490 NEXT I
4495 T=INV(T)
4500 FOR I=1 TO N
4505 Y1(I)=T(I,Z+1)
4510 IF ABS(Y1(I))>1.0E-6 THEN 4520
4515 Y1(I)=0
4520 NEXT I
4525 PRINT "Y1";Y1
4530 REM PUTTING SOLUTIONS BACK INTO X5/**********************
4535 I=0
4540 FOR J=1 TO N1(S)
4545 FOR L=1 TO M
4550 IF S<>1 THEN 4575
4555 IF X5(L,J)=0 THEN 4595
4560 I=I+1
4565 X5(L,J)=Y1(I)
4570 GO TO 4590
4575 IF X5(L,K4(S-1)+J)=0 THEN 4595
```

```
4580 I=I+1
4585 X5(L,K4(S-1)+J)=Y1(I)
4590 IF I=N THEN 4605
4595 NEXT L
4600 NEXT J
4605 PRINT @01:"X5";X5
4610 NEXT S
4615 PRINT @01:"BUILDING OTHER COLUMNS OF C*******"
4620 FOR S=1 TO M
4625 IF N1(S)-1<=0 THEN 4800
4630 FOR I=1 TO N1(S)-1
4635 FOR I9=1 TO N
4640 V9(I9)=0
4645 NEXT I9
4650 V=0
4655 DELETE U1,U2
4660 DIM U1(N),U2(N)
4665 FOR J=N1(S)-I TO 1 STEP -1
4670 FOR L=1 TO M
4675 IF F2(L,N1(S)-I-J+1)=0 THEN 4700
4680 V=V+1
4685 U1(V)=L
4690 REM U2(V)=J
4695 U2(V)=J
4700 NEXT L
4705 NEXT J
4710 U=V
4715 IF S=1 THEN 4740
4720 FOR L=1 TO S-1
4725 IF F2(S-L,N1(S)-I+1)=0 THEN 4735
4730 U=U+1
4735 NEXT L
4740 FOR I9=1 TO N
4745 FOR L1=1 TO V
4750 V9(I9)=V9(I9)+V2(I9,L1)*X5(U1(L1),K4(S)-U2(L1)+1)
4755 NEXT L1
4760 NEXT I9
4765 FOR I9=1 TO N
4770 IF S>1 THEN 4785
4775 C7(I9,I)=V2(I9,V+S)-V9(I9)
4780 GO TO 4790
4785 C7(I9,K4(S-1)+I)=V2(I9,U+1)-V9(I9)
4790 NEXT I9
4795 NEXT I
4800 NEXT S
4805 PRINT @01:"C7"
4810 PRINT @01:C7
4815 DELETE T
4820 DIM C9(N,N)
4825 C9=INV(C7)
4830 STOP
4835 FOR I=1 TO N
4840 FOR J=1 TO N
4845 IF ABS(C7(I,J))>1.0E-4 THEN 4855
4850 C7(I,J)=0
4855 NEXT J
4860 NEXT I
4865 FOR I=1 TO N
```

```
4870 FOR J=1 TO N
4875 IF ABS(C9(I,J))>1.0E-4 THEN 4885
4880 C9(I,J)=0
4885 NEXT J
4890 NEXT I
4895 STOP
4900 DELETE V1,V2
4905 REM   FINAL DETERMINATION OF MATRICES A1, B0 FROM C7 AND C9
4910 DIM A1(N,N),A2(N,N)
4915 A2=A MPY C7
4920 A1=C9 MPY A2
4925 DIM B0(N,M)
4930 B0=C9 MPY B
4935 DIM E(N,P+1),Q8(N,P+1)
4940 E=0
4945 FOR I=1 TO P-1
4950 E(P+I,I)=-1
4955 NEXT I
4960 E(2*P,P+1)=-1
4965 Q8=C9 MPY E
4970 STOP
```

Appendix 2: Theory Of State Variable Feedback Control.

A2.1.1 Introduction.

The following discussion is carried out in the continuous time for compactness and also so as to be able to corelate it with the previous work in linear multivariable control theory. Some part of it is a transcription of Wonham (1978,/78/), and is included to demonstrate the current state of the theoretical development. It is pointed out that the various properties have also been discovered independently by the present author in the discrete time formulation and which have led to the further development described in Appendix 3. It is the later development that has been applied into the practical problem of production control. Moreover, it is strongly believed that definite implications are to be expected in other fields of applications of linear multivariable control theory.

The continuous - time version of the original free uncontrollable system is given as :

$$\dot{x}(t) = A\ x(t)$$

In order to control the first order system in some desirable way as achieving stability, the system is modified into:

$$\dot{x}(t) = A\ x(t) + B\ u(t)$$

and introducing linear state vector feedback,

$$u(t) = F\ x(t)$$

$$x(t) = (A + BF)\ x(t)$$

size of matrices are:

A	=	n X n	Plant matrix.
B	=	n X m	Input matrix.
F	=	m X n	Feedback matrix.

$x(t) \quad = \quad n \times 1$ State variable vector.

$u(t) \quad = \quad m \times 1$ Input variable vector.

The pair (A,B) is changed into (A + BF), B).

The main result of such a transformation is that if (A,B) is controllable then the (A + BF) can be assigned arbitrarily by suitable choice of F. The practical implication is that the new system is controllable. The controllability condition being that the matrix pair (A, B) forms a controllability matrix Γ, whose rank is equal to n, Luenberger (1967,/71/). In other words ,

$$\underset{n \times (n.r)}{\Gamma} = [B, AB, A B, \ldots\ A \quad B] \text{ has rank "n".}$$

A2.1.2 Lemma 1.

For any state feedback F: $\mathcal{X} \to \mathcal{U}$

$$\langle A + BF \mid \mathcal{B} \rangle = \langle A \mid \mathcal{B} \rangle$$

In particular, if (A, B) is controllable, so is (A + BF, B).

Proof:

$$\mathcal{B} + (A + BF)\mathcal{R} = \mathcal{B} + A\mathcal{R}$$

for all $\mathcal{R} \subset \mathcal{X}$ and $F: \mathcal{X} \to \mathcal{U}$

Writing $\hat{A} = A + BF$, we then have

$$\begin{aligned} \langle A + BF/\mathcal{B} \rangle &= \mathcal{B} + \hat{A}\mathcal{B} + \ldots\ \hat{A}^{n-1}\mathcal{B} \\ &= \mathcal{B} + \hat{A}(\mathcal{B} + \hat{A}(\ldots(\mathcal{B} + \hat{A}\mathcal{B}))\ldots) \\ &= \mathcal{B} + A + \ldots + A^{n-1}\mathcal{B} \\ &= \langle A \mid \mathcal{B} \rangle \end{aligned}$$

A2.1.3 Lemma 2.

Lemma 1 is presently repeated when $d(\mathcal{B}) = 1$

Let $0 \neq b \notin B$, then if (A,B) is controllable, there exists $F : \mathcal{X} \to \mathcal{U}$

such that (A + BF, b) is controllable.

Proof:

let $b_1 = b$

and $n_1 = d(\langle A | \mathcal{B}_1 \rangle)$.

Put $x_1 = b_1$ and

$$x_j = A\, x_{j-1} + b_1 \qquad (j = 2, \ldots n_1)$$

then $x_j\,(j \in n_1)$ are a basis for $\langle A | \mathcal{B}_1 \rangle$.

If $n_1 < n$ choose $b_2 \in \mathcal{B}$ such that $b_2 \notin \langle A | b_1 \rangle$, such a b_2 exists by controllability.

Let n_2 be the dimension of $\langle A | \mathcal{B}_2 \rangle$ mod $\langle A | \mathcal{B}_1 \rangle$, i.e. largest integer such that the vectors

$$x_1, \ldots x_{n(1)}, b\,, Ab_2, \ldots \qquad A^{n_2 - 1} b_2$$

are independent; and define

$$x_{n(1)+i} = A\, x_{n(1)+i-1} + b_2\,, \qquad i \in n_2$$

Then $\{x_1, \ldots\ldots, x_{n(1)+n(2)}\}$ is a basis for $\langle A | \mathcal{B}_1 + \mathcal{B}_2 \rangle$.

So continuing, it is possible to obtain eventually independent

$$x_1, \ldots\ldots \qquad x_n$$

and $x_{i+1} = A\, x_i + \tilde{b}_i \qquad i \in n - 1$

where $\tilde{b}_i \in \mathcal{B}$

Choose F, such that

$$BF\, x_i = \hat{b}_i \qquad i \in n$$

where $\hat{b}_n \in \mathcal{B}$ is arbitrary.

Since $\hat{b}_i = B\, u_i$ for suitable $u_i \in \mathcal{U}$, and the x_i are independent, F certainly exists.

Then $\qquad (A + BF)\, x_i = x_{i+1}\,, \quad i \in n-1$

so that $\qquad x_i = (A + BF)^{i-1}\, b\,, \quad i \in n$

and therefore $\qquad \mathcal{X} = \langle A + BF | \mathcal{B} \rangle$.

A2.2 Theorem

The pair (A,B) is controllable if and only if, for every symmetric set Λ of n complex numbers, there exists a map $F : \mathcal{X} \to \mathcal{U}$ such that $\sigma(A + BF) = \Lambda$.

Proof:

(ONLY IF)

First suppose $d(\mathcal{B}) = 1$, $B = b$. It is shown in Appendix 2.1.3 that there is a basis for $\mathcal{X}$ in which A, b have the standard canonical matrices, then A has the characteristic polynomial

$$\lambda^n - (a_1 + a_2\lambda + \dots + a_n \lambda^{n-1})$$

let $\Lambda = \{\lambda_1, \dots, \lambda_n\}$ and

write $(\lambda - \lambda_1)\dots(\lambda - \lambda_n) = \lambda^n - (\hat{a}_1 + \hat{a}_2\lambda + \dots \hat{a}_n\lambda^{n-1})$

On the assumption that (A,b) is in standard canonical form, let f' be the row vector

$$f' = (\hat{a}_1 - a_1, \dots, \hat{a}_n - a_n)$$

then it is clear that the matrix $A + bf'$ is again of canonical form, with a_i replaced by $\hat{a}_i (i \in n)$. This completes the proof when $d(\mathcal{B}) = 1$.

For the general case choose, by Lemma 2 (Appendix 2.1.3), any vector $b = Bu \in \mathcal{B}$ and a map $F_1 \mathcal{X} : \to \mathcal{U}$ such that $(A + BF_1, b)$ is controllable. Regard b as a map $\mathbb{R} \to \mathcal{X}$.

It has been shown of the existence of

$f' : \mathcal{X} \to \mathbb{R}$ such that

$\sigma(A + BF + bf') = \Lambda$.

then $F = F_1 + u f'$.

is a map with the property required.

(IF)

Let $\lambda_i (i \in n)$ be real and distinct, with $\lambda_i \notin \sigma(A)(i \in n)$

Choose F so that $\sigma(A + BF) = \{\lambda_1, \ldots, \lambda_n\}$

Let $x_i \in \mathcal{X} (i \in n)$ be the corresponding eigenvectors:

that is

$$(A + BF)\, x_i \quad = \quad \lambda_i x_i \qquad\qquad i \in n$$

so that

$$x_i = (\lambda_i 1 - A)^{-1} BF\, x_i \quad i \in n$$

now by $\Pi(\lambda)\,(\lambda I - A)^{-1} = \sum_{r=1}^{n} \pi^{(r)}(\lambda) A^{r-1}$

$$(\lambda I - A)^{-1} = \sum_{j=1}^{n} \rho_j(\lambda) A^{i-1}$$

for suitable rational functions $\rho_j(\lambda)$, defined in $\mathbb{C} - \sigma A$.

So $\quad x_i = \sum_{j=1}^{n} \rho_j(\lambda_i) A^{i-1} BF x_i \quad \in \langle A|B\rangle, \; i \in n$

Since the x_i span $\mathcal{X}$, $\langle A|B\rangle = \mathcal{X}$ as claimed.

The result just proved is sometimes called the "pole assignment " theorem, in reference to the fact that eigenvalues of (A + BF) are the poles of the closed system transfer matrix

$$(s1 - A - BF)^{-1} B$$

One direct implication of the above results is that the equivalence between controllablity and eigenvalue assignment by state feedback is established. This has been described and developed in greater depth in Wonham (1978,/78/). Another related approach adopted by Rosenbrock (1970,/50/), Dickinson (1974,/77/) has been to prove that the controllability indices limit the ability to alter the "closed loop" dynamics by such feedback.

Appendix A2.3

Control Canonical Forms.

Let $B = b \neq 0$, that is $\mathcal{B} = \mathrm{Span}\{b\} = \mathfrak{b}$ for some $b \in \mathcal{X}$

The corresponding system equation is

$$\dot{x} = A x + b u$$

where u(.) is scalar valued, i.e. the system has a single control input.

Suppose (A,b) is controllable.

Since $\langle A|b\rangle = \mathcal{X}$ (controllable subspace A b), it follows that the vectors

$\{b, Ab, \ldots A^{n-1} b\}$ form a basis for $\mathcal{X}$, thus A is cyclic and b is a generator.

Let the minimal polynomial (m.p.) of A be

$$\alpha(\lambda) = \lambda^n - (a_1 + a_2\lambda + \ldots + a_n\lambda^{n-1})$$

Introducing the auxiliary polynomials $\alpha(\lambda)$ defined

as
$$\alpha^0(\lambda) := \alpha(\lambda)$$
$$\alpha^1(\lambda) := \lambda^{n-1} - (a_2 + a_3 + \ldots a_n\lambda^{n-2})$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$\alpha^{n-1}(\lambda) := \lambda - a_n$$
$$\alpha^n(\lambda) := 1$$

The corresponding basis

$$e_i = \alpha^i(A)\, b \qquad i \in n$$

then $b = e_n$, and the matrices of A & b are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & & \\ 0 & 0 & 1 & 0 & & \\ 0 & 0 & 0 & 1 & & \\ & \vdots & & & & \\ & \vdots & & & & \\ a & a & a & & & a \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \\ \\ 1 \end{bmatrix}$$

which is the standard canonical (matrix) pair (A,b).

The actual control canonical forms used in this thesis are the ones developed by Brunovsky (1966,/67/). The Prepelita(1971,/68/) algorithm provides the computational approach to obtain such control canonical forms.

Appendix 3.

Synthesis of Feedback Matrix.

In this appendix, it is described how the feedback matrix F is synthesised. In so doing, it is also demonstrated how the structure of the synthesised matrices lead to easy identification of individual sub-systems for individual control.

From the transformed system equations:

$$\bar{x}(k+1) = \bar{A}\,\bar{x}(k) + \bar{B}\,\bar{u}(k) + \bar{E}\,d(k) \qquad \text{---- A3.1}$$

$$\bar{u}(k) = \bar{F}\,\bar{x}(k) \qquad \text{---- A3.2}$$

Then

$$\bar{x}(k+1) = (\bar{A} + \bar{B}\bar{F})\,\bar{x}(k) + \bar{E}\,d(k) \qquad \text{---- A3.3}$$

$$\bar{x}(k+1) = \bar{G}\;x(k) + \bar{E}\,d(k) \qquad \text{---- A3.4}$$

$$\bar{G} = \bar{A} + \bar{B}\bar{F} \qquad \text{---- A3.5}$$

plant matrix governing the closed loop system.

size of matrices;

m = No of inputs in systems.

$n = m \times 3$

$\bar{A} = n \times n$

$\bar{B} = n \times m$

$\bar{E} = n \times (m+1)$

$\bar{F} = m \times n$

$\bar{G} = n \times m$

$x = n \times 1$

$u = m \times 1$

$d = (m + 1) \times 1$

Now the companion matrices $\bar{A}$ and $\bar{B}$ are of the following structures:

$$\bar{A} = \mathrm{diag}(\bar{A}_{k_1}, \quad \bar{A}_{k_2}, \quad \bar{A}_{k_3}, \ldots \bar{A}_{k_m})$$

$$\bar{B} = \mathrm{diag}(\bar{b}_{k_1}, \quad \bar{b}_{k_2}, \quad \bar{b}_{k_3}, \quad \bar{b}_{k_m})$$

where k_1, k_2, ...k_m are control Kronecker invariants, Kalman (1972,/75/), uniquely derived by the pair $\bar{A}$ (m X m) and $\bar{B}$ (m X 1)

.

$\bar{A}_{k_i}$ (m x n) and $\bar{b}_{k_i}$(m x 1) are themselves given as :

$$\bar{A}_{k_i} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ X & X & X \end{bmatrix} \qquad \bar{b}_{k_i} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Submatrix A_{k_i} has entries of 1 in the upper diagonal, and the other non-zero entries are in the third row, marked X.

For the particular case of Chapter 2, when m= 4, $\bar{A}$ and $\bar{B}$ are given as:

$$\bar{A} \qquad\qquad\qquad\qquad \bar{B}$$

$$\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array}
\begin{array}{c} \begin{array}{cccccccccccc} 1&2&3&4&5&6&7&8&9&10&11&12 \end{array} \\
\left[\begin{array}{cccccccccccc}
0&1&0&&&&&&&&&\\
0&0&1&&&&&&\bigcirc&&&\\
0&-1&2&&&&&&&&&\\
&&&0&1&0&&&&&&\\
&&&0&0&1&&&&&&\\
&&&0&-1&2&&&&&&\\
&&&&&&0&1&0&&&\\
&&&\bigcirc&&&0&0&1&&&\\
&&&&&&0&-1&2&&&\\
&&&&&&&&&0&1&0\\
&&&&&&&&&0&0&1\\
&&&&&&&&&0&-1&2
\end{array}\right] \end{array}
\quad
\begin{array}{c} \begin{array}{cccc} 1&2&3&4 \end{array} \\
\left[\begin{array}{cccc}
&&\bigcirc&\\
&&&\\
1&0&0&0\\
&&\bigcirc&\\
&&&\\
0&1&0&0\\
&&\bigcirc&\\
&&&\\
0&0&1&0\\
&&\bigcirc&\\
&&&\\
0&0&0&1
\end{array}\right] \end{array}$$

Therefore from a knowledge of the structured form, it is possible to design a matrix $\bar{G}$ to govern the closed loop response, according to the required assigned eigenvalues. Thus if $\bar{G}$ is written as :

$$\bar{G} = \begin{bmatrix} 0 & 1 & 0 & & & & & & & & & \\ 0 & 0 & 1 & & & & & & & & & \\ X & X & X & & & & & & & & & \\ & & & 0 & 1 & 0 & & & & & & \\ & & & 0 & 0 & 1 & & & & & & \\ & & & X & X & X & & & & & & \\ & & & & & & 0 & 1 & 0 & & & \\ & & & & & & 0 & 0 & 1 & & & \\ & & & & & & X & X & X & & & \\ & & & & & & & & & 0 & 1 & 0 \\ & & & & & & & & & 0 & 0 & 1 \\ & & & & & & & & & X & X & X \end{bmatrix}$$

It can be shown that the entries in the third row of $\bar{G}$ are actually determined by the eigenvalues for that particular submatrix.(Appendix 3.2).

An examination of matrix $\bar{G} - \bar{A}$ and $\bar{B}\bar{F}$ shows that the non-zero entries are in the k_i^{th} row. Moreover each of these rows are identical in each matrix since from equation A3.5, $\bar{G} - \bar{A} = \bar{B}\bar{F}$

Therefore assigning the correct sets of eigenvalues to the governing submatrix $\bar{G}_{k_i}$ (i = 1 to m), is actually determining the entries in the k_ith row (i = 1 to m) equivalent to k_ith row of $\bar{B}\bar{F}$, i.e. each row of $\bar{F}_i$ is synthesised from G_{k_i} .

Each u_i(i = 1 to m) is itself obtained from the i - th row of the feedback matrix $\bar{F}$. The net result is that assigning the required eigenvalues to submatrix G_{k_i} effectively synthesises the control policy u_i , i.e. it is possible control directly individual responses.

These eigenvalues are chosen to be within the range of zero to unity so as to achieve asymptotic stability. The Program for this purpose is given in Appendix 3.3, where feedback matrices are synthesised from zero to unity with invrements of 0.025. These feedback matrices are then stored in appropriate libraries on magnetic discs, where they can be accessed randomly during subsequent simulation runs.

APPENDIX 3.2

To show that matrix $\bar{G}_{k_i}$ (3 X 3) of the following form:

$$G_{k_i} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix}$$

has its values α_1, α_2 and α_3 uniquely determined by eigen values to be assigned.

Proof:

$$\text{Det} \; (\bar{G}_{k_i} - \lambda I) = -\lambda^3 + \alpha_3\lambda^2 + \alpha_2\lambda + \alpha_1$$

$$= 0$$

which is also $\prod_{i=1}^{n} (\lambda_i - \lambda)$

where λ_i, $i = 1, \ldots m$ are the eigen values.

The general results being:

$$\text{Det}(A_n - \lambda I) = (-)^n \{ \lambda^n - \sum_{i=0}^{n-1} \alpha_{i+1} \lambda^i \}$$

The coefficients being uniquely defined by the eigenvalues

.

APPENDIX 3.3: PROGRAM FOR SYNTHESIS OF FEEDBACK MATRICES

```
100 INIT
110 REM
120 REM PROGRAM "SYNT/FED"       26-OCT-81
130 REM
140 O1=32
150 REM PROGRAM FOR PRE SYNTHESIS OF CONTROL POLICIES
160 REM FOR COMBINATION SYSTEM
170 REM ****************************************
180 M=6
190 N=M*3
200 DIM N1(M),K4(M)
210 N1(1)=3
220 N1(2)=3
230 N1(3)=3
240 N1(4)=3
250 N1(5)=3
260 N1(6)=3
270 K4(1)=3
280 K4(2)=6
290 K4(3)=9
300 K4(4)=12
310 K4(5)=15
320 K4(6)=18
330 REM
340 K=1
350 DIM A0(N,N),F1(40*M,N),C9(N,N)
360 REM INPUT MATRICES A AND C  (A0 AND C9)
370 L$="@DISX/A0"
380 REM GOSUB 2100
390 OPEN L$;1,"R",Z$
400 INPUT £1:A0
410 CLOSE 1
420 L$="@DISX/C9"
430 REM GOSUB 2100
440 OPEN L$;1,"R",Z$
450 INPUT £1:C9
460 CLOSE 1
470 FOR C2=1 TO M
480 FOR L=1 TO 40
490 DELETE E1,C1
500 DIM E1(N1(C2)),C1(N1(C2))
510 E1=(L-1)*-0.025
520 C1=0
530 DIM P2(N),M1(N)
540 P2(1)=1
550 L9=0
560 M1(1)=1
570 L9=L9+1
580 L1=L9+1
590 IF M1(L9)=N1(C2) THEN 640
600 M1(L1)=M1(L9)+1
610 P2(L1)=P2(L9)*E1(M1(L9))
620 C1(L9)=C1(L9)+P2(L1)
630 GO TO 570
640 C1(L9)=C1(L9)+P2(L9)*E1(N1(C2))
650 M1(L9)=M1(L9)+1
660 IF M1(L9)(=N1(C2) THEN 580
670 IF L9=1 THEN 700
```

```
680 L9=L9-1
690 GO TO 650
700 REM INVERTING THE  ORDER OF INDICES  **********************
710 DELETE A8
720 DIM A8(N)
730 A8=0
740 FOR I=1 TO N1(C2)
750 IF C2=1 THEN 790
760 V2=K4(C2-1)
770 A8(V2+I)=-1*C1(N1(C2)-(I-1))-A0(K4(C2),V2+I)
780 GO TO 800
790 A8(I)=-1*C1(N1(C2)-(I-1))-A0(K4(C2),I)
800 NEXT I
810 FOR I=1 TO N
820 T9=0
830 FOR J=1 TO N
840 T9=T9+A8(J)*C9(J,I)
850 NEXT J
860 F1((C2-1)*40+L,I)=T9
870 NEXT I
880 NEXT L
890 NEXT C2
900 END
910 REM
920 REM CREATE AND SAVE FEEDBACK MATRICES
930 REM
940 L$="@DIX/F2"
950 REM GOSUB 2100
960 CREATE L$,"UB";1900,10
970 OPEN L$;1,"F",Z$
980 FOR J=1 TO 40*M
990 FOR I=1 TO N
1000 L5=(J-1)*N+I
1010 WRITE &1,L5;F1(J,I)
1020 NEXT I
1030 NEXT J
[illegible]
```

APPENDIX 4 : SUBROUTINE "@DIS/DYN"

```
4000 REM
4005 REM              DYNAMIC SUBROUTINE   "@DIS/DYN"
4010 REM              TO CALCULATE x(k+1)=Ax(k)+Bu(k)+Ed(k)
4015 REM              DATE  26-OCT-81
4020 REM
4025 IF I1(K)<>1 THEN 4060
4030 DELETE T3,T4,T5,T9
4035 X=N(K)
4040 REM                                   CALCULATING   Ax(k)
4045 DIM T3(X,1),T4(X,1),T5(X,1)
4050 X8=X7
4055 T3=A2 MPY X8
4060 IF L>2 THEN 4105
4065 FOR J=1 TO N(K)
4070 T9=0
4075 FOR I=1 TO M(K)+1
4080 T9=T9+Q2(J,I)*Z3(M(1)+I)
4085 NEXT I
4090 T5(J,1)=T9
4095 NEXT J
4100 GO TO 4160
4105 REM                                   CALCULATING Ed(k)
4110 FOR J=1 TO N(K)
4115 T9=0
4120 FOR I=1 TO M(K)
4125 IF U9(I,L-1)<=0 THEN 4140
4130 IF I1(K)=1 THEN 4135
4135 T9=T9+Q2(J,I)*S9(I,1)
4140 NEXT I
4145 REM T9=T9+Q2(J,I)*S9(I,1)
4150 T5(J,1)=T9+Q2(J,I)*S9(I,1)
4155 NEXT J
4160 REM *********
4165 T4=0
4170 V7=M(1)*(K-1)
4175 FOR I=1 TO M(K)
4180 REM V8=I+V7
4185 T4(K4(I),1)=U9(I,L-1)
4190 NEXT I
4195 X7=T3+T4
4200 X7=X7+T5
4205 GOSUB 4225
4210 I1(K)=0
4215 RETURN
4220 REM ********
4225 REM SUB ROUTINE FOR RECONVERTING X-BAR BACK TO X***********
4230 REM                                   CALCULATING x(k) = Cx(k)
4235 T4=C2 MPY X7
4240 FOR J=1 TO N(K)
4245 X9(J,L)=T4(J,1)
4250 NEXT J
4255 RETURN
4260 REM
```

APPENDIX 5 : SUBROUTINE FOR GRAPHICS OUTPUT

```
6000 REM          "@DIS/DRAW"      28-OCT-81
6005 REM          SUBROUTINE FOR DRAWING OUT THE RESPONSES
6010 INPUT T$
6015 PAGE
6025 REM FINDING THE LARGEST L7 VALUE
6030 DELETE Z3
6035 Z3=L7(1,1)
6040 REM           DETERMINATION OF SIZE OF WINDOW
6045 FOR I=2 TO M(K)
6050 IF Z3=>L7(I,1) THEN 6060
6055 Z3=L7(I,1)
6060 NEXT I
6065 Z3=1.1*Z3
6070 Z5=100
6075 02=32
6080 Z2=Z1
6085 K=1
6090 PRINT @02,17:0.9,1.26
6095 REM  CHOOSE LINE TYPE E.G. FULL LINE, DASH LINE, DOTTED LINE
6100 PRINT "I_LINE TYPE ? 1  OR  2  OR  3 OR 4 ";
6105 INPUT Y9
6110 FOR Z=1 TO M(K)
6115 PRINT @1,17:1,1.4
6120 REM ****   --------------    PRINT  U9, CAPACITY RATE
6125 GOSUB 6400
6130 VIEWPORT 0+I,70+I,80+J,95+J
6135 WINDOW -2,Z2,-15,Z3*1.65
6140 REM
6145 MOVE @02:0.5,Z3*1.25
6150 PRINT @02:"** INPUT RATE, STAGE ";Z
6155 AXIS @02:2,Z5,0,0
6160 GOSUB 6285
6165 GOSUB 6340
6170 MOVE @02:0,0
6175 RMOVE @02:1,0
6180 FOR I=1 TO Z2-1
6185 DRAW @02:I,U9(Z,I+1)
6190 GOSUB 6470
6195 NEXT I
6200 REM********* ---------------------PRINT X9(INVENTORY LEVEL)
6205 GOSUB 6400
6210 VIEWPORT 0+I,70+I,64+J,79+J
6215 WINDOW -2,Z2,-1.3*Z3,0.35*Z3
6220 REM GO TO 6495
6225 AXIS @02:2,Z5,0,0
6230 GOSUB 6420
6235 MOVE @02:12.5,-0.7*Z3
6240 PRINT @02:"**INV FLUCTUATION";
6245 GOSUB 6340
6250 MOVE @02:0,0
6255 FOR I=1 TO Z2-1
6260 GOSUB 6470
6265 DRAW @02:I,X9(Z+M(K),I+1)
6270 NEXT I
6275 NEXT Z
6280 RETURN
6285 REM
6290 FOR I=0 TO Z3*1.2 STEP Z5
```

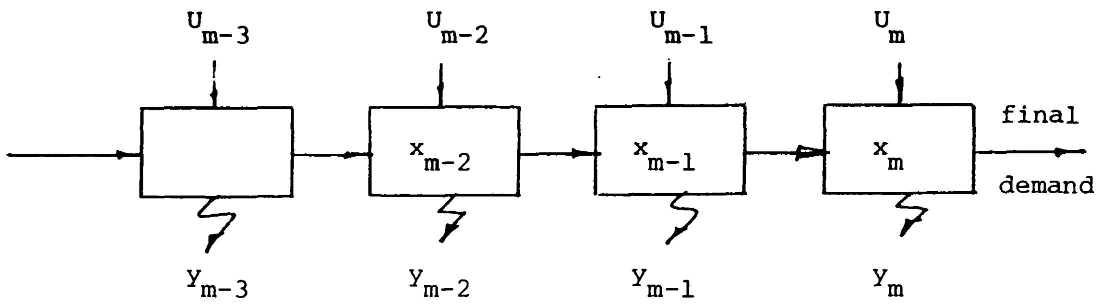
```
6295 IF Z<>1 THEN 6310
6300 MOVE @02:0,I
6305 GO TO 6315
6310 MOVE @02:0,I-2*(I<>0)
6315 PRINT @02:"H_H_H_H_H_";I;
6320 NEXT I
6325 MOVE @02:-1,1.05*Z3
6330 PRINT @02:"  ";
6335 RETURN
6340 REM************
6345 FOR I=0 TO Z2 STEP 2
6350 MOVE @02:I,0
6355 PRINT @02:"H_J_";I;
6360 NEXT I
6365 RETURN
6370 REM *********************
6375 FOR I=0 TO -Z3 STEP -Z5*2
6380 MOVE @02:0,I-2*(I<>0)
6385 PRINT @02:"H_H_H_H_H_";I;
6390 NEXT I
6395 RETURN
6400 REM
6405 I=73*(Z=2 OR Z=4 OR Z=6)
6410 J=-32*(Z=3 OR Z=4)-64*(Z=5 OR Z=6)
6415 RETURN
6420 REM
6425 FOR I=0 TO 0.2*Z3 STEP Z5
6430 MOVE @02:0,I-2*(I<>0)
6435 PRINT @02:"H_H_H_H_";I;
6440 NEXT I
6445 FOR I=0 TO -1*Z3 STEP -Z5
6450 MOVE @02:0,I-2*(I<>0)
6455 PRINT @02:"H_H_H_H_H_";I;
6460 NEXT I
6465 RETURN
6470 REM
6475 GOSUB Y9 OF 6485,6495,6520,6555
6480 RETURN
6485 RDRAW @01:1,0
6490 RETURN
6495 REM
6500 RDRAW @01:0.3,0
6505 RMOVE @01:0.4,0
6510 RDRAW @01:0.3,0
6515 RETURN
6520 REM
6525 RDRAW @01:0.2,0
6530 RMOVE @01:0.2,0
6535 RDRAW @01:0.2,0
6540 RMOVE @01:0.2,0
6545 RDRAW @01:0.2,0
6550 RETURN
6555 FOR O=1 TO 5
6560 RDRAW @01:0.1,0
6565 RMOVE @01:0.1,0
6570 NEXT O
6575 RETURN
```

APPENDIX 6.

CALCULATION OF DISCRETE REJECT VALUES.

In industrial practice, the efficiencies of the production stages are usually known on a historical statistical basis. In this appendix, it is shown how the computation of the discrete reject units is performed given the average efficiencies or conversely the reject rates.

A 4-stage serially linked system as shown in the following figure is used for illustration.



f = Final demand.

r = Reject rate at the stage i, that is inherent of the particular production stage.

u = Input of resource at stage i.

x = Production rate at stage i.

The following equations can therefore be derived:

$$f = x_m(1 - r_m)$$

$$x_m = f / (1 - r_m)$$

$$x_{m-1} = x_m / (1 - r_{m-1}) = f / (1 - r_m)(1 - r_{m-1}) .$$

$$x_{m-2} = x_{m-1} / (1 - r_{m-2}) = f / (1 - r_m)(1 - r_{m-1})(1 - r_{m-2})$$

and so on.

Let the discrete reject values be y at stage i.

$$y_m = x_m \cdot r_m = f.r_m / (1 - r_m)$$

$$y_{m-1} = x_{m-1} \cdot r_{m-1} = f.r_{m-1} / (1 - r_m)(1 - r_{m-1})$$

$$y_{m-2} = x_{m-2} \cdot r_{m-2} = f.r. / (1 - r_m)(1 - r_{m-1})(1 - r_{m-2})$$

or generally the equation is :

$$y_i = f.r_i / \{\prod_{n=1}^{m} (1 - r_n).\}$$

Thus for Chapter 2, section 2.3, where m = 4,

r_1 , r_2 , r_3 and r_4 are 12.5%, 12.5%, 10% and 10% respectively,

and f = 250 units/time period,

y_1 = 50 reject units.

y_2 = 44 reject units.

y_3 = 31 reject units.

y_4 = 28 reject units.